THE PRINCIPLES OF REASONING

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AN INTRODUCTION TO LOGIC AND SCIENTIFIC METHOD

BY

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TO
MY TEACHERS
IN PHILOSOPHY

PREFACE

Within recent years a number of highly significant contributions have been made to logical science. The extant elementary textbooks, however, though in many respects admirable, were written ten to thirty years ago, and such revisions as have been made have not been thoroughgoing enough to take this new literature into account. The teacher of logic is thus confronted with the anomalous necessity of having to use books which are too obsolete to prepare the student for advanced work in this field. My general aim has been to give a fresh treatment of the subject, utilizing, and, as far as possible, bringing within the reach of the beginning student, this valuable new material. For the most part, however, I have ignored the new symbolic logic, since an adequate presentation of it demands a separate course and a separate textbook. The material I have in mind is the rather extensive body of non-symbolic logical literature, which has been accumulating for several years.

In my judgment the central tendency in all this literature is a new theory of the nature of inference. The best exposition of this theory is to be found in the logical writings of the late Dr. Bernard Bosanquet (died 1923). Writing about this great logician's large two-volume *Logic* and his *Implication and Linear Inference,* Dr. John H. Muirhead has recently said: "Taking them together what I think may be claimed for them is that they embody a view of the nature of inference bound sooner or later to revolutionize the entire science of logic" (*Mind,* October, 1923). Using the technical phrases *implicative system* and *inferential whole* as short descriptions of this theory of inference, I

have attempted to rewrite the traditional logic so as to make it clear that the operative unit in all intellectual processes is the implicative system. And in the chapter on the nature of inference I have suggested that Bosanquet's theory of inference is essentially the same as the late Professor Royce's theory of order. But as a matter of fact this theory of inference is really independent of any particular type of metaphysics, and more and more it is being adopted by logicians of other schools than that of the idealistic tradition to which both Bosanquet and Royce belonged.

Some of the material has been presented in a semi-controversial manner in order to provoke discussion in the classroom, as well as individual thought, and to make the student realize that there are some live problems that are awaiting solution in logical science. The charge of sterility so often hurled against logic can only be met by putting the student up against some of the vital modern issues. Even though this may result in making him think that the subject is in an uncertain state, surely this is better than giving him the impression that it is all cut and dried. Too many courses fail to stimulate reflection and discussion. Surely logic should not fail to do this. I have made an honest effort to choose thought-provoking material and to present it in a thought-provoking manner.

In addition to this general aim of utilizing new material in such a way as to make the subject really live, I have had three special purposes in view in writing this book. They are rooted in my conception of what an elementary course in logic can reasonably be expected to give to the student.

In the first place it should lay some sort of a foundation for further work in philosophy, as well as for advanced work in epistemology and logic. In my judgment elementary logic can be made an excellent beginning course for other philosophical subjects. Throughout this book I have introduced material which I thought would orient the

student in the general field of philosophy. For instance, the concluding section on "Recent Tendencies in Logical Theory" is especially intended for this purpose, although it falls strictly within the province of logic.

No other subject has contributed more terms to the general vocabulary which cultured people use than logic. And they are, for the most part, terms which can only be understood by learning them in their context. For this reason alone, if for no other, the attitude of some teachers toward the traditional logic is greatly to be deplored. Throughout I have tried to present intelligible and interesting definitions and explanations of logical doctrines and terms which have become part and parcel of our *educated common sense.* It is for this reason that I have given so much attention to Aristotelian logic. However, I hope I have succeeded in giving it a fresh treatment in accordance with the general aim of utilizing recent discussions.

Then, too, an elementary course in logic should give the student a fair general knowledge of the various methods which are being so fruitfully employed in scientific research nowadays. My treatment of inductive logic is dominated by this consideration, and teachers will find it different from that of any other elementary text. While retaining the valuable content of the traditional inductive logic, especially as presented by John Stuart Mill, I have added considerable new material. Although I have not hesitated to emphasize inductive theory, my primary aim has been to present the whole subject as an *applied logic* or *methodology.*

In fact, throughout the book the more difficult and theoretical have been kept separate from the easier and more practical discussions, for the convenience of the general reader as well as in order that teachers may omit such theoretical portions as are beyond the reach of freshmen and sophomores. For relatively immature students it might

be well to reverse the order of presentation and begin every topic with concrete exercises, illustrations, and discussions, taking up the more theoretical parts later. Thus in Part I some may find it better to begin with definition and division, and then pass to the immediate inferences, omitting the paragraph on the nature of judgment. This may be followed by a discussion of the syllogism and its rules, omitting the chapter on the nature of inference, and then with the hypothetical syllogism, including the dilemma. This will give the class a sufficient background for a consideration of the more theoretical paragraphs and chapters which have been omitted. Similarly in Part II it might be well to begin with Mill's methods, then to take up briefly the general nature of induction, following with a discussion of the methods of probability and explanation. In classes containing few mature students it may be found advisable to omit the controversial parts of the concluding section on "Recent Tendencies in Logical Theory." Since it is easier for a teacher to omit material than it is to add what has been omitted by the author, I have thought best to include some theoretical discussions which are really beyond the powers of the average freshman or sophomore, who has had no previous work in philosophy. Part II is likely to be more interesting to the student than Part I, and hence some may prefer to begin with it.

In preparing this textbook I find myself under obligation to so many that I refrain from specifically mentioning any particular persons. However, in all places where I have used material from other writers, I have tried to give credit. The selected Bibliography in the Appendix includes all of the works from which I have quoted. I hope that it may prove valuable to teachers and to those who wish to do further reading in this field. Although I have added some carefully selected exercises, my own experience leads me to believe that students should be required to exemplify

the various logical operations from their own life and reading.

In conclusion, I venture to suggest that it is time for the pendulum to swing back toward a revival of interest in traditional logic. Undoubtedly every logician owes it to himself and to his profession to be enough of a reformer of logic to keep his science abreast of the best thought in other fields of knowledge. In this sense every teacher of logic should be a reformer. But too many self-styled reformers of logic are making the mistake of eviscerating the content of the elementary course in logic. In some institutions the whole of traditional logic is covered in a few recitations, while in others it is entirely omitted. Eventually teachers of elementary logic will discover the necessity of reviving traditional logic, for it will always remain the only door open to the beginner by which he can enter into possession of the priceless treasures of this age-old, yet perennially interesting, throne room of the temple of knowledge. Hence teachers who have the ultimate welfare of logic at heart will be guided by the watchword "the revival of logic" as much as by that of "the reform of logic."

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OXFORD, OHIO

PREFACE TO THE THIRD EDITION

In this edition the exercises have been changed to make this textbook suitable for use in classes filled with ex-service personnel, all of whom are especially interested in recent additions to human knowledge, and significant changes in our social order. It is hoped that teachers will be able to make effective use of these exercises in freshening up the course in logic and in stimulating student discussions.

I have also revised the statement of the relation of the implicative system to Gestalt Psychology in Chapter I. And I have revised the section on Probability to include diagrams of the Normal Curve of Probability, and samples of isotypes, which are now so important in disseminating statistical information. But the most important addition is the concluding chapter entitled "The Logical Significance of Radar Pips" in which I have subjected some tenets of logical positivism to critical examination. The bibliography has also been brought down to date.

For many helpful suggestions I am greatly indebted to teachers who have used the book and to my publishers. And I owe special gratitude to my son-in-law and daughter, Dr. and Mrs. Charles R. Clark, and to Mr. George Watson, graduate student.

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THE PRINCIPLES OF REASONING

Aristotle, who founded the study, offers no definition of logic, and, at the outset of our inquiry, it is safest to adopt a broad description of the subject and not to attempt a more precise or detailed definition. Some writers, for example, J. S. Mill, make inference the main subject of logic. Yet in Mill's work and in the ordinary textbooks there are topics other than inference, and those not treated merely in relation to inference. Hence we find wider definitions. Whately, for instance, calls logic 'the science and the art of reasoning,' and Mill, at one place, 'the science of the operations of the understanding which are subservient to the estimation of evidence.'

Whatever their differences, however, all accounts seem to imply that *thought* as such is the special object of logical inquiry and that thought owes its existence and its difference from the sciences to some sort of distinction between thought and *things*.

—J. COOK WILSON, in *Statement and Inference*, Vol. I, p. 32.

In the broadest sense, any thinking that ends in a conclusion is logical—whether the conclusion reached be justified or fallacious; that is, the term *logical* covers both the logically good and the illogical or the logically bad. In its narrowest sense, the term logical refers only to what is demonstrated to follow necessarily from premises that are definite in meaning and that are either self-evidently true, or that have been previously proved to be true. . . . Logical, however, is used in a third sense, which is at once more vital and more practical; to denote, namely, the systematic care, negative and positive, taken to safeguard reflection so that it may yield only the best results under the given conditions.

—JOHN DEWEY, in *How We Think*, p. 56.

Logic is a normative science. It deals, namely, with the norms whereby sound or correct thinking is distinguished from incorrect thinking. It consists of two parts,—a general part, called Formal Logic, which defines the universal or formal normative principles to which all thinking must conform, and a special and very extended part called Applied Logic, or Methodology, which deals with the norms of thought in their application to the methods used in various special sciences.

—JOSIAH ROYCE in Ruge's *Encyclopedia of the Philosophical Sciences*, Vol. I, *Logic*, p. 67.

THE PRINCIPLES OF REASONING

CHAPTER I

THE GENERAL NATURE OF LOGIC

Definition of Logic

The term *logic* is the antique and classic name of that branch of human knowledge which studies the intellectual processes involved in reaching any definite truth. The word is derived from the adjective of the Greek word *logos* (λόγος), which means either a thought or the word used in expressing a thought. But in combination with other words *logos* has been given the English form *ology*, which means science. Thus, *biology* is the science of life, *psychology* is the science of the mind, and *geology* is the science of the earth. Following out this idea, the technical term *epistemology* has been formed, and nowadays it is frequently used as synonymous with the term logic. It is a combination of *ology* with the Greek word *episteme* (ἐπιστήμη), which means knowledge. Hence, logic may be defined as the science of knowledge, or more exactly, as the science of the intellectual processes operative in the acquisition and in the creation of knowledge. Some of these processes are those used in proving one statement true by relating it to others or to another, already known or assumed to be true. The study of these constitutes a branch of logic designated by such various names as *formal logic, deductive logic, the logic of proof, Aristotelian logic* and *traditional logic*. In

this textbook, Part I is an exposition of this branch of logic.

On the other hand, accompanying the growth of experimental science there was a development of methods of reasoning which brought to light the actual connections embodied in concrete facts and formulated them as laws of nature. Since the days of Francis Bacon, who vigorously advocated turning away from assumed deductive principles to the actual facts of nature, that branch of logic which deals with such processes of reasoning has been known as *inductive logic.* But to contrast it with the logic of proof this branch has occasionally been called *the logic of discovery.* To-day, however, it is more commonly referred to as *scientific method,* or simply as *methodology.* This branch of logic is sketched in Part II of this textbook, under the title of *Induction or Scientific Methodology.*[1]

It is well to recognize at the outset that the logician is somewhat at a disadvantage, in comparison with the natural scientist, in indicating to the beginning student the precise subject matter of his science. For whereas natural scientists deal with visible or tangible objects, the logician deals with non-visible and with non-tangible, or in general with non-sensuous objects. This fact that the material of logic is of such a nature that it cannot be sensed makes it advisable to explain somewhat in detail just what that material is. This can best be done by approaching the matter from two different angles.

1. *The Subjective Approach.* No student can have reached the stage in his educational development where he is admitted to a course in logic who has not done some thinking for himself, and who has not accumulated a certain amount of knowledge which is peculiarly his own. Your thinking, and the knowledge in which it results, is the subject matter

[1] For a fuller discussion of the development of deductive and of inductive logic, see Ch. XXV.

of logic. When you prove the theorem in geometry that the opposite angles of two intersecting straight lines are equal, you have to think or reason. That and all other thinking or reasoning is the subject matter of logic. Thinking is always of some object, but the object which is thought about is not the subject matter of logic, save only as it enters into and becomes a part of the thinking. When the matter is approached subjectively or from the angle of the individual thinker, *the science of logic is the systematic display or exhibition of the intellectual processes involved in one's own thinking.*

2. *The Objective Approach.* Every student must have some conception of what is meant by the whole body of knowledge, since he is attempting to make some of it his own. All the various sciences form a system of knowledge which is independent of the individual thinker. For while it cannot be said to be your knowledge or mine, that it is real is none the less indubitable. Now, from the objective point of view, *knowledge as knowledge* is the subject matter of logic. *Logic attempts to discover the structure of the whole system of knowledge.*

To make this clearer let us use an illustration. The little one-celled animal known as an amœba is, as a form of animal life, a part of the subject matter of zoölogy. But the amœba may also be considered as a chemical entity, since it is composed of chemical elements. And as such it is a part of the subject matter of chemistry. From still another point of view, an amœba can be regarded as a known object or as constituting a part of the system of knowledge. As such the amœba is a part of the subject matter of logic. And so of every other known object, let it be what it may. To study logic is to study the structure of knowledge, and to study the structure of knowledge is to study the structure of all objects as parts of the whole called knowledge, whereas other sciences deal with the same

objects as parts of some other whole. Logic is interested in the amœba, and in the whole called physical life, and in all other objects of whatsoever kind, as parts of the whole called knowledge.

The Relation of Logic to Psychology

If we define the subject matter of logic as thinking—your thinking, my thinking, any thinking—we are in danger of identifying logic and psychology. But it can be shown by two considerations that these two sciences are really quite distinct: (1) Logic deals only with that part of mental content known as thinking, whereas psychology deals with the content of mind in its entirety. Psychology is the science of all mental phenomena, normal and abnormal, and even of bodily behavior, whereas logic has for its subject matter normal thinking only. (2) The point of view of psychology is entirely different from that of logic. Psychology is only interested in thinking as one among many other features of finite experience. Logic, on the contrary, is interested in thinking, not as a bare existent mental fact, but as an intellectual activity which culminates or fructifies in valid knowledge. One deals with thinking simply as an existential fact, while the other looks away from the existence of thinking to its goal, purpose and meaning.

Treating logic as psychology, dealing with thought processes in the purely descriptive manner of psychology, is a mistake too often made by logicians. Such a procedure always results in making logic a mere adjunct of psychology, and logicians who do this have justly been called "psychological logicians." Psychologizing logic or other sciences is the name the great German logicians, Husserl and Meinong, give to this too frequent habit of introducing the psychological viewpoint into a subject matter in which it has no rightful place. Logic is logic and psychology is

psychology, and nothing is gained by reducing either of these sciences to the other.[2]

The Relation of Logic to Language

If we define the subject matter of logic as that system of knowledge which forms the content of all the sciences, we are in danger of identifying the subject matter of logic with language. This is true because all knowledge, from the human point of view, is necessarily embodied in and transmitted by language. While it is an undeniable fact that "thought could make no progress without embodying itself in language," it is nevertheless a fatal mistake to treat language as the subject matter of logic. The fact that language and knowledge are really entirely distinct has frequently been pointed out, but nowhere has it been made clearer than in John Locke's famous tirade against mere verbosity: "For he that shall well consider the errors and obscurity, the mistakes and confusion, that are spread in the world by an ill use of words, will find some reason to doubt whether language, as it has been employed, has contributed more to the improvement or hindrance of knowledge amongst mankind. How many are there, that, when they would think on things, fix their thoughts only on words! . . . Who can wonder that such thoughts and reasonings end in nothing but obscurity and mistake, without any clear judgment or knowledge?"[3] Knowledge, not language, is the subject matter of logic. Language is only the sensible embodiment of thought. What logic is concerned with is the naked body that is clothed in language. The logician is not studying words, sentences and parts of speech, but the meaning behind words and sentences. That

[2] Bernard Bosanquet gives a difficult but sound discussion of the relation of logic and psychology, with special reference to Husserl, in Ch. VII of his *Implication and Linear Inference.*

[3] John Locke, *Essay Concerning the Human Understanding,* Bk. III, Ch. XI.

is to say, the logician is studying the parts of knowledge and not the forms of language.

The Irreducible Unit of Logical Science

The suggestion that logic studies *the parts of knowledge* raises the interesting question: What is the least conceivable bit of knowledge or the minimum possible amount of knowledge? If we can tell what this least part of knowledge is, we shall have what may well be called the irreducible unit of logical science. An analogy may make the matter clearer. All the physical sciences reduce the objects dealt with to the least possible unit. The unit of biological science is the cell. The unit of physics is the atom, or, according to a more recent theory, the electron. The unit of chemistry is now generally agreed to be the ion. Now is there anything in the science of logic which can be regarded as the irreducible unit of thought or knowledge? Is there a logical electron, or ion, or cell?

The great English logician, Stanley Jevons, taught that the logical idea is the irreducible unit of thought, an idea being a meaning and not a mere psychical image. "An idea or concept is not an image, though it may make use of images. It is a habit of judging with reference to a certain identity."[4] Let us use again the illustration of the amœba. Think *amœba,* and you have an example of an idea in the meaning which your thinking gives you. But you may well answer that you cannot think just amœba by itself, without thinking of some quality which an amœba has, or of some relation of an amœba to something else. And that fact suggests the unanswerable criticism of the view that an isolated idea is the irreducible unit of knowledge. For ideas are not found "lying apart as words lie on a page." Ideas are themselves very complex systems

[4] Bernard Bosanquet, *Logic*, Vol. I, p. 41 (1st ed.).

of thought, as is shown by the statement given above: "An idea is a *habit of judging* with reference to a certain identity." For if every idea is a habit of judging, it necessarily involves numerous other ideas. The attempt to think of an idea by itself, in isolation from all other ideas, turns out to be futile. *One cannot avoid going beyond the idea to something that is more complete.* The irreducible unit of knowledge must be something which is capable of standing alone as knowledge, and this no single idea can do.

Well, then, perhaps judgment, or a single judgment or proposition, is the real unit of knowledge. This is implied in the statement that an idea is a habit of judging. Thus the judgment that an amœba is a one-celled animal is something more than a mere idea. It is several ideas forming together a single meaning. When we know what an amœba is, the idea amœba expands into the judgment: "The amœba is a one-celled animal." Now the fact that every idea expands into a judgment when it is reflected upon, or reveals an act or habit of judging at its source when it is analyzed, has been held to prove that the judgment is the irreducible unit of logical science.

Yet it must be admitted that a single isolated judgment is as far from being knowledge as a single isolated idea. A bare assertion certainly is not knowledge any more than a bare idea. Neither a single judgment nor an idea can hold within its four corners enough of the content of thought to be knowledge. Behind the judgment, as well as behind the idea, is a system of ideas or judgments. You do not get knowledge until you understand the underlying *ground* of the judgment. For Aristotle, logic was the science of proof or evidence, and it should be the same for us to-day. But if logic is the science of proof, the irreducible unit of knowledge must embody in itself the essentials of its proof. To refer again to our illustration: "The amœba is a one-celled animal because, when examined under

a high-powered microscope, nothing can be discerned but a nucleus embedded in a mass of protoplasm.'' Here the judgment is expanded into an inference. You have traced the connections in that tiny little system of reality which is known as the amœba, and in so doing you have constituted a tiny thought-system which is knowledge. Nothing less than such a thought system is knowledge. This system is behind every idea or judgment which refers to the identity in question, for example, the amœba. The mere isolated idea and the single judgment are not knowledge. *Only a thought system, built up by tracing the actual connections within that portion of the real world with which you are dealing, is entitled to the name of knowledge.* It is convenient to name such a thought system *an inferential whole* or *an implicative system.* In recent years logicians of every school have come to recognize that the inferential whole or the implicative system is the minimum possible amount of knowledge and therewith the irreducible unit of logical science.[5]

This conception of an inferential whole, or an implica-

[5] Although the view that the implicative system is the unit of knowledge or the minimum possible amount of knowledge has never been definitely stated, it is implicit in and can be read between the lines of many recent writings on logic. Professor Mellone correctly states the view of the idealistic logicians, Bradley and Bosanquet, when he writes: "The point of view adopted in modern logic is, that in the formation of ideas, in judgment, in reasoning, we have not three separate processes but a development or expansion of one and the same process" (*Introductory Textbook of Logic,* p. 12). This is equivalent to saying that the real underlying unit of knowledge is the implicative system. Indeed, Leibniz appears to have recognized this, as is indicated by his statement: "The least thought of which we are conscious involves variety in its object" (*Monadology* 16). The exceedingly important articles entitled *The Principles of Logic,* by W. Windelband and Josiah Royce, in volume one of Ruge's *Encyclopedia of the Philosophical Sciences,* are best understood from this point of view, as are also the *Essays in Experimental Logic* of John Dewey, and the *Introduction to Reflective Thinking,* of the Columbia Associates in Philosophy.

tive system, is employed again and again throughout this textbook, and the student will do well to grasp it here at the outset. Since exact knowledge consists in a careful organization and correlation of concrete facts into a definite system, it follows that the functional unit of logic must be such a system, howsoever tiny it may be. Each idea which refers to, and each judgment made about such a system, are best conceived as various aspects of it. Moreover, the attempt to fathom the meaning of any particular idea will carry the investigator deeper and deeper into the implicative system which is in question. The following quotation from *John O'London's Weekly* is an excellent illustration of a thinker being carried back into an implicative system in answering a correspondent's question as to the precise meaning of two related words.

"The distinction between 'egoism' and 'egotism' is this: 'Egoism' describes a condition of mind, 'egotism' the habit of expressing that condition. Hence Meredith's title *The Egoist*, not *The Egotist*. The egoist loves himself excessively and sees everything colored by his own thoughts, interests and moods. The egotist is also an egoist, but the word is applied to him in respect of his spoken or written manifestation of this trait of character. The two words, however, have been used more or less indiscriminately by many writers."

This is a good example of an inferential whole. The facts are the thoughts, interests and moods of a human being. All of these varied facts are synthesized and ordered by the ideas, "condition of mind," and "habit of expressing that condition." Egoism is simply one way of referring to this system, and egotism is another. Hence the writer was correct in adding the further comment: "Strictly speaking, there is no difference in meaning between these two words, but there is a difference in their application." For each refers to the same implicative system, but each

expresses a different aspect of that system. Now it goes without saying that various other ideas and judgments could be, and many times actually are, used in reference to this same implicative system.

The Relation of the Implicative System to the Pattern Concept of the Gestalt Psychology

The doctrine of the implicative system is essentially the same as the conception of a configurational pattern so fruitfully employed in the new *Gestalt* psychology to explain the reasoning processes. Professor Wolfgang Köhler devised three experiments to determine the extent to which apes can reason: (1) *The Integration Experiment,* sets a problem that was solved by the animals of how to get a banana by using a long stick to draw it near enough to the cage to bring it within reach. (2) *The Analysis Experiment,* was one in which it was necessary for the animals to get the banana close to the cage from behind an intervening obstacle, so placed that they had to push the banana further away and work it free from the obstacle before pulling it toward the cage. The apes also solved this problem, although with considerably more difficulty. (3) *The Synthesis Experiment,* was one in which the animals had to fabricate a tool in order to get the banana. Two sticks, which could be joined like a jointed fishing pole were put into the cage, and the banana was placed where it could only be reached with the long spliced pole. Only one ape was able to solve this problem.

These experiments indicate that simple perceptual situations can be solved by apes, and that the solution depends upon the animal comprehending how the pattern is to be completed before he performs the action to complete it. If this is true, apes do have a very rudimentary knowledge, and it is in the form of an implicative system. But as Professor Köhler rightly says: "Even behavior with insight

which indicates intelligence must not receive an interpretation too highly intellectualistic."[6]

From the standpoint of logic, a far more important study is the late Professor Max Wertheimer's *Productive Thinking*, a posthumously published work containing the results of several years of psychological experimenting on the higher intellectual processes. Seven chapters are devoted to separate problems. These problems are: (1) The Area of the Parallelogram; (2) The Problem of the Vertical Angles; (3) Gauss's formula: $S_n = (N+1)\frac{n}{2}$; (4) Playing a Game; (5) Finding the Sum of the Angles of a Polygon; (6) Galileo's Discovery of the Law of Falling Bodies; and (7) Einstein's Thinking that Led to the Theory of Relativity.

Dr. Wertheimer, who was one of the originators of *Gestalt* psychology, concludes that thinking consists "in envisaging" and "in realizing structural features and structural requirements." When we proceed in accordance with and when we are determined in our thinking by "these structural requirements," we succeed in improving the factual situation structurally by eliminating and otherwise disposing of "gaps, trouble-regions, disturbances, superficialities, etc."[7] In this manner we are able to delve more penetratingly into the order system involved in the particular situation with which we are confronted, and we can discover by rigorous thinking which aspects of this system are irrelevant and peripheral, and which are fundamental. Such reasoning leads to genuine insight into structural relations, and gradually replaces piecemeal truth with complete

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[6] See Wolfgang Köhler's *The Mentality of Apes* for a detailed account of the experiments. Consult also his *Gestalt Psychology* and *The Place of Value in a World of Facts*, and Kurt Koffka's *The Growth of the Mind*.

[7] Max Wertheimer, *Productive Thinking* (New York and London, Harper & Brothers, 1946), pp. xii, 224. Edited by S. E. Asch, W. Köhler and C. W. Mayer. For quotations see Table III, p. 190.

understanding of the objective pattern of relations, the implicative system involved in any concrete example of right thinking. (For a fuller discussion see Chapter XXIX.)

Logic as an Art

It is customary in an introductory textbook on logic to discuss the question as to whether logic is an art. In considering this question we cannot do better than to quote from H. W. B. Joseph's *An Introduction to Logic*, pp. 9*f.*:

> It is sometimes held that logic is rather an art than a science, or at any rate that it is an art as well. In considering this question, we must remember that there are two senses of the word art. We may say that a man understands the art of navigation when he is skillful in handling a ship, though he may be unable to explain the principles which he follows: or we may say that he understands it, when he is familiar with the principles of navigation, as a piece of book-work, though he may never have navigated a ship. Thus an art may either mean practical skill in doing a thing or theoretical knowledge of the way in which it is best done. In the latter sense art presupposes science; the rules of navigation are based upon a knowledge of astronomical, mechanical, meteorological and physical laws, and presupposes much knowledge of mathematical and other sciences. It is in this sense that logic is called an art; and hence it is clear that if there is an art of logic, there must first be a science, for the study of the nature of sound thinking must precede the giving of instructions for thinking soundly.... The logician's business is not to give rules by following which others or he himself may alter their thought about things, their geometry or chemistry or biology.... His business is to become conscious of the nature of the thinking carried on in those sciences. Logic studies the way in which we already think about things.

In recent years many criticisms have been made of logic on the ground that it has no practical value. Some of the values a student may hope to get from a study of this field of knowledge are briefly summarized in the Preface, but further consideration of these criticisms is postponed to Chapter XXVIII, since it seems best to discuss them after the student has attained some understanding of the subject.

PART I

TRADITIONAL ARISTOTELIAN LOGIC

SECTION I

NAMING AND THE DOCTRINE OF TERMS

CHAPTER II

THE ACT OF NAMING AND THE KINDS OF TERMS

The Intellectual Act of Naming

Looked at from the point of view of an individual thinker the act of naming is the first step in knowledge. At the very beginning of modern logic Thomas Hobbes rightly said: "Reason is attained by industry, first in apt imposing of names."

Naming is that intellectual act, or that thinking process, which takes a particular word or sign and uses it to refer to some identifiable object or quality or relation. As was pointed out in the last chapter, this process depends upon there being in nature a grouping of things, qualities and relations which can be recognized. The act of naming begins with a given content and elaborates it into an intelligible thought system or inferential whole. Every name depends upon or, more accurately expressed, *is*, in its essential meaning, such an inferential totality. Thus in this very first act of knowledge the inferential whole is operative. This is what Windelband meant when he wrote: "Every idea, at every stage in its development, has a multiplicity of moments, however limited, which, while distinct from one another, are yet connected with one another by some sort of relation into a unity."[1] Thus a single idea or name is already a complex whole, and is best described as an inferential whole or implicative system.

The objective element involved in an act of naming, that is to say, the content of the name, may be a definite object or thing, for example, *chair, desk, tree*. From the point of

[1] Wilhelm Windelband in Arnold Ruge's *Encyclopedia of the Philosophical Sciences*, Vol. I, p. 21.

view of logic every such object is an implicative system. Or the content of a name may be a quality or attribute, thought of as distinct from any object and regarded as possessing independent reality like unto a concrete thing or object, for example, *redness*. Or the content of a name may be a *relation*, thought of apart from the group to which it belongs, for example, *greater than, giver of, brother of,* or *resemblance*.[2]

Now to give a name to an implicative system, whatever it may be, radio, television, entropy, relativity or what not, involves knowing it as a distinct unified whole. Thus even in this first stage of knowledge, thought is at work distinguishing definite qualities and relations, ordering them into a system and identifying this system with some linguistic element, for all others as well as for the individual doing the thinking. So through future time that word means that definite implicative system. Bosanquet has rightly emphasized the overwhelming importance for scientific knowledge of this intellectual process of naming. He writes:

> The value of this first step is only to be estimated by experience, now necessarily exceptional, of the attempt to attain knowledge without it. . . . We find such an instance in our attempts to deal with any new region of knowledge of which we have not mastered the rudiments and in which we have neither books nor teachers to guide us. . . . It is probable that we think too little rather than too much of *naming* as a first step in knowledge. To give names which endure is with few exceptions the prerogative of genius. The number of terms which we inherit from Plato and Aristotle is among the most striking proofs of the immense advance which they won for the human intellect.[3]

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[2] The various types of nameable objects themselves have names, and they form together the categories of thought. Here all that we are concerned with is the fact that the content of any name is always a complex system. See the next chapter, on the categories and predicables.

[3] Bernard Bosanquet, *Logic,* Vol. I, pp. 8 *f.* (1st ed.).

In laboratory work in physics, chemistry, biology or psychology, the significance of the act of naming is brought home to the student, in that he is set to perform essentially the same experiments and observations which were employed by the scientist who first discovered and named the object under consideration. The student never forgets what he can be made to discover for himself, *because he goes through the intellectual act of naming in making the discovery.* Our example of the amœba is a case in point. Once a student has found for himself in the field of a miscroscope this tiny animal he knows once and for all the reality to which the name amœba refers. And why? Because he has gone through the intellectual activity which that observer went through who first isolated and named the amœba. Without any question the value of the act of naming underlies the value of the laboratory method in education. To put oneself through the process of renaming an object is really to think the thoughts of master minds after them.

The Name Equivalent to the Logical Term

Names, viewed as distinct from the act of naming, may be defined as words or groups of words used to designate a definite meaning or inferential whole. But the term word must not be taken to restrict names to verbal language, since various symbols are frequently used as significant names. Not only are mathematical symbols examples, but so also are the brands which mark animals, and the signs used by hoboes to mark houses. When a name is first given, the particular word or sign or symbol used is regarded as a mere convention. This was the view of Aristotle. But we must be on our guard against overstating this arbitrariness of names. As William James has well said: "Names are arbitrary, but once understood they must be kept to. We mustn't now call Abel 'Cain' or Cain

'Abel.' If we do, we ungear ourselves from the whole book of Genesis, and from all its connections with the universe of speech and fact down to the present time. We throw ourselves out of whatever truth that entire system of speech and fact may embody."[4]

In a single assertion or *judgment* many different names may occur. But there is a sense in which all of these may be reduced to two. They are the subject and the predicate and they are called terms. A logical term is either the subject or the predicate of a logical proposition or judgment. Hence, the simplest way of conceiving of a judgment is to think of it as consisting of two logical terms united by a copula, that is, some form of the verb *to be,* preferably *is* or *are.* Take for example the proposition: "The rose is a beautiful flower used for decorative purposes." We have here the logical term *the rose* as subject and the logical term *a beautiful flower used for decorative purposes* as the predicate term. But each of these terms is really a single name, even though more than one name is used in each. The word *the* is a name and so is the word *rose,* yet the words taken together form only one name. Consequently the logical term is really equivalent to a name. And many times a single name is a logical term, as in all judgments where the subject or predicate is a single word, for example, "*Vermilion is poisonous.*" Names, then, become logical terms when used in judgments so that, for the purposes of logic, it is better to use the word term to designate them. In a judgment we have two separate thought systems, brought together into a single system by virtue of the fact that each is really in the nature of things only a fragment of the larger system. The judgment or proposition is in turn only a complex name of a single implicative system.

[4] William James, *Pragmatism,* p. 214. For an excellent discussion of symbols see R. M. Eaton, *Symbolism and Truth,* Ch. I.

Kinds of Terms

1. *Concrete and Abstract Terms.* There are two entirely different meanings of the words concrete and abstract. In one sense we mean by concrete what is capable of being perceived by the senses, for example, *an apple, a patch of red.* In this same sense every non-sensuous object is abstract, for example, *thought, goodness, love.*

In the other sense a concrete term is one which includes an object in a definite implicative system and refers to the entire context of that system to which it naturally belongs. Examples of concrete names in this sense are: *this desk, the Washington elm, the man in the moon, the centaur, the absolute, that great Leviathan—the state.* In this same sense an abstract term is one which isolates its object from its natural context. *Sweetness, hardness, dryness, audacity, motion, length, absoluteness* are examples. It should be especially noted, and the examples given are intended to emphasize the fact, that the object referred to by a concrete name may be an imaginary or conceptual object as well as a physical object. So also the quality referred to by an abstract name may be regarded as itself an object. Mellone gives the example, "Unpunctuality is irritating," where unpunctuality is treated as an object. Goodness, regarded as a Platonic form, is another example.

In this second signification, the distinction between abstract and concrete terms is only a relative distinction. Every name refers to some recognizable identity in the actual world of our knowledge. Yet every name is abstract in the sense that what it refers to is, to a certain extent, viewed in separation from the total system to which it belongs. Human knowledge is always fragmentary for the individual thinker. Every name refers to a definite implicative system, but the human mind views it as independent of other implicative systems of which it is really

a member. In the former sense every name is concrete, and in the latter sense every name is abstract.

This explains the so-called *equivocal* or *ambiguous* names of which many logicians make so much. Such names refer to two or more different implicative systems. For example, the word *spade* is the name of a playing card and of an instrument used in digging. Here a confusion between the two implicative systems involved is hardly possible. But an ambiguous abstract word such as *law* or *freedom* must be carefully considered to avoid confusion. For instance, consider the two implicative systems involved in the phrases "law of nature" and "law of the land." Such names are ambiguous until the actual inferential whole intended in the context in which the name is used is understood. Some words are abstract in one context and concrete in another because the underlying implicative system is different or viewed from a different angle in each case. Many discussions in logic, as well as in other subjects, are little more than mere verbal quibbling because the underlying inferential whole is not kept in mind by the disputants. After a long and heated argument a theologian is said to have remarked to his adversary: "Oh, I see, your God is my devil." Ambiguity does not attach to the inferential whole which the name designates, but to the use of words. Consequently it is a matter of language rather than logic.

2. *Singular and General Terms.* A singular name is one which designates one and only one entity. If there is more than one object exactly like the one being named, and the name given is of such a character as to apply to these other similar objects, it is not a singular term. Every singular name is restricted in its meaning to a single instance. Examples are: *the death of Queen Victoria, the center of the material universe, the treaty of Versailles, the Russian Soviet Government*, and all proper names. Even proper names such as John Smith or Tom Brown are

still regarded as singular although there are numerous individuals, human beings, pets, fictitious characters, who have the same name, since every individual is viewed as unique and distinctly different from every other having that name. The only case in which a proper name loses its singular character is when it is used to refer to a class of objects as in the statement: "A Daniel come to judgment." But a general name may also become a singular name. Prefixing *the, this, that* always makes a general term singular, because it limits the meaning to a definite object.

A general or common term is one that may refer equally well to any one of a number of individuals because the individuals are near enough alike that, for practical purposes, it is convenient to ignore their differences. Such names really refer to the whole class of objects having certain definable attributes without specifying any particular object belonging to the class. Examples are: *book, war, submarine, sailor, fair, a long trail, a flying trip to New York.*

3. *Distributive and Collective Terms.* Whenever a term is used in reference to each individual of a class separately, and not to those individuals as making up the whole class, it is said to be a *distributive term.* All general terms are ordinarily so used. They are the names of a number of separate objects, but they may refer to each one of this number separately. Distributive terms are applicable "in the same sense equally to *any one* of an indefinite number of objects which resemble each other in certain qualities" (Jevons). *Policy, planet, rope, automobile* and *university* are all distributive terms.

Collective terms, on the other hand, are those which refer to a group in a sense which is not applicable to the individuals making up the group. A good example is *herd.* The individuals in a herd cannot be referred to as individuals by this term, but must be referred to by some other

term such as *sheep* or *buffalo*. For this reason sheep and buffalo are distributive terms, whereas herd is a collective term. One cannot speak of a single buffalo as a herd. Nevertheless the term herd may be either singular or general in usage, because it may mean a definite herd—*that herd*, or it may mean any herd or any kind of herd, for instance, herds of wild reindeer in northern Canada, herds of goats, herds of cattle. Thus in the sentence: "The dwellers of the deep, in mighty herds, passed by us," the term herd is collective but also general. *Senate* is a collective name used as a singular name when the United States Senate is referred to, but as a general name in the sentence: "All the states have a senate." But note that the collective term, when used as a general term, refers always to the separate groups and not to the separate constituent members of those groups. For instance, in reference to each of the separate state senates, the term senate is distributive, but not in reference to each senator.

Thus, the collective name is logically reducible to a general or a singular name. Many collective names have a distributive aspect and are, therefore, general as well as collective. The word *all* is sometimes confusing because it may mean either all taken together as a group, or each and every member of a group. For example: "All the regiments are ready for action" uses the collective term *regiment* in a distributive sense. But "Madelon is true to all the regiment" uses the collective term regiment in a singular sense, meaning the whole regiment to which Madelon is supposed to belong as a kind of mascot. But if being true to all the regiment is taken to mean true to each and every man, instead of to the regiment as a whole, we have a collective term used as a distributive term in the wrong way, that is, being true to all the regiment should never be taken to mean each and every member of the regiment, but only the regiment as a whole. In every case the actual

underlying implicative system determines the meaning, or, actually is the meaning, and we are always thrown back upon that when we are confronted with the problem of determining whether a term is distributive or collective.

4. *Absolute, Relative and Correlative Terms.* An *absolute term* is the name of any object, quality or relation conceived of as an independent entity. As a matter of fact no object, quality or relation is entirely independent, but it is necessary to treat many objects as absolute for the purposes in hand. Hence, all absolute names are only relatively absolute. Examples are: *the battle of the Marne, Flanders Field, sky, ether, space, time, color and weight.*

Relative terms are names of things viewed as having some definite relation to some other individual or object. Relative names refer either to the relation or relations between such objects, or to the things which stand in that relation. Examples are: *father, cousin, better, west, similarity, equality.*

Correlative terms are names of the two objects or end terms of a dyadic or dual relation. Relations are spoken of as dyadic when they have two terms, triadic when they have three terms, tetradic when they have four terms and polyadic when they have many terms. Ordinarily the names of dyadic relations only are regarded as correlatives, but in the broad sense of the word, correlative may include other relations besides the dyadic. In this case one term is called the *relate* and all the others are said to be the *correlates* of it. For example, if *buyer* be taken as a relate, *seller, commodity,* and *price* would be called correlates of it. Considered together, all four terms would be correlatives. But in the usual logic textbooks the term correlative is restricted to dyadic relations, that is, those having only one relate and one correlate. Examples are: *master and servant, prince and subject, uncle and nephew or niece, instrument and end.*

It is a much disputed question as to whether correlative names are necessarily thought together. Herbert Spencer called the theory that they are necessarily thought together, the *doctrine of correlatives.* Now the fact that each relate and correlate is really a fragment of a system constituted by the relations which hold them together, or since they form an implicative system, it follows that the doctrine of correlatives is true, providing one really thinks through the implications of a given correlative.

It should also be mentioned that the name of the relation itself, as distinct from the terms of the relation, may be taken as an absolute term. For example, the relation of *sovereignty* which underlies the correlatives prince and subject may be so taken.

5. *Positive and Negative Terms.* A *positive term* is usually defined as one referring to the presence of any quality or qualities, whereas a *negative term* is defined as one referring to its absence. Thus John Locke wrote: "We have negative names which stand not directly for positive ideas, but for their absence, such as insipid, silence, nihil, etc., which words denote positive ideas, for example, taste and sound, but with a signification of their absence." Present-day logicians generally agree that names are never purely negative in the sense of referring to the absence of a quality. As Joseph wittily puts it: "The Irishman's receipt for making a gun, to take a hole and pour iron around it, is not more difficult to execute than it would be to frame a term whose meaning consisted simply in the fact that a particular quality was not meant. A term must have some positive meaning in order to be a term at all." [5]

[5] H. W. B. Joseph, *An Introduction to Logic*, p. 41. Royce and Bosanquet take the same position. Royce's detailed setting forth of the meaning of what he calls the not-relation is very suggestive. See his article entitled "Negation" in Hastings' *Encyclopedia of Religion and Ethics* (Scribners). The quotation from Royce below is from this article.

What is the positive basis of negative names? Or, to put it differently, what is the fact, the factual content, the part of the nature of things, referred to by a negative name? Logicians used to answer this question by saying that it is literally everything else in the universe except that which is referred to by the corresponding positive name. For every positive name there is a corresponding negative or opposite, and the two are said to divide the whole of reality or the whole universe between them. To get the exact opposite of a given positive term the best way is to prefix the word *non* or *not* to the positive term in the cases where it is a single word, and to put the word *not* into a relative clause when the term consists of several words. Thus the opposite of black is not, as we usually think, white, but non-black. And the opposite of, "food which is good for an invalid" is, "food which is *not* good for an invalid." The exact opposite of a positive term is its contradictory or true negative. Many supposed negatives are really not opposites but only different in degree. These are called *contraries*. White is the contrary of black because it differs from black only in degree and there are many shades of gray in between. But non-black is the absolute or exact opposite of black. Now what is the actual content of one of these real contradictory or negative terms? Is it all the rest of the universe except what is included in the positive term of which it is the negative? To take a specific example: Is the positive basis of the term non-man *chairs, canoes, lakes, all inorganic objects, all plants, all lower animals*, literally everything except man? Strictly speaking, the only answer to this question is "yes." The positive basis of a negative term is everything except that object. Such negative terms are known as *infinite* negative terms.

Yet even a logician must admit that it sounds like pure unadulterated nonsense to speak of a lake or a canoe as a non-man. This is because the infinite negative term is

rarely, if ever, actually used in practical life. In our constant use of terms in everyday reasoning we tacitly restrict their scope to some definite circle of facts. This comes to expression in the retort, which is frequently made when some one takes exception to what is said: "Yes, but I did not intend to refer to that when I used the words, I was confining myself to so and so." Now this *so-and-so,* to which we tacitly limit ourselves in using terms in ordinary life, is called by logicians *the universe of discourse,*[6] and it has a special value, in that it enables us to explain what is meant by the positive basis of negative names without resorting to the infinite term. For in every usage of a term there is always some fairly definite universe of discourse which is tacitly implied. And this is the positive basis for both the positive and the negative terms used in reference to it. The universe of discourse may vary almost without limit. As Venn says, it may be extended "to embrace the sum total of logical existence," and then the negative term will embrace "the unlimited myriad of entities which people that heterogeneous realm." Or, on the other hand, it may be narrowed to include only a few objects of a similar nature, and then the negative term will have for its positive basis just the ones among those few which are not designated by the positive term. Thus, to use one of Venn's suggestive examples, "We might limit our application of the terms good and non-good to the London cabs with odd numbers." Here the universe of discourse would be London cabs with odd numbers. Those fit for service would be called good and those unfit for service would be called non-good. From this point of view positive and negative terms do not divide the whole of reality between them, but

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[6] This fruitful conception was introduced into logic by the great English logician, De Morgan. John Venn devoted a chapter to its elaboration in his *Symbolic Logic* (Ch. IX), and thereby brought the expression into general logical usage.

only the universe of discourse—*that region of facts which is under consideration in the discussion where the two terms are used.* This makes possible a far more definite interpretation of the meaning of the positive basis of a negative term than can be given with the conception of the infinite term.

An excellent illustration of what is meant by a positive basis for negative names, based on an interpretation of Hamlet's soliloquy, is given by Royce:

> In the opening words of Hamlet's soliloquy, "To be or not to be, that is the question," "not to be" involves a negation of "to be"; both the expressions "to be" and "not to be" refer to possible modes of action. "To be," as Hamlet explains, includes in its meaning "to bear the ills we have"; it names a mode of action which any man who chooses to continue his life decides to adopt; "not to be" involves a course of action, namely, committing suicide, which is treated by Hamlet as the negation of continuing to live.

Thus, in the universe of discourse known as human existence, committing suicide is the positive basis of the negative term "not to be." In like manner, every negative name has a positive basis. It is not easy to state what it is in every case, but nevertheless it is real.

Privative terms are sometimes distinguished from negative terms by the fact that they refer to the absence of a quality which would naturally be present in normal cases, but for some reason does not happen to be present in the case under consideration. The absence, or privation of a positive quality, offers the only explanation for such names. But even privative names have a positive basis over and above this absence of a quality. *Blindness* is an example, and such a name has a very definite positive content. Milton's immortal description of his own blindness is an excellent account of the positive basis of this particular privative term.

...Thus with the year
Seasons return; but not to me returns
Day, or the sweet approach of even or morn,
Or sight of vernal bloom, or summer's rose,
Or flocks, or herds, or human face divine;
But cloud instead, and ever-during dark
Surrounds me....

Thus all negative terms, including the so-called privative negative terms, have a positive basis. They always mean something more than the mere absence of a quality.

Exercise I

1. Make two lists of the following terms, placing all that are concrete sense objects in the first list, and all that are abstract in the second list.

(*a*) the U.S.S. *Missouri*
(*b*) uranium
(*c*) spectroscopy
(*d*) aerial navigation
(*e*) a B-29 bomber
(*f*) lucidity
(*g*) lucite
(*h*) magnetism
(*i*) an atomic bomb
(*j*) negativity

2. Name a universe of discourse, a class, or a system of meaning within which each of the following terms would be logically concrete, and then use the same term in a sentence in which it will be logically abstract with respect to this concrete system.

(*a*) radar communication
(*b*) weather report
(*c*) canyon
(*d*) radioactive element
(*e*) nucleus
(*f*) color inheritance

3. Make two lists of the following terms, placing all general terms in one list and all singular terms in the other list.

(*a*) President
(*b*) Adolf Hitler
(*c*) Admiral
(*d*) judge
(*e*) The United Nations
(*f*) Chief Justice
(*g*) Commander-in-chief
(*h*) General Marshall
(*i*) the Battle of the Bulge
(*j*) World War

4. Restate each of the following collective terms to make it a singular instead of a general or distributive collective term:

(*a*) marine detachment
(*b*) firing squad
(*c*) battleship squadron
(*d*) international court

(*e*) home-coming celebration
(*f*) a cartel in business
(*g*) a conference of statesmen
(*h*) cabinet meeting
(*i*) chaplain corps
(*j*) a board of trustees

5. Using the terms in question 4, make statements to show how each term would need to be changed in order to make it applicable to individual members of the collection. For example, John is corporal in a marine detachment.

6. Correlative terms here listed have one or more missing members to be supplied in each case by the student and to be designated as the relate or relates of these correlates.

(*a*) gunpowder
(*b*) shell
(*c*) lessee
(*d*) automobile salesman
(*e*) pastor
(*f*) patient
(*g*) toothache
(*h*) football

7. Give the contradictory of each of the terms in the lists under questions 1 and 2.

8. Select from any of the above lists four terms, and place each in a definite universe of discourse. Then give the limited negative within the universe of discourse you selected.

9. Make lists of five each of the following kinds of terms: contraries, correlatives, singulars, collectives, absolute terms, contradictories, abstract in the first sense, concrete in the first sense, and general terms.

10. Use the word *all* in a sentence where it is ambiguous. Then use it to mean only *each* and *every*. Then use it to mean only a collective whole.

11. Note that the same term may fall into several of the classes of terms. For example, the term *elephant* is concrete in the first sense, may be either concrete or abstract in the second sense, is general, distributive, absolute, and positive. Put each of the following terms into each of the classes in which it logically belongs, as has just been done with the term *elephant*.

(*a*) Iwo Jima
(*b*) saber-tooth tiger
(*c*) cyclotron
(*d*) photo-electric cell
(*e*) denuded area
(*f*) parent
(*g*) reciprocity
(*h*) basket-ball team

CHAPTER III

THE CATEGORIES AND THE PREDICABLES

What a Category Is

In distinguishing between the various kinds of terms in Chapter II we had to make use of fundamental notions such as *object, quality,* and *relation.* These are certain basic kinds of reality which are dealt with in knowledge. Now the question arises whether they constitute the only kinds of existence there are, so that ultimately every name can be referred back to one of these three. What are the different types of knowable entities? is the question we have to answer in working out a list of *categories.* Hence we may define a category as *the most general kind of existence or reality which any mentionable object can have.* When the thought content of any term whatsoever has been referred back or placed under its proper category, thinking about that particular thought content has reached its culmination.

Or we may regard the categories from the point of view of their function or use in thinking. For we use them in every act of thought. They are the indispensable prerequisite of any thinking whatever. Thoroughgoing analysis always brings to light certain *field ideas* which constitute the background of all thought. Long before an individual becomes conscious of them as facts he is using them in his thinking. That is to say, in every-day life we use them without knowing that we use them, without separating them out and making them definite objects of thought. ***We think with them long before we think about them.*** **Thus, for** example, the conception of *substance and attribute*

enters in some fashion or other into practically every item of human knowledge. "We use the conception of substance and attribute at every step, but we do so unconsciously" (Lossky). Now it is the business of logic to isolate these field ideas called categories. The logician must bring the instruments of thought before the mind so that they can be made objects of study.

How absolutely fundamental categories are, can be made clear by an analogy. In the zoölogical laboratory you study microscopic animals and parts of animals. To enlarge these tiny organisms, so that their activities and structure can be accurately observed, an instrument known as the microscope is indispensable. You are not studying the microscope when you are looking through it at an amœba, but you are studying the amœba and its activities in the drop of water which constitutes the field of vision. You have to disregard the instrument which makes the amœba visible in order to study the behavior of the animal. Yet without that instrument you could not know what microscopic animals are, because you could not see them, you could not observe their unique structure and their interesting habits. Now the categories stand in a similar relation to all objects whatsoever as the microscope stands to these tiny animals. Without field ideas or categories, no one can think about an object. They are the forms or structural beams that run through all reality, as well as through all thought about reality. They give order and shape to whatever you think. Indeed it is literally true that they give "to airy nothing a local habitation and a name." The objects of our thought simply cannot be objects of thought without taking on the form of some definite category. With this statement of what a category is we are ready to return to the question from which we started: *How many categories are there?* Are *substance* and *attribute, quality* and *relation* the only categories or field ideas necessary to thought?

Can everything we think about be reduced to one or the other of these?

Aristotle's List of Categories

Now in answering these questions, it must be admitted that different logicians give different lists of the categories. What one logician regards as a basic category another will subsume under something else. So it is very difficult to give a list which cannot be criticized as containing too many or too few. But suppose we hold that there are other ultimate kinds of existence or being than substances, qualities and relations. What are the other kinds? To answer this question it suits my purpose best to give the list worked out by Aristotle, even though his list has been severely criticised by Immanuel Kant and others.

1. *Substance.* Anything thought of as relatively independent of everything else, and as holding within itself various differences or modifications called attributes, is a substance. All sense objects, that is, animals, plants, manufactured objects like chairs, desks, houses, all chemical elements, etc., are thought of as belonging to the category of substance. Aristotle regarded this as the most fundamental category. Certain it is that we cannot think without it. The expression reality, which has been used often in this textbook, may be regarded as synonymous with the category of substance in its widest extent. In this sense the category of reality may be regarded as the one absolutely ultimate category. Now substances are of two different kinds. I have mentioned concrete sense objects. But we commonly treat all names of *kinds* or *types* of objects as belonging to the category of substance. For example, the superdreadnaught, U. S. S. *Texas,* belongs to the category of substance, but so also does the kind name *ship.* Endless disputes are indulged in by logicians over the nature of substance, both as to individual substances

and as to kinds or universals or types. But this is really a question of systematic philosophy or metaphysics, or at least of advanced rather than a question of elementary logic. It is sufficient here to remark that there are about every object aspects which do not belong to the category of substance, either in the sense of individual substance or in the sense of kind or universal substance. All that can be taken away from a substance is regarded as being something other than substance.

2. *Quantity*. The measurable aspect of substances is called quantity. This category is exceedingly fundamental. We cannot get along in practical life without using the category of quantity repeatedly. Thus *large* and *heavy*, and all degrees of these and all degrees of anything whatever are thought with the category of quantity. What is a pound? We seem to be at a standstill when we say that it is a certain definite quantity.

3. *Quality*. Much of an object reduces on analysis to sensible qualities. Synonyms for quality are *characteristic*, *attribute*, or *mark*. Every substance may be said to consist of a number of different aspects or attributes, and these are called its qualities. *Hardness, softness, solidity, blueness, agility, swiftness* are examples of qualities. When we are asked to tell what these are we cannot go further than to say that they are just qualities. They are not substances and they are not quantities, although they may admit of degree and hence may be measurable or have a quantitative aspect. But in general quantity and quality are regarded as opposite kinds of existence.

4. *Relation*. A relation is a character which an object has as a member of a collection or whole. In modern logic this category is universally recognized as of primary importance, and some logicians would reduce most of Aristotle's categories to this one. Our discussion of relative terms in Chapter II was really a discussion of this category,

and the student is referred to that section for examples of relations.

5, 6. *Time and Place* (*Space*). Here again we have difficulty in defining what is meant. To paraphrase St. Augustine's words: "We know what space and time are, when no one asks us what they are, but if any one has the audacity to ask us to say what they are, we do not know what they are." That is because they are really ultimate ideas in terms of which other things are defined, but which cannot themselves be satisfactorily defined. But modern logicians are at one in reducing space and time to relations, the relation of extension is space and the relation of duration is time. Thus, space and time are really the same as spatial and temporal relations. But even though we admit that this does not enlighten us as to what space and time in themselves are, we all know that many things cannot be defined except as being either a space or a time. Thus, yesterday belongs to the category of time, and the north pole is a place. Again, *when* falls under the category of time but *where* and *there* fall under the category of place.

7. *Situation.* When we speak of an object being in a certain situation we really mean something different from being in a certain place, although situation obviously involves being in a place. A perfectly round sphere, for example, is always in the same situation no matter what place it is in, whereas a man may be regarded as either erect or reclining while occupying exactly the same place. For this reason Aristotle distinguished the category of situation from that of place. What is *horizontal* and what is *perpendicular?* You can only answer by saying that they are situations.

8. *State or Condition.* The manner in which a thing capable of change exists at a certain time, as distinct from the manner in which it exists at another time, is called its

state or condition. Thus *ice* is a state or condition of water, *war* and *peace* are states or conditions of nations, and *fever* is a state or condition of the body. State or condition Aristotle regarded as an ultimate form of being. The only way you can define fever, when you go as far as you can go, is to say that it is a state or condition.

9, 10. *Activity and Passivity.* A great many names have to do with motion. *Cutting, fighting, running* are all reducible to the category of activity. We can hardly define motion except as an ultimate category of our thinking and of existing things. Many logicians reduce the category of motion or activity to the relation of duration or time. There is no question but what motion and the stream of time are closely related, but are they really identical? *Passivity* refers to an object as a recipient of activity. Hence both these Aristotelian categories can be reduced to *motion* or *change.* Examples of the category of passivity are *being horsewhipped,* or *being tarred and feathered.*

We realize the great significance of this list of categories when we take a simple sentence such as: "Bill's bay horse is running at breakneck speed down Main Street, hitched between the shafts of a new buggy," and refer each of the names in it to its proper category. For this shows how much we use the Aristotelian categories in every-day life without being conscious of the fact. Thus, the word *Bill* belongs to the category of substance in the individual sense because it is the name of a man, but the possessive indicates a relation between Bill and the horse, namely, the relation of ownership. *Bay* is a quality, *horse* is an individual substance, *running* is an activity, *breakneck speed* is a state or condition of the horse, *down* is a relation, *Main Street* is a place, *hitched* is passivity or condition, and *between* is a relation. The shafts are individual substances, but *of* defines their relation to the buggy, which is another individual substance. *New* is a quality and the word *is*

has in it a temporal meaning, which places it under the category of time. We never utter a statement of any kind in which we do not employ a certain number of these field ideas or categories.

The Predicables

In addition to the categories, there is another set of terms which are of special importance in traditional logic. They are known as the *predicables*. The best approach to an understanding of them is to take any individual substance and analyze it into its various qualities and relations. For instance, a *desk* may be analyzed as follows: table, especially adapted for convenience in writing, having a smooth top surface, made of wood, five years old, in a certain office, between two windows, etc., etc. Each of these separate names, including the name *desk* itself, may be referred back to one or another of the Aristotelian categories. But over and above the relation of each of these to its category, *there is a relation between each of them and the individual substance, desk.* It is this later relationship which is expressed by the predicables. They are the names of the different possible types of relations between the various parts into which an individual substance can be analyzed and that substance itself. Or we may also say that they are the names of *the most general relations which a predicate* (a quality, relation, etc.) *can have to its subject* (the individual substance in question). The names of the predicables are: *genus, species, definition, property* and *accident.* For species we sometimes use the expression *differentiæ,* meaning the differentiæ of the species. Now the genus and the differentiæ of the species constitute together the *definition* of a thing, and these three are to be distinguished from property and accident by the fact that they are *the essential attributes* of the thing in question, whereas the property and accident may both be conceived

as absent without the object ceasing to be. Strictly speaking, an individual substance is its definition. For genus means the qualities which the substance has in common with other substances of the same general type, and differentiæ of the species means the qualities which distinguish it from other objects of the general class, and these two sets of characteristics are the definition, and constitute together the individual substance.

Our example of the desk will make this clearer. The genus is *table* and every desk must have the essential attributes of a table in order to be a desk. The differentiæ are: *Especially adapted for convenience in writing* and *having a smooth top,* and these characteristics are just as essential to a desk as are the attributes of a table. Now a property is a characteristic which every object of the species in question actually has but which is not absolutely essential to it. Hence, before the use of other materials came into vogue, *made of wood* could have been regarded as a property of a desk. An accident, on the other hand, is a non-essential attribute which is sometimes present and sometimes absent. Thus, *five years old* is an accident of desk because a desk may be a hundred years old or be just newly made and still be a desk. Now, in the same way, we can say of every predicate whatsoever, meaning by predicate the various components of the individual substance in question, that it is either a genus or a differentia of the species or a property or an accident of the subject. This is the meaning of the Aristotelian predicables. To be sure, one may question whether any attribute is really non-essential to an object, and insist that property and accident be made a part of the essential attributes. This would break down the rigid separation which Aristotle makes between essential and non-essential attributes. But in practical life we need to distinguish between the essence of an object and its relatively unimportant features, and

for this purpose the predicables of Aristotle serve very well.

The Relation between Aristotle's Categories and Predicables and the Modern Conception of Class.

We can best understand the relation between the *Aristotelian categories* and the *Aristotelian predicables* by noting that the predicable called the *genus* always becomes a category when the analysis of the object in question is carried to the limit. As was pointed out above, the end of thought is reached when a given object is fully analyzed, and each part is placed in its proper category or categories. This is what is always involved in stating the essential attributes of an object. It follows that the forming of general or universal conceptions, such as are the names of the Aristotelian categories, is one of the most important functions of human thinking, and the ultimate goal of all scientific research.

Modern logicians use the term *class* for the Aristotelian term category.[1] To think is to form classes, and to place individual objects in their proper class or classes. Hence, the relation of a member of a class to the class takes the place of the Aristotelian notion of a category whose nature is embodied in objects which belong to it. It follows that modern logicians treat the genus as a class. By so doing the distinction between categories and predicables is blurred. In fact, the conception of class and of membership in a class takes its place, and this is held to be a decided improvement over the Aristotelian logic.

In answer to this contention it is well to point out that the conception of class is not only highly ambiguous but positively misleading, and for these reasons it is a very poor substitute for the term category. For a subject to

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[1] See *Introduction to Reflective Thinking*, Ch. II, Secs. 3 and 4.

belong to a certain category one thing is essential, namely, that it embody in itself the nature of that category, but for an object to belong to a class any one of three different relations is satisfactory. For by a class is meant some sort of totality of things, and three different meanings of membership in a class satisfy this notion.

In the first place, membership in a class may mean *inclosure,* to be inclosed in a container, just as apples are in a barrel or pigs in a sty. In that case the relation is wholly external. That is to say, the container has no actual inner connection with the things contained in it, because they can be taken out without ceasing to be what they are and without the container ceasing to be what it is. Pour the apples in a pile on the ground and they are still apples, and the barrel is still a barrel. To put it in Aristotle's terminology, the inclosure relation is an accident of the apples. They can be apples without being inclosed in anything. Now if membership in a class means this and nothing more, the real logical relation between an individual object and its category or categories is not grasped at all. By the relation between an object and its category or genus, Aristotle meant something other than mere inclosure.

In the second place, membership in a class may mean forming a part of an *aggregation.* The aggregation or collection in question may be the most homogeneous or the most heterogeneous group of objects conceivable. A pile of apples or bricks is a good illustration of a homogeneous aggregate, and a miscellaneous collection of articles on a bargain counter in a department store may serve to illustrate the heterogeneous type of aggregation. Such a total aggregation would disappear if all the members were taken away from it, so that the relation between a class and its members in this sense is more fundamental than in the case where membership means inclosure. The barrel remains after the apples are taken away, but the pile vanishes into

nothingness when the apples are put back into the barrel. Nevertheless, the real logical relation which Aristotle had in mind in his conception of belonging to a category is missed entirely, when membership in a class is interpreted to mean forming a part of an aggregate.

The third meaning of membership in a class is the real logical meaning. It involves the notion that the member embodies, in some particular manner, the nature of the class in question. Thus, every apple in a barrel belongs to the class of apples as well as to the barrel, and every apple in a pile belongs to the class of apples as well as to the collection which we call a pile. It is the relation between *an apple,* any apple you please, and those numerous characteristics or attributes which we think of as essential to an apple, which is the logical relation Aristotle was trying to grasp. All these characteristics constitute the class *apple.* Any particular apple is what it is, by virtue of the fact that it possesses by nature the general characteristics which constitute the class *apple.*

When we use the conception of membership in a class in this third sense, it is equivalent to the Aristotelian notion of belonging to a category or being in a certain genus. But the word class is so certain to carry with it one of the other meanings that it is a poor substitute for the Aristotelian notion of category. The logical meaning of category is difficult to grasp, but it is at least free from the ambiguity with which the word class is infected. Consequently, the modern substitution of class for category is really a step backward.

Exercise II

1. The following sayings of great philosophers are inscribed on the walls of the Reading Room of the Hoose Library of Philosophy, at the University of Southern California. Place each word in each inscription under its proper Aristotelian category. If any word cannot be placed under an Aristotelian category, try placing it under one of the twelve categories of Kant, listed in

the table under question 3. Or, name a category you think is needed for the word in question and give a reason for your choice.

(*a*) "It is difficult to know thyself, it is easy to advise others." Thales
(*b*) "For the most part the things divine escape us because of our unbelief." Heraclitus
(*c*) "Mind is infinite and self-ruled." Anaxagoras
(*d*) "The fatherland of the wise and good is the whole world." Democritus
(*e*) "Man is the measure of the universe." Protagoras
(*f*) "No evil can befall a good man either here or hereafter." Socrates
(*g*) "The first and best victory is to conquer self." Plato
(*h*) "Dear is Plato, dearer still is truth." Aristotle
(*i*) "Thou hast made us for thyself, and our hearts are restless until they find their rest in thee." Augustine
(*j*) "If you live according to nature you will never be poor." Epicurus
(*k*) "The creation of time is a revelation of the eternal acting of God." Albertus Magnus
(*l*) "To perfect the understanding is nothing else but to understand God." Spinoza
(*m*) "It is God who is the final reason of things." Leibniz
(*n*) "I have been like a boy playing on the seashore while the great ocean of truth lay all undiscovered about me." Newton
(*o*) "Thinking is the action and not the essence of the soul." John Locke
(*p*) "Westward the course of empire takes its way, time's noblest offspring is the last." Berkeley
(*q*) "Mind is supreme and the universe is but the reflected thought of God." Kant
(*r*) "A nation which has no metaphysics is like a temple possessing no holy of holies." Hegel
(*s*) "Great men are those who see that spiritual is stronger than any material force." Emerson

2. In the following passage Immanuel Kant suggests a number of criticisms of Aristotle's list of categories. State each of these criticisms in a single sentence in your own words.

It was an enterprise worthy of an acute thinker like Aristotle to try to discover these fundamental concepts; but as he had no guiding principle he merely picked them up as they occurred to him, and at first gathered up ten of them, which he called *categories* or *predicaments*. Afterwards he thought he had discovered five more of them, which he added under the name of *post-predicaments*. But his table remained imperfect for all that, not to mention that we find in it

some modes of pure sensibility (*quando, ubi, situs,* also *prius, simul*), also an empirical concept (*motus*), none of which can belong to this genealogical register of the understanding. Besides, there are some derivative concepts, counted among the fundamental concepts (*actio, passio*), while some of the latter are entirely wanting. —*Critique of Pure Reason,* translated by Max Müller (The Macmillan Company), p. 67.

3. Following is Kant's own table of categories. Mention at least three differences between Kant's list and that of Aristotle. Do you think that Kant avoided all of the criticisms he made of Aristotle in the above passage? Explain your answer.

TABLE OF CATEGORIES

I. *Of Quantity* 1. Unity 2. Plurality 3. Totality	III. *Of Relation* 7. Of Inherence and Subsistence (*substantia et accidens*) 8. Of Casualty and Dependence (cause and effect) 9. Of Community (reciprocity between the active and the passive)
II. *Of Quality* 4. Reality 5. Negation 6. Limitation	IV. *Of Modality* 10. Possibility, Impossibility 11. Existence, Non-existence 12. Necessity, Contingency

4. Select from the list of sentences under question 1 at least one example of each of the Kantian categories.

5. Jeremy Bentham gave the following definition of a community:

A community is a fictitious body composed of individual persons who are considered as constituting, as it were, its members.

This definition has been criticised by Iverach as follows:

No body of any kind is constituted by the members alone. Any unity has to be looked at from two points of view, and is never the sum of its parts. Regard must be had to the wholeness of the whole as well as to the parts. Society or a community is not a fictitious body, of which individuals are the fictitious parts or members; a people is a unity, an organic whole, and the individuals are so in relation to the whole.

What idea of class dominates Bentham's definition? What idea of class dominates the criticism of Iverach, and his own definition as expressed in the last sentence? Is either of these identical with the implicative system? If so, which and why?

CHAPTER IV

THE INTENSION AND THE EXTENSION OF TERMS

What Intension and Extension Are

We have just discovered an important dual character of the term *apple*. For we have seen that it may mean *apple in general* or, on the other hand, it may mean *any or all particular apples*. Now, this is true of every term whatever. Every term has both of these aspects. It means certain definite qualities or attributes and it also refers to certain objects, or, in the case of a singular term, to one object which has those qualities. When we think of any one or of all of these qualities which constitute anything's essence, without especially thinking of an existing object in which these qualities are embodied, the term in question is used *intensively*. On the other hand, when we refer to an object having those qualities, without any special reference to the qualities themselves but only to the actual concrete object in which they are embodied, we use the term *extensively*. For example, if I say, "an apple is the *fruit of a pyraceous tree*," I use the term apple intensively, because no particular apple is intended but the nature of apple in general. But if I say, "These apples are delicious," or "All the apples in Corso's fruit store are cheap," or, "All the apples raised in Ohio last year have been sold," I use the term extensively, because I am not telling what an apple is, but only that there are certain objects called apples.

This distinction between the specific meaning of a term and the embodiment of that meaning in one or more concrete examples is one of the most important distinctions in logic. It may be made clearer by an analogy. Inventors and artists use models to give body to their ideas before

they actually make the object for public use or exhibition. Thus one sees in Saint Gaudens' studio at Cornish, New Hampshire, the model the great sculptor used when he produced his famous statue of Abraham Lincoln. When an artist finally gets the model so that it fittingly embodies his conception of his subject, he is ready to cast the statue in bronze, and, needless to say, as many different statues can be made as he wishes. The model may be regarded as analogous to the intensive meaning of a term, whereas the various statues that are actually cast are comparable to its extensive meaning. Or to come just a little closer to the real meaning of intension, let us say that the completed model is the first extension of the Saint Gaudens Lincoln, and that the conception of the statue in the artist's mind, from which this completed model was constructed, corresponds to the intension. Then all the replicas of that statue, the one in Lincoln Park in Chicago, the one in front of Bascom Hall at the University of Wisconsin, and any others there may be would form the rest of the extension of that term—the Saint Gaudens Lincoln. In the same way those qualities essential to the constitution of any object whatsoever form together the complete intension, and those objects in the real world which embody those qualities in a particular way form together the complete extension. Each object of the kind in question may be, nay, must be, unique and different from every other. Nevertheless each embodies in itself the characteristics of its kind. The ability to grasp in its fullness this distinction between the intensive and the extensive meaning of a term may be said to be one of the first and most necessary stages in the development of a logical mind.

In order to tie up this discussion of intension and extension with the discussion of definition and division which is to follow, it should be stated that the intension of a logical term is equivalent to its definition. Logical division

is in like manner connected with extension. For by the complete intension of a term we mean *all* the essential attributes, qualities and relations involved in the constitution of *any* object which is properly designated by that particular term. There may actually be just one or an innumerable host of such objects, and extension has reference to this *extent* of the objects. The complete extension means all of the objects having the essential attributes necessary to constitute an object of that kind. Whether the term is used to refer to one, or some, or many, or all of the objects constituting its complete extension it is still said to be used extensively. Similarly, whether the term is used to mean one or many of the essential attributes involved in the constitution of an object, it is still used in intension. But in each case *complete* intension and *complete* extension means all.

A synonym for intension is *connotation*. While some logicians attempt to distinguish the intension from the connotation, the terms are generally regarded as synonymous. And a synonym for extension is denotation. The advantages of these synonyms is that they each have verbal forms. Hence we can say that the term apple *connotes* certain attributes and *denotes* certain objects. Consequently these terms are frequently used instead of intension and extension, and the student must remember that they are equivalent terms.

Alleged Non-Intensive and Non-Extensive Terms

It must be especially emphasized that every term whatsoever has both an intensive and an extensive meaning, both a connotation and a denotation. This has been denied. It has been held that there are non-connotative terms and it has also been held that there are non-extensive terms. Let us begin by considering the alleged non-connotative terms.

John Stuart Mill argued at some length to prove that all proper names are non-connotative. In other words, he held that such names really have no meaning. This seems true when they are taken in abstraction from the object which they name, usually a human being, or an animal, or an important place. For instance, the term *John Jones* appears to have no intension until it is used to refer to some particular person, but when it is once so used its intension is supplied to it by the nature of that person. But there is a sense in which the term John Jones has an intension, even when not used to refer to a definite person. Its intension then is not the qualities of any person, but the qualities of proper names. Thus, the intension of John Jones, when no particular person is meant, would be *a name which is frequently employed to designate a human being.* Even though the intension of a proper name differs when it is used in abstraction from any particular person, nevertheless it has an intension even then. But it is true that the intension of a proper name differs for every person who happens to have it. Thus the intension of John Smith is one thing when the name means Captain John Smith of our colonial history, and quite another thing when it means one of the boys who fought in Belleau Woods. Consequently proper names have a different extension for each individual who is known by them. In other words they have but a single extension because they are really singular names. This is one reason why they have been supposed to have no intension.

On the other hand, names of imaginary or fictitious objects have been denied an extensive meaning. This denial is due to a failure to grasp the full significance of the distinction. Take as an illustration the term *fairy.* Its intension is: imaginary beings or spirits, generally represented as of a diminutive and graceful human form, but capable of assuming any other, and as playing pranks,

frolicsome, kindly, mischievous, or spiteful, on human beings or among themselves. Now, it is urged, that since a fairy is an imaginary being there can be no actual object having these qualities and hence no extension. But the expression *playing pranks among themselves* indicates an extensive meaning. And this extensive meaning would be: all fairies which have even been imagined to exist, or all the fairies mentioned in the story of Cinderella. As Bosanquet has well said:

> Imaginary ideas, the content of absolute fictions, have their extensions in the instances, particulars, or units, or in the aspect of unity which they naturally imply. Chimeras, four-dimensional space, Gulliver's voyage to Lilliput, have all the same complementary aspects of meaning and particularity (intension and extension) that are involved in man, horse, or triangle (Vol. I, p. 48).

The Distribution of Terms

Another very important logical distinction is connected with the distinction between intension and extension, and it will be better understood when we come to apply it later on if it is taken up here. It is the distinction between a distributed and an undistributed term. The distinction really refers primarily to extension but terms that are used intensively have to be given a definite distribution in order to perform the logical operations to which we are coming. The rule is that all terms which are used in intension, instead of in extension, are treated as being undistributed, because there is no way of determining the extent of a term which is not used in extension. Hence, to be sure that we are not saying more than we should say, we have to assume that terms used in intension are undistributed. *For undistributed means terms used to refer to only a part of their extension, and distributed means terms used in their fullest or complete extension.* Thus, the term, *all apples* is distributed because none are left out, but the term, *some apples*

is undistributed because a part of the class of apples is not included by the word *some.* Now let us take an example of a term used in intension, instead of in extension, and show why it must be taken to mean some rather than all, or in an undistributed rather than in a distributed sense. In the proposition, *All equilateral triangles are equiangular* the term *all equilateral triangles* is obviously used in extention and is distributed. The word all shows this. But the term *equiangular* is not used extensively. Nothing is said to indicate whether it means some or all because the term is used intensively. We know from geometry that it means all, but *we do not know this in logic and we have to treat it as meaning some.* For the purposes of formal logic every term which is used in an intensive rather than in an extensive sense has to be treated as undistributed.

Since singular terms have only one object to which they refer, they are always distributed, no matter whether they are used intensively or extensively. Thus, the *center of the material universe* is a distributed term. And since, as we saw above, every proper name is a singular term in the sense that it has a different intension for every object to which it refers, these terms, too, are always distributed. One can never say, *some Socrates.* We shall use this distinction between distributed and undistributed terms many times. But now let us return to intension and extension.

The Alleged Inverse Ratio between Intension and Extension

Generally speaking, the less the intension of a term the greater the extension. If only one or two attributes are needed to constitute an object, there are likely to be a great many more such objects than where there are a number of qualities essential to constitute the object. Now the intension of a term may be increased by adding some new characteristic. When this is done the extension tends to

decrease. For example, the term *man* is relatively rich in intension, that is to say, it would require a great many qualities to express completely the nature of man. But when the attribute *civilized* is added to the term man the intension is increased enormously. Now what happens to the extension in that case? It becomes much less. There are far fewer civilized men than there are men. Now another set of qualities may be added to the term civilized man, namely *educated*, civilized man. This still further increases the complexity of qualities necessary to the object, and consequently there are far fewer objects which can be comprised under the term. The number of educated, civilized men is much less than the number of civilized men, so that while the intension is greater, the extension is less.

The question arises as to whether this is always true. Do intension and extension vary inversely? The answer is "no." A very slight increase in the intension may decrease the extension enormously. Thus, add to books the attribute *bound*, and very little addition is made to the intension, whereas the extension is very greatly modified. Moreover, qualities cannot be measured mathematically, but objects can be enumerated. Hence it is possible to get a definite figure for the variation in the extension of a term, but it is not possible to determine the precise amount of variation produced by adding a particular quality or set of qualities. There can be no mathematical ratio in the variations. All we can say is that generally speaking, when intension is increased, extension is decreased. On the other hand, to increase the extension may not have any effect on the intension or it may modify it only slightly.

Some logicians make the mistake of thinking that mentioning the *kinds* of objects is giving the extension of a term. For instance, they say that the extension of *man* is Caucasian, Mongolian and Negro, instead of saying that the

extension is *all men of all kinds.* This is a mistake because using terms such as Mongolian is giving another intensive term, the extension of which would be all Mongolians. This blurs the distinction between intension and extension. The extensive meaning of a term is not another intensive term, but all of the objects having the qualities specified by the original term. The intension of man is simply increased by adding the term, Mongolian.

To the elementary student the intension of a term is hard to grasp because we usually tell what a thing is by giving an example of it, instead of analyzing it into its specific attributes. To define accurately is exceedingly difficult, but to point out is comparatively easy. This was Socrates' problem in ancient Athens. When he asked for a definition of justice, he was given an instance of an act of justice. He asked for the intension and he was given the extension of the term in question. Now we know the objects before we know their full nature. Only a student of botany knows fully the nature of a leaf, its intricate structure and its constituent elements. But every one supposes that he knows what a leaf is, because every one has seen leaves all his life. It is comparatively easy to tell what a leaf is by pointing to one, and saying: "That is a leaf." It is very difficult to tell what a leaf is by giving all of the essential attributes in the constitution of a leaf, because we can only learn what these are by a careful study under the microscope of the structure of a leaf. Hence Mackenzie is entirely right in saying: "The denotation (extension) of a term may be sufficiently apparent without any definite apprehension of its connotation."[1] Few, indeed, are those who know the essential intensive meaning of the commonest words of everyday life. Yet every one assumes that he knows what they mean, because he has in mind instances

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[1] J. S. Mackenzie, *Elements of Constructive Philosophy*, p. 69.

of the things. One of the greatest values of logic is that it teaches us how limited is our knowledge, and how prone we are to deceive ourselves into thinking that we know what an object is because we have seen instances of it.

EXERCISE III

1. Use each of the following terms in a sentence in which the meaning of the term is intensive, and then use the same term in a sentence in which the meaning is extensive.

(*a*) uranium
(*b*) radar
(*c*) helicopter
(*d*) altimeter
(*e*) hydramatic drive
(*f*) gene
(*g*) ion
(*h*) Grand Canyon
(*i*) zero temperature
(*j*) economic inflation

2. Put an appropriate word before each of the following terms that will increase its intension. State what effect this has on the extension in each case.

(*a*) radiogram
(*b*) rocket
(*c*) navigator
(*d*) eskimo
(*e*) evolution
(*f*) smog
(*g*) atmosphere
(*h*) astronomy
(*i*) nucleus
(*j*) nostalgia

3. Use each of the following sets of terms to prove that intension and extension do not vary in inverse ratio:

(*a*) ship, steamship, Diesel powered steamship, the Queen Elizabeth.
(*b*) airplane, twin-motored airplane, naval twin-motored airplane, naval air transport twin-motored airplane.
(*c*) radio, automobile radio, automobile twelve-tube radio.
(*d*) radar, airplane radar, bomber airplane radar, B-29 airplane radar.

4. Make series, as in question 3, out of the following terms, and tell which word in each series has the greatest intension and which word the most extension. Note that series may begin either with least intension and end with greatest intension, or with greatest intension and end with least intension: military organization, radar, U.S. Supreme Court, radar set, educational institution, airborne radar set, U.S.S. *Wisconsin*, navy, Army Board, civil court, court martial, high school, battleship squadron, public high school, supreme court, light-weight airborne radar set, Roosevelt High School, trial of Premier Tojo.

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CHAPTER V

DEFINITION AND DIVISION

The Nature of Definition

A term is said to be defined when its complete intensive meaning is given. This involves embodying in language the results of a scientific analysis of the constitution of whatever is being defined. It assumes that every definable object, or entity, has a complex structure the nature of which can be ascertained by careful inspection and analysis. A complete definition is difficult to obtain because it involves giving an exhaustive analysis of the inferential whole in question. As Mill well said:

> The definition of a name is the sum total of all the *essential* propositions which can be framed with that name for their subject. All propositions the truth of which is implied in the name, all those which we are made aware of by merely hearing the name, are included in the definition if complete, and may be evolved from it without the aid of any other premises; whether the definition expresses them in two or three words, or in a larger number. It is, therefore, not without reason that Condillac and other writers have affirmed a definition to be an *analysis*. To resolve any complex whole into the elements of which it is compounded, is the meaning of analysis; and this we do when we replace one word which connotes a set of attributes collectively by two or more which connote the same attributes singly, or in smaller groups.[1]

The purpose of defining is to fix the universal meaning, or the intensive qualities, of the thing defined so that they can be communicated to another person, or so that the individual thinker may have a permanent thought content

[1] J. Stuart Mill, *Logic*, Bk. I, Ch. VIII, Sec. 2.

with which to carry on his own thinking about objects. The distinction is frequently made between *verbal* and *real* definitions. The former are definitions of words and the latter of real existing objects. The only value of this distinction is to emphasize the fact that true definition is always real, and has to do with actual existences. The aim of every intellectual act of defining is to express as accurately as possible the actual nature of the thing being defined, no matter what it may be. Verbal definitions are useful as tentative or provisional definitions, but thought aims to replace them by real definitions and should do so as soon as possible. Definition is really valueless when it fails to grasp the actual content, the intensive meaning of what is being defined.

As Mill pointed out, *description* is not *definition,* because description *pictures* the object instead of giving its actual thought content or intensive meaning. Definition is the result of accurate analysis, painstaking investigation, thorough study. Description, on the other hand, gives an outside view, catches what is most apparent to the senses, presents an imaginative and superficial account of the thing being defined, and thus fails to grasp the inner essence with which thought is chiefly concerned.

Kinds of Definition

1. *Aristotelian Definition.* In the discussion of the predicables and of intension, we have already referred to the Aristotelian type of definition. It is that method of defining which proceeds by stating the genus nearest to the object being defined, and by giving the essential differentiæ which set this object off from others belonging to the same genus. The object is said to belong to a class of objects known as the species, and the differentiæ are the essential attributes of the species. Here what was said above about the real logical meaning of membership in a

class or species must be kept in mind. The relation in question is internal and vital, not extraneous and unessential. Each of a number of species belongs to the genus and has all the essential attributes of the genus, but it also has unique attributes which differentiate it from other species in the same genus. In Aristotelian definition a term is defined by simply mentioning the genus, without specifically mentioning its attributes, and then going on to a more detailed account of the differentiæ. All the essential attributes of the species have to be specifically mentioned, but it is regarded as sufficient simply to name the genus.

Practically all dictionary definitions are of this kind. The student should look through some of the definitions given in any standard English dictionary in order to appreciate how fundamental the Aristotelian method of defining is. The following examples may be given: (*a*) "A *dodecahedron* is a solid having twelve faces." Here the word *solid* is the genus, and the phrase *having twelve faces,* differentiates the dodecahedron from other solids. What it takes to constitute a solid is not specifically mentioned. (*b*) "A falling star is one of a class of meteors which appear as luminous points shooting or darting through larger or smaller areas of the sky, and which are often followed by trains of light." (*c*) "A faucet is a device fixed in a receptacle or pipe to control the flow of liquid from it by opening or closing an orifice." (*d*) "A dendrite is a stone or a mineral in which are figures resembling shrubs, trees or mosses." Now, this last definition gives us one of the inferential wholes for which the term dendrite stands. There is an entirely different one brought out in the following definition: "A dendrite is one of the protoplasmic processes of a nerve cell." Each of these definitions of a dendrite illustrate the Aristotelian type of definition, and yet each is stating an entirely different intensive meaning

2. *The Logical Comprehensive Type of Definition.* This type of definition aims to correct the defect of the Aristotelian type. Merely to mention the genus, without stating its essential characteristics, is to leave out a part of the most essential attributes of the thing defined. Hence, every Aristotelian definition is in so far incomplete. Logical comprehension is that method of defining which, in contradistinction from the Aristotelian type, *gives a complete list of essential marks or attributes, without using the distinction between genus and differentia.* All essential marks are treated as being on the same level, and are aggregated together in the definition without regard to their relations with one another. While this type of definition is especially serviceable in defining highly artificial classes, such as some of those dealt with in mathematics, it is really not as good a type of definition for general purposes as the Aristotelian. It makes the definition more cumbersome to include the differentiæ of the genus along with those of the species, and it is not really possible to avoid some word which may be taken as a genus. Every definition has to assume some ideas which are not defined and a logically comprehensive definition is no exception to this rule. As a matter of fact, it is simply an Aristotelian definition which uses the most remote instead of the nearest genus. For example, when I define a dodecahedron as a magnitude or quantitative whole, having length, breadth and thickness, and twelve faces, I give the differentiæ of solid as well as of dodecahedron. But I am forced to use the word magnitude or quantity, and this becomes the genus instead of solid. It is hardly possible to give a definition which does not use some word which may be regarded as the name of a genus. Logical comprehension is really only an extension of Aristotelian definition, and in practice it is neither necessary nor advantageous to state the essential attributes of the genus.

3. *The Genetic Type of Definition.* This is the name given to that type of definition which traces a thing to its origin, or tells how a thing is constituted or produced, instead of placing it in its nearest genus. According to this method I define a dodecahedron when I tell how to construct one. The rule for the production of an individual of the kind in question is equivalent to the genetic definition.

Mathematical definitions are frequently of this kind. For example: "A sphere is a solid figure formed by the revolution of a semicircle about its diameter, which remains fixed." But many natural objects are also definable by the genetic method. Hence it is frequently employed in the natural sciences, and it is undoubtedly occasionally preferable to the Aristotelian type. "To define *heat* as a force in nature recognized in the phenomena of fusion and evaporation (this is an Aristotelian definition) tells us less about its real nature than the statement that it is a form of energy possessed by bodies *derived from* an irregular motion of their molecules (this is a genetic definition). To define water as a fluid which descends from the clouds in rain (again the Aristotelian type) is less adequate for scientific purposes than the chemical definition of it as a fluid formed by adding one part of oxygen to two parts of hydrogen."[2]

The Rules of Definition

There are five important rules which must be observed in formulating a logical definition. These rules have come down through the ages, and they refer especially to the Aristotelian type of definition.

1. The basic rule is that the definition *must state all the essential attributes of the object which is being defined.*

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[2] J. E. Creighton, *An Introductory Logic* (New ed., Macmillan Co.), p. 74.

This rule is frequently violated. Dostoeffsky's statement: "Man is a creature who can get used to anything, and this, I think, is the best definition of him," is a poor logical definition because it violates this rule. Of course his purpose was well served by this definition since this one aspect of man, namely, being able to adapt himself to any condition, is the part of man's nature in which he was most interested. But man is much more than a creature who can get used to anything.

2. The definition *must include no more and no less than the object or implicative system which is being defined.* It must neither be too broad nor too narrow, but commensurate with that which is being defined. "To define fish as an animal that lives in the water, would be too broad because many insects, etc., live in the water; to define it as an animal that has an air bladder, would be too narrow because many fish are without any. Or again, if in a definition of money you should specify its being made of metal, that would be too narrow, as excluding the shells used as money in some parts of Africa; if, again, you should define it as an article of value given in exchange for something else, that would be too wide, as it would include things exchanged by barter, as when a shoemaker who wants coal, makes an exchange with a collier who wants shoes." (Whately.)

3. The definition *must not be obscure and confused.* Since accuracy, clearness and precision is the goal of definition every obscure and ambiguous definition defeats its own purpose. There are many causes contributing to ambiguity and obscurity. Sometimes it is due to the use of an ambiguous word or phrase, or one which is harder to understand than the one we are defining. Sometimes it is due to the use of figurative expressions, especially metaphors. Sometimes too many technical expressions are incorporated in the definition. Yet this latter is always justi-

fiable, within due limits, in scientific treatises. No student has a right to condemn a definition for being too technical until he can give a definition which expresses the exact meaning in non-technical language. Technical terminology is essential and justifiable and this rule is not directed against its use. On the other hand, a scientist who is writing for the general public or for elementary students owes it to his readers to define each technical term when he first introduces it, and this is too rarely done. Examples of violations of this rule are numerous, but the following may serve: "Architecture is frozen music." "Life is a continuous adjustment of internal relations to external relations." (Spencer.) "Virtue is a passive symbol of an idea." "Religion is an experience of some mystical thrill of emotional psychology."

4. The definition *should not be circular.* The term being defined or a synonym of it should not appear in the definition. This is obvious and yet it is a failing that is all too common. Every teacher knows how frequently students use this device to conceal their ignorance of the true answer to a question. When we violate this rule we come out of the same hole we went into, as a friend of mine wittingly expressed it. To define a preacher as one who exercises the functions of a clergyman would be a circular definition.

5. The definition *should not be negative when it can be positive.* Now, it must be admitted that a good way to lead up to a positive definition is to tell what the object under consideration *is not.* As a rhetorical device this method is frequently very valuable, but logically it is bad procedure. What a definition is intended to do is to tell the actual positive nature of what is being defined. Defining a moral man as one who is not intemperate, does not lie, or steal or covet is an easier way than defining a moral man by giving the positive features which distinguish

morality from immorality, but it is logically a bad definition because telling what a moral man is not may fail entirely in conveying an idea as to what he is. This is really a corollary of rule one, since giving the essential attributes involves stating the positive nature of the thing defined.

Sometimes a corollary of rule two is added for a sixth rule as a kind of counsel of perfection. It is that the definition *should contain nothing superfluous.*

The Importance of Definition

It is easy to exaggerate the importance of definition, and many make the mistake of doing so. Superficial people often suppose that all disputes about really profound questions could be solved by laying down a definition. The fact of the matter is that definition is the end of knowledge rather than its beginning, and for that reason no one can give a good definition until he has made a thorough investigation of the nature of the entity being defined. Consequently any definition with which we begin an investigation is provisional and more or less arbitrary. It solves no problems. It only opens up the field of investigation. Professor Hocking, after defining religion as anticipated attainment, rightly remarks: "This precursory definition of religion serves the purpose of such definitions—not to solve problems, but rather to open them." Most of the definitions given at the beginning of a discussion or textbook are of this sort.

To warn the student against the danger of exaggerating the importance of such precursory definitions let me quote the significant words of one of England's ablest statesmen, Edmund Burke. In his philosophical discussion entitled "On Taste," he wrote:

> I have no great opinion of a definition. . . . For, when we define, we seem in danger of circumscribing nature within the

bound of our own notions, which we often take up by hazard or embrace on trust, or form out of a limited and particular consideration of the object before us; instead of extending our ideas to take in all that nature comprehends. . . . A definition may be very exact, and yet go but a very little way towards informing us of the nature of the thing defined; but let the virtue of a definition be what it will, in the order of things, it seems rather to follow than to precede inquiry, of which it ought to be considered as the result.

Yet, although what Burke says is true, definition is frequently highly important. Many needless disputations between eminent men are due to the fact that the object under discussion has not been clearly defined by each of the disputants. In such a branch of human knowledge as law, definition frequently assumes an overwhelming significance. In the words of a distinguished French professor of law: "The definition dominates juridical science. It is discovered in the writings of the legislator and the judge as well as in those of the jurist. Its function is no longer simply to clarify debate. It has become an active factor in the formation of law. It directs the progress of the law. Definition is an intellectual and rational force in juridical development." To be sure, definition has often been overemphasized by lawyers. "When the definition had conquered the domain of the law, when it had forced itself upon the lawmakers, as well as upon the lawyers, it behaved like an absolute monarch, who may have the last word when he wishes, but whose authority is at the same time lax and capricious. It has the air of governing everything; in reality, it allows itself to be governed by circumstances. It is supposed to be obeyed, but provided that proper forms are observed, many liberties can easily be taken with it."[3] Thus to the lawyer who knows how to use it, definition is admittedly a most effective instrument.

[3] Tourtoulon. *Philosophy in the Development of Law*, pp. 328, 343.

This is just as true in general as it is of legal science. Although definition is not a cure-all and end-all, and is really only a subordinate part of logical doctrine, still "there is no need to throw to the dogs all that is not fit for the altar of the gods."

What Logical Division Is

Just as definition has to do with the intensive meaning of a term, so division has to do with its extensive meaning. Now, obviously division is practically restricted to those terms which comprise a large *variety* and a comparatively large *number* of objects in their extension. And this means that in division we are really dividing the genus into its species and the species into subspecies. Logical division is thus practically restricted to naming the different species which make up a certain genus and naming the different subspecies which make up the separate species or, as they are also called, *genera.* The subspecies with which a division stops are called *infimæ species,* the genus with which it starts the *summum genus,* and the intermediate species are called *subaltern genera* because they each constitute a new genus in respect to the subdivisions into which they are further divided. The *proximum genus* of any species is that next above it. Every genus is said to be *superordinate* to those below it, *coördinate* with those on the same level with it and *subordinate* to all of those above it in the series of species or genera. We may, then, define logical division as the intellectual process of breaking up a genus into its subordinate genera.

Breaking up is analyzing, and we seem to have the same process here that we had in definition. And there is a sense in which this is true. For each separate genus in the series of genera has a separate intensive meaning as well as an intensive meaning common to all the genera. Hence it is true to say that logical division is seoarating out of a

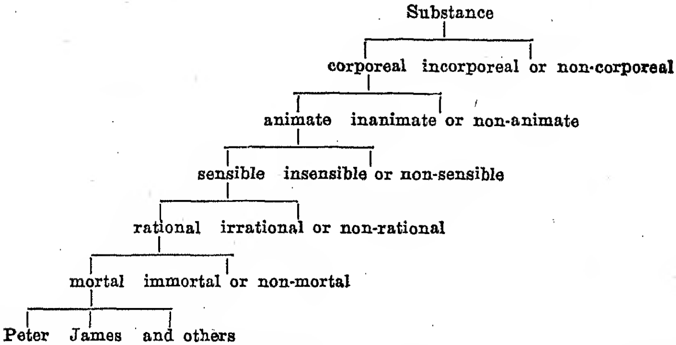
complex group of qualities the respective constituent differences, and since these are intensive, division seems to be the same as definition. But since the purpose of division is *to arrange the actual objects* having those differences in some sort of orderly manner, we are justified in holding that the chief concern in division is the extensive rather than the intensive meaning.

We meet in division a distinction similar to the one we met with in definition, under the name of *verbal and real* definition. Here we distinguish *artificial* from *real* division. In the former we simply arrange things in any way that happens to suit our purpose, or to be convenient for us. Many very useful divisions are of this purely arbitrary and artificial character. But in scientific research into the infinite variety of nature the aim is to get at the real distinctions in nature. Since thought should always delve for the actual existing differentiations between various natural or real objects, a purely artificial division is only of relative and transitory value, whereas a real division, although the complexity of nature makes it most difficult even for the scientific genius to reach, is nevertheless of permanent and eternal value.

The Chief Kinds of Division

1. *Dichotomy.* The method of division which Plato used in the investigations carried on in the Academy was dichotomy. It is based on the law of excluded middle which we considered when we dealt with the basic laws of thought. Dichotomy assumes that the whole universe can be satisfactorily divided by prefixing the word *non* to a given term, so that the positive term forms one side of the division and the negative term forms the other part, after which the positive term is further divided into a positive and a negative side, and so on indefinitely. This type of division is also called *binary classification.* An illustration

will make the process clearer. I use for that purpose the famous *Tree of Porphyry.* It comes down to us from the great Neo-Platonist logician of that name.



Here, starting with a term, our first division is into its negative and positive aspects; the negative aspect is left to one side and the positive aspect is further divided into positive and negative aspect. This is continued until the purpose of the division in question is accomplished.

There are two bad features of binary classification, or dichotomy. In the first place, it restricts the division to two coördinate classes, and many times division demands that there be several coördinate classes. Moreover, there is no way of knowing whether the two classes really are coördinate, since we do not know what may be included under the negative term. It is one of the infinite negative terms which may mean anything. Or to put it differently, the method of dichotomy is really an attempt to include in the division what we do not know as well as what we know, whereas a real division must be based on what we know. Hence Bacon was right when he said: "These dichotomists would wrest whatsoever does not aptly fall within their dichotomies." In other words, the dichotomist has to make everything come within his dichotomies whether they natu-

rally do fall within them or not. Consequently, the method of dichotomy is not highly valued to-day.

Moreover, as the quotation from Bacon suggests, the logical defects in the method of dichotomy frequently lead to serious practical consequences. Molders of public opinion, such as politicians and clergymen, know how readily immature and poorly trained minds are victimized by catchy but superficial dichotomies. Thus the current dichotomies, such as Darwinian and Non-Darwinian, Fundamentalist and Non-Fundamentalist, Bolsheviki and Non-Bolsheviki, "Cluckers" (members or adherents of the Ku Klux Klan) and "Non-Cluckers," really become brands by which individuals are designated, even though their views may really place them entirely without and above either of the opposing groups. For no great man is ever bounded by the nutshell of a sect or party. As we grow in wisdom and reach maturity of judgment on social, political and religious issues, we come to see that "the habit of indulging in sharp but facile dichotomies is probably the most serious error into which we can fall." (M. R. Cohen.)

2. *Classification.* Since it is not possible by the method of dichotomy to express the complex relations in nature which gradually and insensibly pass over into each other, we are thrown back upon *classification* as the only scientifically valuable method of division. Here the aim is to form separate classes according to the natural lines of demarcation in the subject matter under investigation. As many coördinate classes are named as the actual structure of that subject matter reveals to careful and exact observation, and in the same way these larger classes are broken up into as many subdivisions as the nature of the objects being classified seems to demand. Modern logicians regard classification as one of the most essential parts of logic and in Part Two we shall see how essential it is in inductive procedure.

A very simple example of a classification may be taken from the science of mathematics where *conic sections* are classified as the hyperbola, the parabola, the ellipse and the circle. Every conic section must be one or the other of these four types. It is readily seen that a dichotomy here would be worthless. For example, it might be hyperbola and non-hyperbola, but non-hyperbola would leave us completely in the dark as to the character of the other conic sections which are not hyperbolas. But when these are given the positive designations of parabola, ellipse and circle we know precisely what they are.

Among the numerous attempts to classify the sciences, perhaps the most famous is Auguste Comte's so-called *hierarchy of the sciences.* It is an arrangement of the chief sciences in a linear series on the principle that the order followed in their historic development was from the most abstract to the most concrete and from the most simple to the most complex. Thus, his classification is really based on two principles of division, but he held that the development actually followed both of these principles in a strictly parallel manner. Hence his hierarchy is as follows:

I. COSMOLOGY
   (*a*) Mathematics
   (*b*) Astronomy
   (*c*) Physics
   (*d*) Chemistry

II. BIOLOGY
   (*a*) Botany
   (*b*) Zoölogy (including physiology and psychology)

III. SOCIOLOGY
   (*a*) Social statics
   (*b*) Social dynamics

IV. ETHICS

Although this celebrated hierarchy has had a by no means insignificant influence on modern thought, it is now

generally regarded as more speculative than scientific. However, Comte thought that it was strictly scientific.[3]

Cuvier's classification of the animal kingdom dominated zoölogy from 1817 until it had to be revised in the light of the Darwinian hypothesis. In outline form it is as follows:

THE CLASSIFICATION

I. VERTEBRATES.—Animals having a backbone
- (*a*) Mammals
- (*b*) Birds
- (*c*) Reptiles
- (*d*) Fish

II. MOLLUSCS.—Animals having a soft body protected by a shell
- (*a*) Cephalopods
- (*b*) Pteropods
- (*c*) Gasteropods
- (*d*) Acephala
- (*e*) Brachiopods
- (*f*) Cirropods

III. ARTICULATA.—Worms and insects having a ringlike body, a nervous system and respiratory organs
- (*a*) Annelides
- (*b*) Crustacea
- (*c*) Trilobita
- (*d*) Arachnida
- (*e*) Insects

IV. RADIATA.—This is Cuvier's miscellaneous class, which has been spoken of as his "wastebasket" for animals not included in the other three groups
- (*a*) Echinodermata
- (*b*) Entozoa
- (*c*) Acalepha
- (*d*) Polypi
- (*e*) Infusoria

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[3] For a more recent and logically superior classification of the sciences, see the "outline map of scientific knowledge" on page 223, which I have taken from J. Arthur Thomson's *Outline of Science*.

Aside from its historical interest Cuvier's classification has the negative value to the student of logic of showing that a real scientific classification can neither be made nor fully comprehended by one who does not have a detailed knowledge of the field in question. For a full comprehension of this classification an extensive knowledge of the whole science of zoölogy is needed. Each word in the classification stands for countless animals, all of which have common characteristics along with differences. These differences would have to be expressed by further subdivisions. And the greatest weakness in this famous classification is in the fourth division concerning which Cuvier knew the least. "The lower groups of which he knew least, and which he threw into one great heterogeneous assemblage, the *Radiata*, have been altogether remodeled and rearranged" (Huxley).

Classification is extremely important in systematizing scientific knowledge, but like definition it, too, is really the goal of a scientific investigation rather than its beginning. For example, the table of elements in chemistry is really a classification of the elements which was only made possible by years of painstaking experimental investigations. For this reason some sciences are spoken of as classificatory sciences, for example, zoölogy, botany and philology.

The Rules of Division

There are three rules of division which a good logical classification will not violate.

1. The division *must be exhaustive,* or, in other words, nothing that is included in the genus or class of objects which is being divided can be omitted from the separate genera into which it is divided. The fallacy which results from a violation of this rule is called the fallacy of *incomplete division.* An inexperienced classifier is likely to omit

unique objects which do not easily come within the main classes. To make sure that such an omission is not made it is sometimes a good plan to make a class called the *miscellaneous class,* which is equivalent to the negative class in dichotomy. An illustration of the fallacy of incomplete division would be a classification of the ships of the United States Navy into battleships, transports, gunboats, destroyers, and submarines. For such a division leaves out mine layers, colliers, hospital ships, etc. These might all be included in a miscellaneous class of ships.

2. The different species or classes included under the genus or main class *must be mutually exclusive,* that is to say, the different divisions must not overlap. Such an overlapping is usually the result of making a class coördinate with other classes which should be made subordinate to one or the other of these classes. Dr. Johnson's amusing division of the inhabitants of Scotland into, "Scotchmen and dammed Scotchmen," is an illustration of this fallacy. A division of trees into forest trees, ornamental trees, fruit trees and apple trees would be an example of this fallacy, because apple trees are fruit trees.

3. A division *must be based upon one fundamental principle of division.* If more than one principle is used there is a fallacy of *cross-division,* since it becomes necessary to divide the constituent members first by one principle and then the same members again by another principle. This is all right if each principle is carried through separately, since it is legitimate to use more than one principle in a division, but it is illogical to use two different principles to obtain coördinate classes. One must be subordinated to the other. Thus, the personnel of the United States Navy can be divided according to rank or according to function, but if I start out using one principle and shift over to the other, the result is the fallacy of cross-division.

There are some principles of classification which can be used to classify many different heterogeneous collections of items. For example, the Dewey Decimal System of classification, which was invented by Melvil Dewey for use in classifying books in libraries, has not only come into general use for that purpose but it has also been widely adopted for other purposes. As applied to libraries, the main outline of the system is as follows:

THE DEWEY DECIMAL SYSTEM OF CLASSIFICATION

000 *General Works*

010 Bibliography.
020 Library Economy.
030 General Cyclopedias.
040 General Collections.
050 General Periodicals.
060 General Societies.
070 Newspapers.
080 Special Libraries. Polygraphy.
090 Book Rarities.

100 *Philosophy*

110 Metaphysics.
120 Special Metaphysical Topics.
130 Mind and Body.
140 Philosophical Systems.
150 Mental Faculties. Psychology.
160 Logic.
170 Ethics.
180 Ancient Philosophers.
190 Modern Philosophers.

200 *Religion*

210 Natural Theology.
220 Bible.
230 Doctrinal Theology.
240 Devotional and Practical.
250 Homiletic. Pastoral.
260 Church. Institutions. Work.
270 Religious History.
280 Christian Churches and Sects.
290 Non-Christian Religions.

300 *Sociology*

310 Statistics.
320 Political Science.
330 Political Economy.
340 Law.
350 Administration.
360 Associations and Institutions.
370 Education.
380 Commerce. Communication.
390 Customs. Costumes. Folklore.

400 *Philology*

410 Comparative.
420 English.
430 German.
440 French.
450 Italian.
460 Spanish.
470 Latin.
480 Greek.
490 Minor Languages.

500 *Natural Science*

510 Mathematics.
520 Astronomy.
530 Physics
540 Chemistry.
550 Geology.
560 Paleontology.
570 Biology.
580 Botany.
590 Zoölogy.

600 *Useful Arts*

610 Medicine.
620 Engineering.
630 Agriculture.
640 Domestic Economy.
650 Communication.
660 Chemical Technology.
670 Manufacturers.
680 Mechanic Trades.
690 Building.

700 *Fine Arts*

710 Landscape Gardening.
720 Architecture.
730 Sculpture.
740 Drawing. Design. Decoration.
750 Painting.
760 Engraving.
770 Photography.
780 Music.
790 Amusements.

800 *Literature*

810 American.
820 English.
830 German.
840 French.
850 Italian.
860 Spanish.
870 Latin.
880 Greek.
890 Minor Languages.

900 *History*

910 Geography and Description.
920 Biography.
930 Ancient History.
940 Modern — Europe.
950 Modern — Asia.
960 Modern — Africa.
970 Modern — North America.
980 Modern — South America.
990 Modern — Oceanica and Polar Regions.

Try to classify books on athletics, eugenics, and measles, and you will find amusing violations of rule 2. It would be practically impossible to avoid violating that rule when dealing with large aggregates of miscellaneous objects. Another system of classifying books is the alphabet system used in the Library of Congress, but it is not as useful as the Dewey Decimal System. Professor Pickering's alphabet system of classifying the stars, however, is now more widely used than the old system of classifying them according to magnitude, for Pickering's system is based on the stage of evolution of the various stars. Alphabet systems

of classification are widely used for other purposes also.

Sympodial classification is the name given to classifications based on the successive branchings of phenomena. Genealogies are good examples. *Phylogenetic classification* corresponds to genetic definition. It is a classification based on the principle of common descent and is frequently used in classifying plants and animals. Such a classification might also be sympodial, but it would not be necessary for a sympodial classification to be phylogenetic.

Exercise IV

1. Define each of the following terms by naming the nearest genus and indicating the differentiæ of the species.

(*a*) piano	(*d*) submarine	(*g*) streptomycin
(*b*) photograph	(*e*) ionosphere	(*h*) hormone
(*c*) marble	(*f*) vitamin	(*i*) blockhouse

2. Define by the genetic method:

(*a*) photomicrograph	(*d*) suntan	(*g*) cinema
(*b*) flashbulb exposure	(*e*) skis	(*h*) radarscope
(*c*) meteorite	(*f*) television	(*i*) thermometer

3. Define by the logically comprehensive method:

(*a*) hypnotism (*b*) salamander (*c*) wax

4. "Fascism is a political philosophy which exalts the state as such, whereas Nazism exalts the state only when it is a racial state, and Communism does not exalt the state at all." Criticize these three definitions by pointing out clearly which of the five rules of a good definition are violated by each.

5. Criticize the following definition from the standpoint of the first rule of a good definition, especially with respect to technicalities. "Antimetabolites are such substances as alphatocopheral quinone and pantoyltaurine which attack amino acids and vitamins." Then use the following information in formulating an understandable definition of antimetabolites: Vitamin B-1 is a valuable ingredient of many foods and it is destroyed by nutritional mischiefmakers or antimetabolites found in other foods, which contain an enzyme that attacks vitamin B-1 and amino acids.

6. Use the five rules of a good definition to test each of the following definitions, indicating at least one rule that is violated:

(*a*) Law is what makes men sure of the bearings of their actions.
(*b*) The economic ideal is the mastering of nature.
(*c*) Business is business.
(*d*) Social right is that which is not economically wrong.
(*e*) The social ideal is the subordination of nature to mind.
(*f*) Economic right is that which is not economically wrong.
(*g*) Law is the formulation of rules of conduct that will satisfy both economic and social ideals.
(*h*) A crowd is the product of the social instinct.
(*i*) A spontaneous crowd is an unorganized gathering of people held together by primitive emotions of imitation, sympathy, and suggestion.
(*j*) A herd is a browsing, planless flow of mentality, with crises of massive madness.
(*k*) Money is what men use to finance their business deals.

7. State three different principles of division which could be used in classifying the voters of the United States. Use one of these principles to make a classification of voters. Then indicate how the fallacy of cross-division would arise by the use of one of the other two principles in conjunction with the one you selected.

8. Use the Dewey Decimal System of classification to classify the voters of the United States.

9. Following are two classifications of attempts to define ethical terms. How does the first differ from the second? Change one of the two to make it a complete dichotomy. State what you take to be the fundamental principle of division used in each classification, and explain why.

First classification

- (A) Attitude theories
  - (*a*) Naturalistic
  - (*b*) Non-naturalistic
- (B) Consequence theories
  - (*a*) Naturalistic
  - (*b*) Non-naturalistic

Second classification

- (A) Naturalistic theories
  - (i) Attitude theories
  - (ii) Consequence theories
- (B) Non-naturalistic theories
  - (i) Attitude theories
  - (ii) Consequence theories

10. Classify the members of the solar system in a way that will illustrate the fallacy of incomplete division, indicating what members are not included in your classification. Then create a class for those members your classification omits, and name it.

SECTION II

JUDGMENT AND THE IMMEDIATE INFERENCES

CHAPTER VI

JUDGMENT AND THE KINDS OF PROPOSITIONS

The Nature of Judgment

In the last section on naming and the doctrine of terms we had to make use of a precursory definition of judgment in order to make clear what logic means by terms. That definition was that a judgment consists of two logical terms, known as the subject and predicate respectively and symbolized by S and P, united by a copula which is always some form of the verb *to be,* preferably *is* or *are*. Although this is a good preliminary definition of judgment, and one which will be frequently used below as a working formula, nevertheless it must here be supplemented by a deeper analysis of the real nature of the logical judgment.

From the point of view of the individual human being, judgment may be defined as an intellectual activity, a process of differentiating and putting together again the facts of experience. In this sense, judging involves all of the following mental processes: comparison, resemblance, identity, difference, and relation in general. Any act of comparing or identifying or discriminating or establishing a relation would be included under the intellectual activity of judging. Such a conception of judgment throws the emphasis on its psychological aspect. Every human being is continually making judgments in this sense. Indeed, a person's mind may be said to consist primarily in his judging activity. Analyze your own mental life, and you will find that you are always making judgments when you are mentally alert. Of course much of our waking life is spent in mere daydreaming, in the haphazard associating of ideas, and this must not be compared with that alertness

of mind which characterizes judging. But for a man to play football, box in a boxing bout, fly an airplane, conduct a business, manage a social settlement institution, a university or a church, or to do well any of the other manifold deeds which constitute the very stuff of life, nothing is more important than the intellectual ability to size up a situation, and to propose rapid and sure solutions of the difficulties which that situation involves. This quick and sharp mental alertness, which is behind all efficient action, is judgment in the subjective sense, as is shown by our common expression concerning an efficient person: "So-and-so is a man of good judgment." The term *judgment* is used in this sense in the following remarkable passage, in which John Locke distinguishes the witty man and the man who has a good memory, from the man who has good judgment:

> There is some reason for that common observation that men who have a great deal of wit and prompt memories have not always the clearest *judgment* or deepest reason; for, wit lying most in the assemblage of ideas and putting those together with quickness and variety wherein can be found any resemblance or congruity, thereby to make up pleasant pictures and agreeable visions in the fancy, *judgment* on the contrary lies quite on the other side, in separating carefully one from another, ideas wherein can be found the least difference, thereby to avoid being misled by similitude and by affinity to take one thing for another.[1]

While the above conception of judgment is important in logic, from the point of view of knowledge as a whole, judgment is rather the final and clearly formulated product of such a sustained intellectual activity. And frequently it is the product, not simply of the intellectual labors of one man or one generation of thinkers, but of a long line of great thinkers. The body of judgments which

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[1] John Locke, ***Essay Concerning the Human Understanding***, **Bk. II, Ch. XI, Sec. 2.**

make up scientific knowledge are the products of untold mental exertion on the part of countless thinkers, many of whom are nameless. In this sense, a judgment is a statement of fact which is independent of the individual thinker. The product of a judging activity has a stability and an independence of its own. Sometimes this meaning of judgment is expressed by the word *proposition,* instead of by the word judgment. This is a useful term, because it throws the emphasis on the logical aspect of the judgment rather than on the psychological aspect.

Take as an illustration of a judgment or proposition "He is one who had better consult a physician." This may be looked at as the act of somebody's mind, an act of observation let us say. But it is also a statement of fact. As such it is a logical proposition. It expresses an actual connection between a given individual's health and another person, called a physician, who is supposedly able to alter the condition of the man's health. This content is tied together into a definite system by the judging activity, and the content as so tied together into a definite system is the judgment or logical proposition. Thus, any particular proposition is an expression in language of an actual connection or relationship within a systematic whole. The complete set of propositions necessary to express the full nature of that system would be what we have so often called an inferential whole or an implicative system.

This notion of systematic connection is the essential element in the nature of judgment, and in the following list of definitions of judgment it comes to expression in each definition. "Judgment discloses a relation between two things (Avicenna). Judgment connects together or separates from each other two notions (Wolff). Judgment connects many possible cognitions into one (Kant). Judgment connects the attributes connoted by terms (Mill). Judgment connects subject and predicate in definite form

(Lotze)."[2] The inmost essence of judgment is this tying of discrete or separate elements into a unity.

Three other important characteristic features of logical judgments or propositions may be briefly mentioned. (1) *Necessity.* The intellect is subject to constraint when it judges. This is what really distinguishes judgment from mere daydreaming. The objective fact determines the nature of the judgment or logical proposition, and the mind is constrained to judge the way the fact is. As Bosanquet has well remarked, this constraint comes to expression in such popular expressions as "I am obliged to think," and "I cannot but believe." And it also comes to expression in the well-worn formula: The logic of the situation was ignored by the writer or speaker. (2) *Universality.* Every judgment is also universal in the sense that any intelligent being would make the same judgment, given identically the same facts. The logical proposition is independent of human spontaneity and prejudice. It is a kind of objective standard, and every intelligence must recognize it as being demanded by the situation. This universality of judgment follows as a direct corollary of its necessity. (3) *Truth.* Every judgment claims truth. Unless a man in making an affirmation or denial aims at truth the judgment in which the affirmation or denial is expressed is not really a judgment. No proposition is a logical proposition which does not claim truth.

The Chief Types of Propositions

There are a number of different ways of classifying propositions, but it suits my purpose best to make the following division.

Categorical propositions are straightforward assertions or denials: "Towser is Fred's dog." "The moon is made

[2] See J. Brough's article entitled "Judgment," in Hastings' *Encyclopedia of Religion and Ethics.*

up of elements." "No comets have circular orbits." "Some dogs are vicious animals." "Some Turks are not fanatical." These are examples of categorical propositions. The distinction may also be made between *pure* and *modal* categorical propositions. The above examples are all pure categorical propositions. For a modal proposition qualifies the assertion or denial *by stating the mode or manner in which P is connected with S.* For example: "All that lives *must* die." "It *may* rain to-morrow." "He *might* come yet." "An intemperate man will *probably* be sickly." The italicized words are the qualifying modal expressions. We shall not deal further with modal judgments, since they can readily be reduced to pure categorical propositions by putting the modal expression in the predicate term.

Over against categorical propositions stand *conditional* propositions. These make an assertion or denial subject to a condition or proviso. There are two kinds, *hypothetical* and *disjunctive.* The usual sign of the hypothetical proposition is: "*If* so-and-so, *then* so-and-so." "If he is intemperate he will be sickly." There is a categorical element in every hypothetical proposition. For to use the example just given, while it is not asserted that he is intemperate nor that he will be sickly, it is asserted that there is a connection between being intemperate and being sickly. Thus, although the hypothetical proposition does not affirm or deny its *antecedent* or *consequent,* these being the names of the two parts of an hypothetical, nevertheless *it does affirm a relation between them.* In this sense it is categorical. In fact, any categorical proposition can be expressed hypothetically, and any hypothetical proposition can be expressed categorically. The disjunctive proposition is usually expressed by the words *either* this *or* that. Its parts are called alternatives. Here, too, there is a categorical element. Neither this nor that is asserted, but one

or the other is asserted, so that other alternatives are excluded. For example: "Either the policeman shot the student, or the testimony of the witness is unreliable." Here it is not asserted that the policeman shot the student, and it is not asserted that the witness's testimony is unreliable, but it is asserted that one or the other alternative is true. Which one is not asserted. When the separate alternatives of a disjunctive proposition, or the antecedent and consequent of the hypothetical, are either affirmed or denied, each of the various affirmations and denials is a pure categorical proposition. We shall postpone further consideration of conditional propositions until we take up hypothetical syllogisms in section four.

Returning, now, to pure categorical propositions, we must subdivide them according to two different principles: *quality* and *quantity*. By quality we mean the characteristic of a proposition as *affirmative* or *negative*. Thus: "Socrates was the wisest man in Athens," is an affirmative proposition. "No Armenians can escape the Turkish massacres," is negative in quality. By the quantity of a proposition we mean its extent. This is determined by whether the subject term is distributed or undistributed. Propositions are called *universal* when the predicate is affirmed or denied of *all* of the subject, and they are called *particular* when the predicate is affirmed or denied of only part of the subject. Thus: "*All* horses are quadrupeds," is a universal proposition, whereas, "*some* horses are quadrupeds," is a particular proposition.

Singular propositions are propositions having singular terms for subject and predicate. Since there is only one object in the extension of such a term, all singular propositions are treated as universal. Thus the proposition: "The sun is the center of the solar system," is universal because S and P as here used are singular terms, and obviously *all* of each is meant. It follows that there are

no particular propositions in which a singular term is the subject.

Both universal and particular propositions may be either affirmative or negative in quality. This gives a total of four kinds of propositions, which are extremely important in traditional logic, and we shall be dealing with them exclusively through the rest of this and through the next section, or down to Chapter XII. They are symbolized as follows: The universal affirmative is called the *A* proposition and the particular affirmative is called the *I* proposition. These symbols are the first two vowels in the Latin word *affirmo,* meaning to affirm. The universal negative is called the *E* proposition and the particular negative is called the *O* proposition. These symbols are the vowels in the Latin word *nego,* which means to deny. The *A* proposition is affirmative in quality and universal in quantity; the *E* is negative in quality and universal in quantity; the *I* is affirmative in quality and particular in quantity; and the *O* is negative in quality and particular in quantity.

The following classification is a schematic summary of the kinds of propositions:

PROPOSITIONS

I. CONDITIONAL
- A **Hypothetical** (*If* . . . . . . *then* . . . .)
- B **Disjunctive** (*Either* . . . . *or* . . . .)

II. CATEGORICAL
- A **Modal** (may, must, probably, etc. . . .)
- B **Pure** (a straightforward assertion)
    - (1) *Universal*
        - *A* (*a*) Affirmative (All, every, any, etc.)
        - *E* (*b*) Negative (No, and the so and so is *not*)
    - (2) *Particular*
        - *I* (*a*) Affirmative (Some so and so are . . . .)
        - *O* (*b*) Negative (Some so and so are not . . . .)

Logical Form of Propositions

In order to perform the various logical operations, which are explained in the next chapter, it is necessary that the proposition be in correct logical form to begin with. And that means that it must be *unambiguous* with regard both to quantity and quality. Otherwise expressed, to put a proposition in logical form is to write it so that one of the symbols *A, E, I,* and *O* may be used correctly to designate it. If the proposition has a double meaning it will even be necessary to reduce it to two or more logical propositions, one of which may be, for instance, *A* and the other *O*. There are various types of expression which cause trouble. I shall consider only the four most important types.

1. *Propositions having other verbs than is or are as copula.* Take as an example: "All the German battleships have been destroyed by the Allies." All such propositions, including those mentioned above which contain modal verbal expressions, are readily reducible to logical form by throwing the verb into a relative clause, and using either *is* or *are* for the copula. Thus the example just given is reduced to logical form by writing: "All the German battleships are the ships which have been destroyed by the Allies." "Socrates declared knowledge to be virtue," becomes: "Socrates is one who declared knowledge to be virtue." Note that it is necessary to supply a word synonymous with the subject term in order to have something for the relative clause to qualify.

2. *Propositions beginning with "not all," or beginning with "all" and having "not" after the copula.* Take the following examples: "Not all who mock their bonds are free." "All is not lost." "All lawyers are not formalists." Such propositions have the appearance of being universal, but they are not. In reality they are all *O* propositions. They are neither universal negatives nor uni-

versal affirmatives, but particular negatives. Thus, "Not all who mock their bonds are free," really means, "Some who mock their bonds are not free." "All is not lost," means, "Something is not lost." "All lawyers are not formalists," means, "Some lawyers are not formalists." Of course an *I* proposition, for example, "Some lawyers are formalists," is also implied in such forms of expression, but the central meaning is expressed in the particular negative proposition.

3. *Exclusive Propositions.* These are perhaps the most difficult to express in logical form. They are propositions introduced by such expressions as "none but" and "only." "None but the wise are good." "Only ignorant persons hold such opinions." In the same group belongs such a proposition as: "He jests at scars who never felt a wound." For here a part of the subject is written after the predicate, and the statement is equivalent to: "Only he who never felt a wound is one who jests at scars." Now to reduce such propositions to logical form three different methods may be employed: (*a*) Take the *opposite* of the subject as the subject of an *E* proposition, and leave the predicate as it stands in the original assertion. Thus, "None but the wise are good," becomes, "No non-wise are good." "Only ignorant people hold such opinions," becomes, "No non-ignorant persons are those who hold such opinions." "Only he who never felt a wound is one who jests at scars," becomes, "None of those who have felt wounds are those who jest at scars." (*b*) Another way to put exclusive propositions into logical form is to take the predicate as the subject of an *A* proposition, and qualify it by the original subject. That is to say, the original subject becomes the predicate of the *A* proposition which is taken as the correct logical form. Using the same examples, this method gives us: "All the good are wise," "All who hold such opinions are ignorant,"

and, "All who jest at scars are those who never felt a wound." (*c*) A third method of dealing with such propositions is to keep both the subject and the predicate the same and make two particular propositions: an *I* and an *O*. Thus: "Some of the wise are good," and, "Some of the wise are not good." "Some ignorant persons are those who hold such opinions," and, "Some ignorant persons are not those who hold such opinions." Now this method is the weakest for two reasons. In the first place, we really have no right to the *O* proposition. It may be true that, "All of the wise are good." We know only that some are. That some are not remains nothing but a possibility. The *I* proposition is really all we are entitled to take when we use this method, because the *A* and the *O* are either one possible. And in the second place, it is always better to get all you can out of an assertion in putting it into logical form. Assuming that an *I* or an *A* are both possible, we should use the method which will give us an *A* or universal proposition. For these reasons few logicians recognize this third method.

4. *Exceptive Propositions.* Another type of proposition is that introduced by such expressions as "all except" and "few, save." For example: "All of the members of Congress from Wisconsin except LaFollette and Berger, voted for the war." "Few, save the poor, feel for the poor." These are really negative: "LaFollette and Berger are not those who voted for the war." "Some (meaning most) non-poor are not those who feel for the poor." The student will observe that an exceptive proposition can easily be changed into an exclusive one: "Only LaFollette and Berger voted against the war." "Only the rich fail to feel for the poor."

While this is not an exhaustive division of the various types of propositions which have to be reduced to logical form the student who masters the reduction of these more

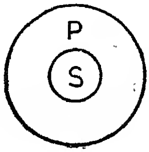
common and more difficult forms should have no trouble in dealing with the others. If he finds, for instance, *S* and *P* reversed, as in "Blessed are the merciful," he will at once recognize that this means, "All of the merciful are those who are blessed." Practice alone will make one proficient in comprehending the real logical meaning in a sentence in which it is not clearly expressed. And even though many students inevitably look upon such exercises as tedious and tiresome, and regard the intellectual labor expended in putting propositions in logical form as wasted energy, this is because they do not know how much it may be made to mean to them. Professor Mellone is unquestionably right when he says that such exercises, carefully performed, constitute one of the most valuable mental disciplines in the study of elementary logic.

The Distribution of S and P in A, E, I and O Propositions

The chief purpose of putting propositions into logical form may be said to be to make explicit the distribution of the subject and the predicate terms of each of the four types of propositions. In the case of the subject the distribution becomes obvious as soon as the proposition is put into logical form, but the rule is that *all universal propositions distribute their subjects, and all particular propositions leave their subjects undistributed.* It is not so easy, however, to see what the distribution of the predicate term is, because predicate terms are used in intension rather than in extension. But the rule for the predicate term is: *All negative propositions distribute their predicates and all affirmative propositions leave their predicates undistributed.* The ability to determine rapidly and surely the distribution of terms in propositions is so essential to an understanding of what follows that the student will do well to memorize these two rules. Note that the rule for the distribution of the subject is based

on *quantity,* whereas that for the predicate is based on *quality.*

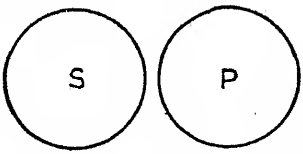
This matter of distribution is made much clearer by considering the formal relation of *S* and *P* in each of the four types of propositions. The great mathematician and logician, Euler, devised circles for this purpose. Figure



A = All *S* is *P.*

FIGURE I

I represents the *A* proposition. Since all of *S* falls within *P*, *S* is said to be distributed, in that all of the members of the class *S* are talked about in the proposition. *P* is undistributed because only those members of *P* which are

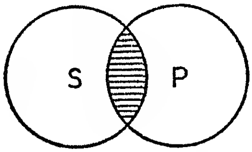


E = No *S* is *P.*

FIGURE II

also members of *S* are talked about. The part of *P* which falls outside of *S* is not under consideration in the *A* proposition. But in Figure II, which represents the *E* proposition, the whole of *S* and the whole of *P* are talked about, since all of *P* is asserted to fall outside of all of *S*. It is denied that any of the members of the class *S* are members of the class *P*. Now for all of the members of a class to be under consideration means that the term standing for that class is distributed. Figure III represents the *I* proposition. Here none of the members of the class *S* outside of class *P* is talked about nor are any of the members of the class *P* which are outside of the class *S*. The content

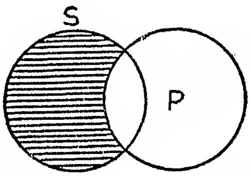
of the judgment is restricted to those members which are common to S and to P. Since only a part of S and only a part of P are under consideration both terms are undis-



$I =$ Some S is P.

FIGURE III

tributed. In Figure IV, however, all of P is talked about, because when we say that some S is not P we always mean that it is not any of P or not anywhere in P. Obviously S is here undistributed because we are only concerned with



$O =$ Some S is not P.

FIGURE IV

that part of S which is not in P. Hence the O proposition distributes only the predicate.

There is danger in using the circles of Euler to illustrate the relation of an object or instance to its class or genus or kind. This was referred to above in the discussion of the three different meanings of membership in a class. It must not be thought that the relation between a class and its members is extraneous or arbitrary, in the sense that the members can be separated from the class without affecting them or it. But if he is on his guard against this danger, the circles of Euler, by giving a visible representation to the formal relation of S and P in the four types of propositions, assist a beginning student in getting a firmer grasp of the meaning of distribution.

Exercise V

1. Designate each of the following propositions with its proper symbol (*A, E, I,* and *O*), and mark the distribution of each term with these symbols—*S-dis.* or *undis., P-dis.* or *undis.* Tell which of the universal propositions (*A* and *E*) are singular propositions.

(*a*) All communists are enemies of capitalists.
(*b*) Some enemies of capitalists are communists.
(*c*) No radios are radars.
(*d*) Some radars are not radios.
(*e*) The atomic bomb is a terrifying invention.
(*f*) World War II is not completely over.
(*g*) Some doctors are army officers.
(*h*) Some statesmen are immovable.
(*i*) No defenseless people are unafraid.
(*j*) All Angelinos are Californians.
(*k*) The Washington Monument is not in New York City.
(*l*) The conquest of Iwo Jima is a remarkable military achievement.

2. Put each of the following propositions into logical form and then designate each with the proper symbol and mark the distribution of each term.

(*a*) Only U.S. Marines fought at Iwo Jima.
(*b*) None but Japanese regulars defended Iwo Jima.
(*c*) All atomic bombs have not been exploded.
(*d*) All radioactive elements are not of military value.
(*e*) Few Americans understand Russians.
(*f*) Most Americans understand Englishmen.
(*g*) All except members of the orchestra must pay an admission fee.
(*h*) Only veterans are eligible for government loans.
(*i*) Not all enlisted women are Navy personnel.
(*j*) Home is not always the happiest place on earth.

3. In the following list distinguish between the disjunctive and the hypothetical propositions, and indicate the antecedent and the consequent of each hypothetical proposition.

(*a*) Either the Japanese had to be defeated, or they would have invaded our West Coast.
(*b*) If the Germans had won the Battle of the Bulge, they would have won the war.
(*c*) Plutonium is either an isotope of uranium, or a separate element.
(*d*) If plutonium is a separate element, it can be separated from uranium.

(*e*) If our statesmen are not alert, it will not be possible to prevent another World War.
(*f*) Either the Democratic or the Republican party will win the next election.
(*g*) If industrial strikes are not controlled, our economic system will be destroyed.
(*h*) Either the United States must support the United Nations, or there must be union now with England.
(*i*) If the national budget is not balanced, the national debt will impoverish the people.

The propositions listed under questions 1 and 2 in this Exercise may also be used to illustrate the various immediate inferences expounded in the next chapter. Give the opposites of each logical form. Convert each logical form which will yield a valid converse. Obvert each logical form. Give the partial and full contrapositive of each logical form from which it is possible to derive these immediate inferences. Give the partial and the full inverse of each of the universal propositions. Designate each logical form and each immediate inference with its proper symbol, namely, *A, E, I,* or *O.*

CHAPTER VII

THE IMMEDIATE INFERENCES

Definition of Immediate Inference

There are a number of ways of interpreting a proposition which aid in bringing to light its underlying implicative system. These various processes are called *immediate inferences.* Taking a single proposition, these processes make divers changes in quantity or in quality or in the order of S and P for the purpose of expressing the underlying implicative system in as many different ways as possible. If the process or processes are correctly performed, the additional propositions which are thereby obtained have as much truth as the original proposition. Some logicians refer to these processes as purely formal interpretations of the meaning of a proposition, and hold that they add nothing to our knowledge. But since it is their purpose to make explicit the different aspects of the implicative system which is embodied in the proposition, they are not really entirely formal, but frequently aid the thinker in getting a better grasp of the real meaning of a given proposition. In giving a new formulation to a proposition, there is no question but what the underlying implicative system is set in clearer relief.

There are seven immediate inferences: (1) Immediate inference by opposition. (2) Conversion. (3) Obversion. (4) Contraposition. (5) Inversion. (6) Immediate inference by added determinants. (7) Immediate inference by complex conception. The first three are the most important. Contraposition and inversion are combinations of conversion and obversion. The sixth and seventh are merely verbal and are easily explained.

Opposition and Immediate Inference by Opposition

In order to explain what is meant by immediate inference by opposition we must first understand the meaning of opposition. Propositions are said to be opposed when they have the *same subject and predicate but differ in quantity or in quality or in both.* When they differ only in quantity they are called *subalterns,* when they differ only in quality they are called *contraries* or *sub-contraries,* *A* and *E* being called contraries and *I* and *O* sub-contraries, and when they differ in both quantity and quality, they are called *contradictories.* A singular proposition has only one opposite, called the contradictory. The ancient *square of opposition* will make these relations clearer.

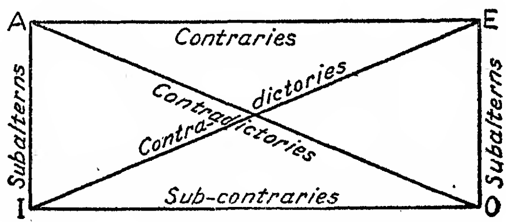


FIGURE V

A. All men are mortal.
E. No men are mortal.
I. Some men are mortal.
O. Some men are not mortal.

We are now in a position to understand immediate inference by opposition. We get such an immediate inference when we assume any one of the four opposite propositions, *A, E, I* or *O,* to be either true or false and draw a conclusion as to the truth, falsity or doubtfulness of the other three. If *A* is assumed true we know that *I* is true because what is true of all must be true of some, but we also know that *E* and *O* are false because what is affirmed of all cannot be denied either of all or of some without

violating the law of contradiction. But if we assume *A* to be false, then only *O* is true since *I* or *E* may be either true or false. Hence we say they are doubtful. The student can work out the truth, falsity or doubtfulness of the opposites when any one of the propositions is assumed true, from Table I, and, when any one is assumed false, from Table II.

TABLE I

	A is	*E* is	*I* is	*O* is
If *A* is true	true	false	true	false
If *E* is true	false	true	false	true
If *I* is true	doubtful	false	true	doubtful
If *O* is true	false	doubtful	doubtful	true

TABLE II

	A is	*E* is	*I* is	*O* is
If *A* is false	false	doubtful	doubtful	true
If *E* is false	doubtful	false	true	doubtful
If *I* is false	false	true	false	true
If *O* is false	true	false	true	false

Only contradictory propositions are strict opposites in the sense that when one is true the other is false, and *vice versa*. This may be expressed in a practical maxim: "To prove a given proposition false, establish the truth of its contradictory." Thus if you said that, "All of the football playing of Jimmie Johnson is stellar playing," you would be refuted by any one mentioning a single poor play made by Jimmie Johnson. On the other hand, if you said, "Some of Jimmie Johnson's football playing is stellar," you could only be refuted by the establishment of the universal negative: "None of his playing is stel-

lar.'' A common failing of naïve and inexperienced thinkers is the making of hasty and loose generalizations. Those who form the habit of measuring the length and breadth of their assertions avoid making sweeping universal statements which are not warranted by the actual facts, and for this reason their opinions carry more weight. They are said to be more judicial. Here is a good practical lesson to be learned from the study of immediate inference by opposition.

Conversion

To convert a proposition is to interchange the subject and predicate. The original proposition, which is known as the *convertend,* must first be put in strict logical form. Its converse is obtained by transferring all of *S* to the position of the predicate and all of *P* to the position of the subject. The quality of the converse is always the same as that of the convertend, that is, negatives convert into negatives and affirmatives into affirmatives. Thus: ''No bacteria are harmless,'' converts into: ''No harmless things are bacteria.'' And: ''Some elements are composed of helium atoms,'' converts into: ''Some things composed of helium atoms are elements.'' Assuming the convertend to be true, the truth of the converse follows.

There are two kinds of conversion—*simple conversion,* examples of which were just given, and *conversion by limitation,* or, as the older logicians called it, *conversion per accidens.* The distinction between the two kinds is made necessary by the rule for conversion: No term can be distributed in the converse which is undistributed in the convertend. Perhaps a clearer way of stating this rule is that one is forbidden to refer to all of the members of either *S* or *P* in the converse, if only some of their members are referred to in the convertend. One can pass from all in the convertend to some in the converse, but *one can-*

not pass from some in the convertend to all in the converse. Simple conversion means keeping the proposition the same in quantity, and this is possible for *I* and *E* propositions because the distribution of both *S* and *P* is the same in each of these two types of propositions. That is to say, in *I* both terms are undistributed and in *E* both are distributed. Hence there is no danger of changing the distribution when *S* and *P* are reversed. Note that the examples given above are *I* and *E*. Now, since a singular term is always distributed, the singular *A* is also converted by simple conversion. For example: "Socrates was the wisest man in Athens," converts into: "The wisest man in Athens was Socrates."

Conversion by limitation or *per accidens* applies only to the universal A propositions, that is, to *A* propositions having more than one member in *S* and *P*. For example: "All primitive people are believers in magic," converts into: "Some believers in magic are primitive people." When *P* becomes the subject of the converse it has to be limited and the proposition becomes *I*, for in the convertend *"believers in magic,"* being the predicate of an affirmative proposition, is undistributed. It means only some believers in magic and this undistributed sense must be retained in the converse. Now for the same reason that A (universal as distinct from singular) converts by limitation, *O* cannot be converted. For *S* is undistributed in *O*, and when it becomes the predicate of the converse it necessarily becomes distributed as the predicate of an *O* proposition. Hence the conversion of *O* is always a violation of the rule. For example: "Some swans are not black," would necessarily become: "Some black things are not swans," and we have no right to pass from some swans in the convertend to all swans in the converse, for we have already made it clear, with the circles illustrating distribution at the end of the last chapter, that

the predicate of an *O* proposition includes all of its members. But note that the singular *E*, which also has the not after the copula and consequently resembles an *O* in form, can be converted by simple conversion. For example: "This bird is not the bird you intended to kill," converts into: "The bird you intended to kill is not this bird." Here, however, the subject, *this bird*, refers only to one object and is therefore distributed. It is this fact which makes the proposition a singular *E* instead of an *O*. Now it goes without saying that a universal *E* can be converted by limitation, since what has been denied of all can be denied of some.

There is an indirect method of converting *O*. It can first be obverted into *I*, and then the *I* can be converted by simple conversion. This is sometimes called *conversion by negation*, but it is really equivalent to the partial contrapositive and this cannot be understood until obversion is explained.

Obversion

It will help to make immediate inference by obversion clear if we remember that four of each of the four types of propositions are possible, when we take account of both the positive terms and their corresponding negatives or opposites. This will be obvious from Table III, in which the four possibilities for *A, E, I* and *O* propositions are exemplified. The student must note that an *A* can have a negative subject and a positive predicate without ceasing to be an affirmative proposition, and that an *E* can have a positive subject and a positive predicate, and still be a negative proposition. Understanding obversion is absolutely dependent upon grasping this distinction between a term being negative and a proposition being negative. Table III is intended to bring this distinction into clear relief.

TABLE III

Quality of Proposition	Proposition		Quality of Term
POSITIVE	*A.* All *S* is *P*. *All men are mortal.*	1	*S* positive *P* positive
	A. All non-*S* is *P*. *All non-men are mortal.*	2	*S* negative *P* positive
	A. All non-*S* is non-*P*. *All non-men are non-mortal.*	3	*S* negative *P* negative
	A. All *S* is non-*P*. *All men are non-mortal.*	4	*S* positive *P* negative
NEGATIVE	*E.* No *S* is *P*. *No men are mortal.*	1	*S* positive *P* positive
	E. No non-*S* is *P*. *No non-men are mortal.*	2	*S* negative *P* positive
	E. No non-*S* is non-*P*. *No non-men are non-mortal.*	3	*S* negative *P* negative
	E. No *S* is non-*P*. *No men are non-mortal.*	4	*S* positive *P* negative
POSITIVE	*I.* Some *S* is *P*. *Some men are mortal.*	1	*S* positive *P* positive
	I. Some non-*S* is *P*. *Some non-men are mortal.*	2	*S* negative *P* positive
	I. Some non-*S* is non-*P*. *Some non-men are non-mortal.*	3	*S* negative *P* negative
	I. Some *S* is non-*P*. *Some men are non-mortal.*	4	*S* positive *P* negative
NEGATIVE	*O.* Some *S* is not *P*. *Some men are not mortal.*	1	*S* positive *P* positive
	O. Some non-*S* is not *P*. *Some non-men are not mortal.*	2	*S* negative *P* positive
	O. Some non-*S* is not non-*P*. *Some non-men are not non-mortal.*	3	*S* negative *P* negative
	O. Some *S* is not non-*P*. *Some men are not non-mortal.*	4	*S* positive *P* negative

This table shows that there are sixteen propositions possible when we take into account the various differences in quality of the two terms. These differences are important in obversion, since we have to substitute non-*P* for *P* and *P* for non-*P*, as the case may be. For example, A1 obverts into E4.

In order to obvert any proposition, *we must first change the quality of the proposition,* from *A* to *E* or *vice versa,* or from *I* to *O,* or *vice versa,* and then *substitute for the predicate of the original proposition its contradictory,* that is to say, the *term* having the opposite quality. If in the original the predicate term is positive we substitute non-*P* in the obverse and *vice versa.* Obversion, then, may be defined as *asserting the same meaning in the opposite quality.*

Consider carefully the following examples of each of the four propositions in which the predicate term is positive:

A. All good clothes are expensive.
Obverse *E.* No good clothes are non- or inexpensive.

E. No bad eggs are edible.
Obverse *A.* All bad eggs are non- or inedible.

I. Some people are normal.
Obverse *O.* Some people are not non- or abnormal.

O. Some mushrooms are not poisonous.
Obverse *I.* Some mushrooms are non-poisonous.

Since, in each pair of propositions, one is the obverse of the other, the above examples serve equally well to illustrate the process for propositions having negative predicate terms, if the first of each pair is treated as the obverse of the second, instead of the second being treated as the obverse of the first. *I* and *O* usually cause the most difficulty. When an *O* has a positive predicate it obverts by taking the *not* out of the proposition and putting it into the predicate term. When the predicate is a single word, this is done by prefixing *non* with a hyphen to the term. In other cases the word not is put into a relative clause. Thus: "Some men are not those who believe in

peace at any price,'' becomes in the obverse: ''Some men are those who do not believe in peace at any price.'' These two propositions are identical in meaning, but the first is an *O* and the second is an *I* proposition. The first has a positive predicate and the second has a negative predicate. It is important that the student grasp this significant distinction between these two statements. Now when the *I* with a positive predicate is obverted *two negatives* are necessary. One of these changes the proposition from positive to negative form and is written immediately after the copula. The other changes the predicate into a negative term. It is either prefixed to or put inside of the predicate in such a way that other words stand between it and the not just after the copula. Take this example: ''Some barns are suitable for dairy cows,'' of which the obverse is: ''Some barns are not non- or unsuitable for dairy cows,'' or it may also be written: ''Some barns are not barns which are not suitable for dairy cows.'' The most natural form would be the first with unsuitable taken as the opposite of suitable. Sometimes, however, the second form is necessary because prefixing the *non* or *un* may make just the word to which it is prefixed negative instead of the whole of the predicate term.

Contraposition

Immediate inference by contraposition does not involve anything different in principle from obversion and conversion. The *partial contrapositive* is obtained by converting the obverse. The *full contrapositive* takes another step, and obverts the converse of the obverse of the original. There are thus two steps in reaching the partial contrapositive, and three in reaching the full contrapositive. If we start with *A*, we get by obversion *E*, by simple conversion *E* (partial contrapositive), and by obversion again *A* (full contrapositive). If we start with *E* we get by

obversion *A*, which converts by limitation into *I*, and obverts into *O*. However, if we start with a singular *E*, we can reach a singular *E* for the full contrapositive. If we start with *I* we get by obversion the *O* proposition which cannot be converted because of the rule of distribution. There is, then, no contrapositive for *I* for the same reason that there is no converse of *O*. But if we start with *O*, we get by obversion *I*, by simple conversion *I* and by obversion again we get *O*. This can be expressed as follows: *AEEA, EAIO, IO—, OIIO*. Taking the examples given above under obversion, let us derive the full contrapositive:

	A.	All good clothes are expensive.
Obverse	*E*.	No good clothes are inexpensive.
Partial	*E*.	No inexpensive things are good clothes.
Full	*A*.	All inexpensive things are things which are not good clothes
	E.	No bad eggs are edible.
Obverse	*A*.	All bad eggs are inedible.
Partial	*I*.	Some inedible things are bad eggs.
Full	*O*.	Some inedible things are not things which are not bad eggs.
	I.	Some people are normal.
Obverse	*O*.	Some people are not abnormal.
Partial		(Not obtainable because O cannot be converted).
Full		(Cannot be reached because no partial to obvert).
	O.	Some mushrooms are not poisonous.
Obverse	*I*.	Some mushrooms are non-poisonous.
Partial	*I*.	Some non-poisonous things are mushrooms.
Full	*O*.	Some non-poisonous things are not non-mushrooms.

Facility in reaching the full contrapositive of a statement comes only with practice. The student must remember to begin with obversion, and in converting, *all* of the subject and *all* of the predicate must be transposed. The

full contrapositive can be obtained directly by taking for the subject non-*P* of the original, and for the predicate non-*S* of the original, and making a proposition of the same *quality* as the original. When this is done the student must not forget that when the original is *E* universal, the full contrapositive is *O*, due to the fact that the obverse of *E* universal, being *A* universal, converts by limitation into *I*. After deriving the contrapositive by going through the separate steps, practice deriving it directly.

Inversion

The inverse of a proposition is one having non-*S* for its subject and the same predicate as in the original. It is only obtainable when the original proposition is universal, either *A* or *E*. *I* and *O* do not yield an inverse. It is reached by using conversion and obversion in succession until the desired form appears. If the original proposition is *A* we begin with obversion, then convert, then obvert, then convert, then obvert, so that we have five steps. In this case we take the full contrapositive of *A* and convert it into *I*, and then obvert the result into *O*. Using the example given above, we get the following steps:

	All good clothes are expensive.
Obverse	No good clothes are inexpensive.
Partial contrapositive	No inexpensive things are good clothes.
Full contrapositive	All inexpensive things are things which are not good clothes.
Converse of full contrapositive and full inverse	Some things which are not good clothes are inexpensive.
Obverse of this and partial inverse	Some things which are not good clothes are not expensive.

In this partial inverse there is a change in distribution of the term expensive. Only *A* singular avoids this.

If we start with *E* we have to begin with conversion,

then obvert and then convert again in order to get the inverse. Using the example given above:

	No bad eggs are edible.
Converse	No edible things are bad eggs.
Obverse	All edible things are things which are not bad eggs.
Converse and partial inverse	Some things which are not bad eggs are edible.
Obverse and full inverse	Some things which are not bad eggs are not inedible.

Immediate Inference by Complex Conception and by Added Determinants

These are not especially important forms of immediate inference. We obtain an immediate inference by complex conception by using the subject and the predicate of the original proposition to qualify the same word in the inferred proposition. Thus: "Logic is useful," might be made more complex by saying: "Logical writings are useful writings." This would be an example of an immediate inference by complex conception. Added determinants qualify both the subject and the predicate with the same words, and claim that the truth of the original implies the truth of the new proposition. Thus: "A Negro is a man," might become: "A badly persecuted Negro is a badly persecuted man." This would be true, but most immediate inferences by added determinants and by complex conception are false. It depends altogether on the content of the proposition. Thus, to borrow an illustration from Joseph, if it is true that, "A shark is not a mammal," it is also true that, "The anatomy of a shark is not the anatomy of a mammal," but it is not true that, "The food of a shark is not the food of a mammal." It cannot be too much emphasized that real inference is not a purely formal process, but takes into consideration

the content as well. And, as Joseph points out, the real value of immediate inference by complex conception, and by added determinants, is to be found in the fact that they show with great clearness how important the content is in determining the truth of a proposition obtained by these processes. The other forms of immediate inference are far more formal, as is shown by the fact that they are readily obtained when symbols, as well as when concrete concepts, are used as subject and predicate. On the other hand, it is not possible to use the symbols (S and non-S and P and non-P) in immediate inference by complex conception, and by added determinants.

Summary of Results

It is convenient and helpful to sum up the immediate inferences in the following table, which is taken from the fourth edition of J. N. Keynes's *Studies and Exercises in Formal Logic,* p. 140. S' and P' are equivalent to non-S and non-P.

TABLE IV

		A	I	E	O
I	Original proposition	SaP	SiP	SeP	SoP
II	Obverse	SeP'	SoP'	SaP'	SiP'
III	Converse	PiS	PiS	PeS	
IV	Obverted Converse	PoS'	PoS'	PaS'	
V	Partial contrapositive ...	$P'eS$		$P'iS$	$P'iS$
VI	Full Contrapositive	$P'aS'$		$P'oS'$	$P'oS'$
VII	Partial Inverse	$S'oP$		$S'iP$	
VIII	Full Inverse	$S'iP'$		$S'oP'$	

Note that the table does not cover the exceptions necessary for singular propositions, nor does it include immedite inferences by complex conception and by added determinants. But it does include the obverted converse.

which is the second step in the inversion of an *E*. (See the example on page 101.) Note that the obverted converse of a singular *A* proposition would be a singular *E*, and not an *O*.

Exercise VI

1. Give the opposites of each of the following propositions. Assume the original true and state what follows about each of the other opposites. Assume the original false and state what would follow about each of the other opposites.

(*a*) All atom bombs are dangerous explosives.
(*b*) No radioactive elements are worthless.
(*c*) Some radars are defective.
(*d*) Some radarscopes are not easily read.
(*e*) The Battle of Leyte Gulf was one of the decisive naval battles of the War in the Pacific.
(*f*) The Volcano Islands are not the most important islands in the Pacific.

2. First convert each of the following propositions which can be converted and explain why those you think cannot be converted cannot be. Then obvert the converse of each. Designate each proposition with its proper symbol and mark the distribution of *S* and *P* in each proposition.

(*a*) Some war relics are dangerous objects.
(*b*) No hospitals are pleasant places.
(*c*) Some insurance companies are insolvent.
(*d*) Some statesmen are not unhampered.
(*e*) The United Nations is the successor to the League of Nations.
(*f*) The League of Nations is not functioning.

3. First obvert, then give the partial contrapositive if possible (if not, explain why not), then give the full contrapositive of each of the following propositions. In this list there is one of each of the sixteen types given in Table III, p. 96. Designate each type, for example, an *A* of the *S; P* type, or of the Non-*S; P* type or of the Non-*S;* Non-*P* type or of the *S;* Non-*P* type, and so on for the four *E* types, the four *I* types, and the four *O* types.

(*a*) All sailors are skilful.
(*b*) No wine is unintoxicating.
(*c*) Some unwise men are army officers.
(*d*) Some friendly girls are not Red Cross workers.

(*e*) All soldiers are unafraid.
(*f*) No admirals are generals.
(*g*) Some rockets are not unautomatically propelled.
(*h*) Some guns are unmounted.
(*i*) Some ships are airplane carriers.
(*j*) No unfavorable reports are uninteresting.
(*k*) Some discharged servicemen are unhappy.
(*l*) All inharmonious music is unappreciated.
(*m*) All non-commissioned officers are enlisted personnel.
(*n*) No ungrateful persons are those who escaped death.
(*o*) Some unmolested persons are not innocent.
(*p*) Some unwarranted beliefs are not irreligious beliefs.

4. Taking the first proposition in each of the following lists as the original, state what each successive proposition is in relation to the original. Designate each proposition with its proper symbol and mark the distribution of *S* and *P* in each proposition.

List A

(*a*) No ideologies are unphilosophical concepts.
(*b*) Some unphilosophical concepts are not ideologies.
(*c*) No unphilosophical concepts are ideologies.
(*d*) All ideologies are philosophical concepts.
(*e*) Some ideologies are unphilosophical concepts.
(*f*) All ideologies are unphilosophical concepts.

List B

(*a*) All fissionable materials are radioactive elements.
(*b*) Some radioactive elements are fissionable materials.
(*c*) No fissionable materials are non-radioactive elements.
(*d*) All non-radioactive elements are non-fissionable materials.
(*e*) No non-radioactive elements are fissionable materials.
(*f*) Some fissionable materials are not radioactive elements.

List C

(*a*) Some radars are not used in airplanes.
(*b*) All radars are used in airplanes.
(*c*) Some instruments that are not used in airplanes are radars.
(*d*) Some radars are instruments that are not used in airplanes.
(*e*) Some instruments that are not used in airplanes are not non-radars.

5. Select two *A* and two *E* propositions from any of the above lists, and derive the partial and full inverse of each.

SECTION III

INFERENCE AND THE SYLLOGISM

CHAPTER VIII

THE NATURE OF INFERENCE AND ITS RELATION TO THE SYLLOGISM

The Nature of Inference

Nothing in the experience of educated people is more common than inferences. They make up the very warp and woof, the fabric and texture of the intellectual life. Every book, magazine and newspaper you read is filled with inferences. Every conversation, public discussion, debate, address, sermon, and classroom lecture is largely constituted out of them. And practical success in life depends more than anything else on the ability to draw the right inference at the right time. In short, examples of inference are innumerable, but when we attempt, as we must in logic, to turn attention to the intensive meaning of the term inference, instead of toward its extensive meaning, there is no term in the whole science of logic more difficult to analyze or more baffling to define. There are, however, two controversies among contemporary logicians which afford fairly good pathways into the nature of inference.

1. In explaining the immediate inferences I pointed out that many logicians hold that they are not real inferences. To take a specific example, from the proposition: "No insane people are normal," may be inferred by obversion: "All insane people are abnormal." Most present-day logicians hold that this is not a genuine inference, because it tells us no more about insane people than we were told in the original proposition. Inference, they say, involves *novelty,* movement of thought from what we know to what we do not know. Thus, inference is held to be a movement of thought from old facts to new facts. It is the bridge

which the mind builds across the chasm which separates the realm of human knowledge from the realm of human ignorance. "What we know," said Laplace, "is little, but what we do not know is immense," and inference is the intellectual process by which we add to the little we know by biting off bits from the immensity of the universe which we do not know. But the so-called processes of immediate inference all move around in the realm of what we already know. At best they are simply *interpretations* of what is already known. "It is essential to inference that there shall be a real transition from one fact to another—that the conclusion reached shall be different from the starting point" (Creighton). But in the immediate inferences "the new proposition is the result of a verbal interpretation of the original one, and restates the same fact in a different way" (Creighton). Hence they exhibit no real movement to a new fact, but only clarify and bring the mind to a fuller realization of the meaning and bearings of the original proposition. It should be pointed out that calling the immediate inferences interpretations, instead of real inferences, does not get us anywhere unless the distinction between interpretation and inference is made sharp and precise. For at least one eminent logician has used the term interpretation to designate the very highest form of inference.[1]

However, when we fall back on our conception of the inferential whole or implicative system as an underlying functional unit behind the two statements involved in an immediate inference, and of which each is an aspect or fragment, we have to insist that real inference is exhibited in the immediate inferences, however insignificant it may be. For it is by means of this underlying system that we

[1] See Josiah Royce's article entitled "Mind," in Hastings' *Encyclopedia of Religion and Ethics*, from which the quotations below are taken by permission of Charles Scribner's Sons.

are able to pass from the original proposition to its obverse or converse or contrapositive. Hence, in this controversy, I agree with Rieber, who justly condemns definitions of inference which say that the conclusion must be *another*, or a *new*, or a *different*, or a *fresh* proposition, without making clear what is meant by *otherness, newness, difference* or *freshness*, since these terms are full of ambiguity. He seems to me to be entirely right when he says:

> "The immediate inferences have been so denominated because thought seems to pass from one judgment to another without the assistance of a middle term. But this conventional distinction between mediate and immediate inference is sadly defective in fundamental insight. A common ground is required quite as much in immediate as in mediate inference, as a means of bridging the gaps between the two judgments. We pass from one particular judgment to another particular judgment only because both are embedded in a universal. All thought is from particular to particular via the universal; and, moreover, the certainty and value of the conclusion in any form of reasoning—immediate or mediate—depends upon the grip we have upon the universal."[2]

It is this universal to which I have so often referred under the name of the inferential whole or implicative system. Passing from one aspect or phase of such an underlying system to another phase or aspect, by means of the pervading unity which orders these and all elements of that system, is what is meant by inferring.

2. Now let us approach the nature of inference from the controversy initiated by William James' famous distinction between knowledge by acquaintance and knowledge by description.[3] The former is supposed to be non-inferential

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[2] Charles S. Rieber, *Footnotes to Formal Logic*, p. 111.

[3] The history of these terms is obscure. James evidently did not coin them, because they are found in the form, "knowledge of acquaintance," and "knowledge about," in J. Grote's *Exploratio Philosophica*, Part I, p. 60, from which they were taken by Joseph (*An Introduction to Logic*, p. 68). Royce, in the article entitled "Mind,"

and the latter is inferential. "In the simplest possible case one who listens to music has 'knowledge of acquaintance' with the music; the musician who listens in the light of his professional knowledge has not only 'knowledge by acquaintance,' but also 'knowledge about'; he recognizes what changes of key take place and what rules of harmony are illustrated. A deaf man who has learned about the nature of music through other people, in so far as they can tell him about it, but who has never heard music, has no 'knowledge by acquaintance,' but is limited to 'knowledge about.' Knowledge of acquaintance is also sometimes called 'immediate knowledge.'" (Royce.)

The distinction, which goes back to Aristotle, between intuitive and demonstrative knowledge represents another attempt to distinguish a kind of knowledge which is non-inferential from inferential knowledge. Certain basic postulates or axioms are held to be self-evident or to be known intuitively without inference. Then other conclusions are drawn from these postulates, and inference is defined as this process of drawing conclusions from self-evident or intuitive principles.[4]

There are, then, according to many modern logicians, two kinds of non-inferential knowledge which may serve as the starting point for inference. In this way logicians attempt to lift themselves by their own bootstraps by setting up non-inferential types of knowledge to use for the

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in Hastings' *Encyclopedia of Religion and Ethics,* attributes the terms to James. But the distinction itself is at least as old as Hobbes' *Leviathan.* He there said: "There are of knowledge two kinds; whereof one is knowledge of fact, and the other knowledge of the consequence of one affirmation to another. The former is nothing else but sense and memory, and is absolute knowledge. . . . The latter is called science and is conditional." Ch. IX, and elsewhere. Bertrand Russell uses the expression "knowledge by description" for "knowledge about." *Problems of Philosophy,* Ch. V.

[4] For a discussion of axioms and the laws of thought, see Ch. XXVII.

purpose of defining inference. For however valuable the distinction between "knowledge by acquaintance" and "knowledge about" may be for psychological purposes, it is likely to be misleading in logic. All knowledge is inferential. The starting point for inference is always some fact or bit of knowledge, but this is itself through and through inferential. The postulates of thought are themselves the great structural connecting links in the whole system of knowledge. We reach them by inference and use them in further inferences. All thinking is inferential. The proof for this is that by no possible means can the inferential element be taken out of knowledge without knowledge itself being thereby destroyed. As Royce rightly insists: "In the actual cognitive process of the individual human being, 'knowledge by acquaintance' never occurs quite alone, since, when we know something perceptually or by acquaintance, we also always have more or less 'mediate' knowledge. For example, one who listens to music, but who also considers the person of the artist, the relation of the music to the programme, the name of the composer, or the place of this experience in his own life, has in his knowledge that which is more than the immediate hearing of the music." And that something more is the universal, the inferential whole or implicative system which is the very life blood of knowledge and the vital essence of inference.

Immanuel Kant defined inference as "that function of thinking by which one judgment is derived from another." This is not a bad definition if we keep in mind that every known fact is itself a judgment grounded in other judgments. In other words, every known fact is an implicative system. The process of mind or the movement of thought by which we pass from one element to another by means of our knowledge of their relation to the system is inference. Therefore, the most important element in inference is the

pervading universal which controls all of the elements of which it is constituted. "The notion of inference seems to demand on the one hand that there shall be difference between the concrete fact reasoned from and the concrete fact reasoned to; and, on the other hand, that there shall be—whether dimly or clearly seen—a universal which connects them."[5]

This definition brings out very well the two essential elements in inference.

The meaning of inference, then, must be looked for in a pervading universal or order system. But what is a pervading universal? What is an order system? Given a collection or assemblage of individual objects of any kind whatsoever, be they existential such as an army, or purely conceptual such as the whole numbers of arithmetic, it is said to be an order system or an *array*, if a thinker is able, from a knowledge of qualities and relations of some of the members of the collection, to tell something about the characters of some or all the other members, without having to examine each separately and individually. Orderliness is possessed by any group of objects of which a knowledge of some enables one to pass to a knowledge of others in the group. This is the simplest and the most general definition of an order system recognized by modern logicians. As Royce puts it: "Order belongs to sets of individuals, to collections, to arrays of things, persons, deeds, or events." When a person is sure he is dealing with such an order system, it is possible for him to select and to concentrate his attention upon a relatively small portion of the facts involved in the system in question, and to ignore other portions; since he can use the information he has about those he studies as a basis of inference as to the nature of some or all of the others in the system. Consequently, in-

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[5] A. Sidgwick, *The Use of Words in Reasoning*, p. 305.

ference is impossible apart from such an order system. Indeed, the maxim, "Order is Heaven's first law," is literally true. The whole universe, whether viewed as a physical or space-time world or as a conceptual or ideal world, must be thought of as swarming with a great variety of order systems, and as forming an order system itself. "Order is known to us through inference; that is, the orderly is that which corresponds, in the real or the ideal world, to what we infer when we systematically draw conclusions. . . . If order is only one aspect of the spiritual world, it is an indispensable aspect. Without it life would be a chaos and the world a bad dream. Loyalty would have no cause and human conduct no meaning."[6]

It is, then, order systems which make inference possible. Inference is the burrowing of the human mind into the heart of such systems. Take, for example, the following newspaper account of how a defective boy thought out for himself, independently of his teacher, who had always regarded him as an absolute imbecile, the way to construct an ordinary footstool.

When eighteen months old, spinal meningitis left Vincent deaf, dumb and blind. At Randall's Island he was pronounced an imbecile, and as such was sent to the Port Jefferson Home. Vincent had been at the Home about one year when Sister Augustine became convinced that the boy was not an imbecile. Sister Augustine taught the class of blind children, and each day Vincent sat near her desk in the classroom. One day he was playing with three sticks of wood, and getting three pins from the teacher, made the homely model of a footstool. Taking Sister Augustine by the hand, he led her to his room in the Home and there placed his model beside the stool in his room. This little action convinced Sister Augustine that the boy was not an imbecile.

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[6] Josiah Royce, in the article entitled "Order," in Hastings' *Encyclopedia of Religion and Ethics.* Quoted by permission of Charles Scribner's Sons.

And well she might be convinced, for the boy had demonstrated his ability to penetrate into an actual implicative system, and he who can do this is far from being an absolute imbecile. In a short time she had taught him to read, to write and to talk, and each of these activities involved his comprehending still other order systems.

A noted logician has recently predicted that this new conception of the nature of inference, as elaborated in the logical writings of the late Bernard Bosanquet, is destined to revolutionize the science of logic.[7] However that may be, it is in favor of the theory that a similar conception has been reached more or less independently by some of the ablest logicians of our day, among whom may be mentioned a Russian—N. Lossky, a German—W. Windelband, an Italian—B. Croce, an Englishman—F. H. Bradley, and two Americans—Josiah Royce and John Dewey. To be sure, there are important differences in the detailed working out of the conception of inference of each of these logicians, and the latter, in particular, is quite different from the others, but generally speaking, their theory is consistent with the doctrine of the implicative system here presented.[8] Bosanquet has rightly characterized the older theory of inference, which continues to dominate the logical writings of some contemporary logicians, as a *linear theory*. In contrast, the new theory may be called an *organic theory*. According to the linear view, knowledge advances

[7] John H. Muirhead, *Mind*, Oct., 1923. See my Preface.

[8] For references, see the Bibliography in the Appendix. The differences between Dewey and the idealistic logicians are too technical to introduce here. The essential point is that Dewey confuses the strictly logical process of scientifically analyzing actual implicative systems with the temporal and practical process of intelligently meeting a given situation. Although I have elsewhere (*Journal of Philosophy*, April, 1917) criticized Dewey's theory of practical judgments, I think that he has rendered a great service to logical science by his insistence upon the inferential character of all knowledge and by his steadfast refusal to adopt a dualistic position.

"precept upon precept, line upon line, here a little, there a little," like a long pavement which is extended indefinitely by placing one brick or stone next to another. For on this theory an inference is not a living unity, in which the various elements interpenetrate and reciprocally modify one another, but a dead connection of static blocks which man's mind has abstracted from the stream of experience and crystallized. According to the organic theory, the whole of human knowledge is a living organism, and each system within it is comparable to a living cell. Growth in knowledge consists in a further penetration of man's mind into the actual order systems of nature, and continues until the human inferential whole is worked into an identity with this actual order system. Hence, all knowledge is inferential in this sense, and the implicative system is the functional unit of logical science.

The Relation of Inference to the Syllogism

The *syllogism* is Aristotle's greatest contribution to logic. It is his technical formulation of the procedure which he thinks is always followed in demonstrative, as distinct from intuitive, reasoning. For him, then, *all inference, when logically formulated, should take the form of the syllogism.* We shall make a detailed analysis of syllogistic reasoning in the next two chapters. All that it is necessary to do here is to explain briefly what a syllogism is in order to deal with the question as to whether all inference must take this form.

A syllogism is a set of three propositions, two of which are called premises because they contain a common term, and the third the conclusion which constructs a new proposition out of the two terms not common to the premises. The reason the third proposition is called the conclusion, is that it is supposed to be obtained by means of the comparison made in the premises between the two terms of the

conclusion and the common term, which is always called the *middle term*. The real unifying principle in the syllogism is the middle term, since the terms put together to form the conclusion can only be so combined because each has already been related to the common term in the premises. The following is an example of a syllogism:

All men are mortal.
Enoch is a man
Enoch is mortal.

Here the predicate, mortality, is affirmed of Enoch because it has already been asserted of all men, and Enoch has been qualified by the predicate man. Enoch and mortality are two aspects of the inferential whole here called man. And we only know Enoch to be mortal on the ground that both Enoch and mortality are essential elements in the order system called man. In this sense, then, the syllogism does represent the actual procedure involved in inference. It was probably for this reason that Hegel said that all things are syllogisms. What he meant was that all things are implicative or inferential systems, and that we can pass from one to another part of an order system only by following out the actual relations in the system. Only to this extent can we agree with the theory that all inference is syllogistic. The syllogism can be interpreted in terms of the principle of the implicative system. As Bosanquet has rightly said:

> The syllogism at its best is not a mere marshaling of trains of predicates, which remain apart and unmodified. The syllogistic process, properly understood, and taken in instances which reveal its full import, is an operation in which the terms come together, modify one another, and construct a systematic whole, within which the conclusion is obvious and explains itself. If you say, "Oxygenated blood is bright; the blood in the arteries is oxygenated blood, therefore the blood in the arteries is bright," you have brought together your terms in the conception of the circulation of the blood, and your conclusion shows a system in which

the terms are factors, their union is rationally explained, and their meaning developed. Such a term as *bright* acquires a new meaning in the construction.[9]

In other words, the real implicative system behind the middle term *oxygenated blood* is the circulatory system of the human body, so that this is an excellent illustration of the way in which syllogistic inference depends upon an ordered system of facts.

Yet it does not follow that all thinking should be thrown into syllogistic form. The attempt to do this is what vitiated scholastic logic in the Middle Ages. So much effort was wasted in reducing thought to syllogistic form that thinking itself became sterile. It had to be rejuvenated by the great scientists, such as Galileo and William Gilbert. Surely, even a formal logician must admit that the important thing is that men should go forward with their thinking into the actual concrete order systems of the world. The demand that each step in the advance of a scientific investigation be given a technical syllogistic formulation is absurd. But although this is true, the student of logic will find that a full comprehension of syllogistic logic aids one in reaching an insight into the real meaning of inference, and an understanding of the place in inference of the universal or underlying system which comes to expression as the middle term of the syllogism. Such a study will also pave the way for a consideration of other forms of inference.

In the syllogism, the logician has an ideal and abstract order system which is very remote from the concrete world of the physical, biological and social sciences, and more like the conceptual world of mathematics. This may be expressed by saying that in the system which a syllogism

[9] Bernard Bosanquet, *Implication and Linear Inference* (Macmillan), p. 27. A single clause has been omitted to simplify the passage.

embodies, the premises may be actually false, and yet the conclusion may follow logically from them. In this case the conclusion is said to be *materially* false but *formally* true. Some logicians hold that logic is primarily concerned with this *internal relation of consistency* between propositions forming a purely conceptual system. This is the real meaning of the expression *formal logic*, and syllogistic logic is undoubtedly susceptible of this interpretation. There is a great advantage and an equally great defect in restricting oneself to a purely formal treatment of syllogistic logic. The advantage is that it enables the logician to escape the necessity of testing the validity of his science by an appeal to concrete facts. The defect is that it inevitably leads to a vicious and unwarranted separation of the logical laws and relations from the world of concrete reality. It is largely because of this chasm between the entities dealt with in logic and those met with in more concrete bodies of knowledge that students find syllogistic logic dry and uninteresting. When dealing with the syllogism it is not possible wholly to avoid making the separation. But if students could only see that underneath the abstract logical relations dealt with in the doctrine of the syllogism is the infinitely rich and intricate world of human experience, and that the great logicians have simply been trying to comprehend the logical order underlying this factual world, much of the reproach from which syllogistic logic suffers might be removed. In any case, the syllogism is only one form of inference, and when we take up in Part II the various inductive processes, we shall be considering forms which bring us closer to the order systems dealt with in the sciences. However, it will be necessary to return again to this question of the relation of inference to the syllogism, when we take up the basic principle of the syllogistic type of inference at the end of Chapter X.

CHAPTER IX

THE STRUCTURE AND RULES OF THE SYLLOGISM

The Logical Form of the Syllogism

We have already explained that putting a proposition into logical form involves a clear indication of both its quantity and quality, as well as having the subject and predicate terms properly set apart from each other by some form of the verb *to be,* which is known as the copula. Now the first step in putting an argument or an explicit inference into logical or syllogistic form is to reduce it to three constituent propositions, or sets of three, expressing each in correct logical form. We have the elements of a syllogism when we get three propositions which contain just three terms. Hence each term appears twice in the syllogism.

The next step is to arrange the propositions in the right order. This is best done by starting with the conclusion. It is usually sufficiently obvious, owing to the use of such expressions as *therefore,* which shows that the proposition following is to be taken as the conclusion, and *for, since* or *because,* which indicate that the conclusion precedes the proof. Once the conclusion has been discovered, its predicate is taken as the *major term,* and the proposition containing it, either as subject or predicate is called the major *premise.* When the syllogism is written in correct logical order this premise stands first. The other term in the major premise is called the *middle term.* The subject of the conclusion is known as the *minor term,* and the proposition which contains it, along with the middle term, is called the *minor premise.* It should be written between the major and the conclusion.

Take the following argument to exemplify the process: "Revolutions in science do not destroy human knowledge, because they extend it, and what extends human knowledge cannot destroy it." The word *because* indicates that the proposition: "Revolutions in science do not destroy human knowledge," is the conclusion. It is ambiguous with regard to quantity, and the copula must be some form of the verb to be. So we begin by putting the conclusion into logical form, thus: "No revolutions in science are things which destroy human knowledge." Now let us get the major premise. It is the premise which contains "things which destroy human knowledge" as one of its terms, and is, therefore, the proposition: "What extends human knowledge cannot possibly destroy it." This is also ambiguous with regard to quantity, and must be put into the following form: "Nothing which extends human knowledge is a thing which can destroy human knowledge." We next proceed to the minor premise. It is the remaining proposition: "Revolutions in science extend human knowledge," and it is obviously intended to be universal, so that writing it in correct logical form gives us: "All revolutions in science are things which extend human knowledge." Writing all of these propositions together we get this syllogism:

Major premise: Nothing that extends human knowledge is a thing which destroys human knowledge.
Minor premise: All revolutions in science are things which extend human knowledge.
Conclusion: No revolutions in science are things which destroy human knowledge.

Mood

We are now prepared to understand the meaning of *mood.* It is a principle by which different varieties of syllogisms are distinguished. The principle is that of the quantity and quality of the propositions making up the

syllogism. There are as many different moods as there are possible combinations of *A, E, I,* and *O* propositions. The student will find, by working it out, that there are sixty-four different moods. Many of these are not valid, that is to say, they violate one or more of the rules of the syllogism. Yet we must consider that there are sixty-four different kinds of syllogisms when we distinguish them by the principle of mood. Later we shall have to determine which ones of the sixty-four possible moods are valid.

All that is necessary to remember here is that one way of classifying syllogisms is according to mood, and that mood is determined by the quantity and quality of the propositions making up the syllogism. Thus the mood of the example given above is *EAE*. Now when an argument is put in correct syllogistic form the proper vowel should always be placed before each proposition. When this is done the student will avoid making mistakes based on a misconception of the quantity and quality of the proposition. No one can understand a syllogistic argument until he is sure of the mood. This is a comparatively easy, but an indispensable step in the comprehension of any particular piece of syllogistic reasoning. That is why it is always necessary to have each proposition in the syllogism in correct logical form.

Figure

There is another principle by which different kinds of syllogisms are distinguished, and that is the position of the middle term in the premises. This is what is meant by *figure*. There are as many different figures as there are possible positions of the middle term, namely, four. When the middle term is the subject of the major premise and the predicate of the minor, the figure is said to be the first or perfect one, because this is the most natural way to express an argument, and because this figure yields a valid con-

clusion in any of the four propositions, *A*, *E*, *I* and *O*. No other figure will yield a valid conclusion in each of the four types. When the middle term is the predicate of both premises the figure is said to be the second. This figure yields only negative conclusions: *E* and *O*. When the middle term is the subject of both premises the figure is said to be the third. This figure yields particular conclusions: *I* and *O*. These were the only figures recognized by Aristotle. After his time the fourth possible position of the middle term, namely, as predicate of the major and subject of the minor premise, was used to define the fourth figure. This yields conclusions in *E*, *I* and *O*, but not in *A*. The results of this paragraph may be symbolized as follows:

Figure I	Figure II
M is *P*	*P* is *M*
S is *M*	*S* is *M*
S is *P*	*S* is *P*

Figure III	Figure IV
M is *P*	*P* is *M*
M is *S*	*M* is *S*
S is *P*	*S* is *P*

Thus we find in the doctrine of the syllogism an analogous classification to that found in the doctrine of propositions. Whereas propositions are classified by the two principles of quantity and quality, syllogisms are classified according to the two principles of mood and figure.

The Rules of the Syllogism

Eight rules are usually given as governing syllogisms, but the first two are not so much rules as statements of the number of propositions and terms needed to constitute a syllogism, and the last two are really corollaries of the third, fourth, fifth and sixth. These four rules are the

essential ones in determining the *validity* of a syllogism. For the word *valid* is used to designate a syllogism which conforms to the rules, and *invalid* or *fallacious* are expressions used to designate a syllogism which does not conform to the rules.

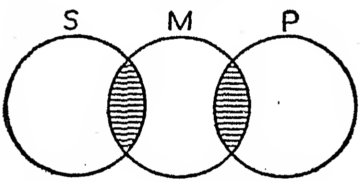
1. *Every syllogism must and can have only three propositions.* This rule needs no explanation, nor is the student likely to commit the fallacy of having more than three propositions when he has once understood the structure of the syllogism.

2. *The sum of the different terms in the three propositions must be three;* or, in other words, *each term must appear twice.* The fallacy here is that of having more than three terms, but since it is usually found as a middle term which is used in two different senses, one in the major and one in the minor, it is sometimes spoken of as the fallacy of *ambiguous middle.* Strictly speaking, the fallacy of ambiguous middle is a special form of the fallacy of *four terms.*

We dealt with ambiguity in the discussion of naming. It is really a fallacy which belongs to terms rather than to the syllogism. But the presence of ambiguous terms in an argument does vitiate and impair its validity, so that in this sense it is also a syllogistic fallacy. The following exemplifies this fallacy: "All interference is subject to a fifteen-yard penalty, and since static in the air is interference, therefore it is subject to a fifteen-yard penalty." This fallacy is very frequent in arguments in which abstract terms are used without the one who uses them laying down and sticking to a precise meaning. That is why many trivial philosophical discussions appear to the initiated to be mere verbiage, but to the uninitiated to be the very substance of profundity. No really profound thinker ever juggles his terms. To stick to the same meaning throughout is the essence of logical thinking.

3. *The middle term must be distributed in at least one of the premises.* The fallacy here is known as *undistributed middle.* Many amusing examples are given: "All monkeys are animals, and all men are animals, therefore all men are monkeys." "All clowns are men, the class president is a man, therefore he is a clown." These two examples are in the second figure, where the fallacy always appears when both premises are affirmative.

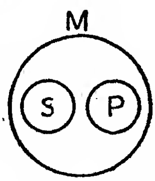
The reason why the middle term has to be distributed is that otherwise S may be compared to one part of M and P to an entirely different part of M in the premises, so that there is no real universal to use as the connecting link between S and P. Such a case is really equivalent to four terms. Figure VI will make this clearer. Or we might



Some M is P
Some S is M
Some S is P

FIGURE VI

have all of S and all of P in M without S and P being related to each other, as shown in Figure VII. Likewise one might be entirely within M and the other only partly within M, without their being related to each other. But as



All P is M
All S is M
All S is P

FIGURE VII

soon as the middle term is distributed, the fallacy disappears. For instance, when we say: No P is M, and some

S is *M*, we can infer that some *S* is not *P*. That part of *S* which is *M* cannot be *P*, when none of *P* falls within *M*.

4. *No term can be distributed in the conclusion, which is undistributed in its premise.* There are two fallacies involved in the violation of this rule, since either *S* or *P* may be distributed in the conclusion when undistributed in the premise. They are known as *illicit major* and *illicit minor*, according to which term is involved. Now it must be especially emphasized that *passing from a distributed to an undistributed use of a term is not a fallacy.* For what has been said concerning *all* in the premise, one is justified in saying about *some* in the conclusion. But when one has only been passing judgment about *some* in the premise he has no logical right to extend his conclusion to *all.* This was made clear in explaining conversion.

The following is a fairly common form of the fallacy of *illicit major,* it being in the first figure with the minor premise negative:

> All beverages containing more than five per cent of alcohol are intoxicating.
> No beer contains more than five per cent of alcohol.
> No beer is intoxicating.

The major term is *intoxicating,* and, being the predicate of an affirmative proposition, it is *undistributed* in the major premise, but, being the predicate of a negative proposition, it is *distributed* in the conclusion.

Now consider this illustration of the fallacy of *illicit minor*:

> All airplanes are lightly constructed.
> All airplanes are speedy vehicles.
> All speedy vehicles are lightly constructed.

Here the term *speedy vehicles* is *undistributed* in the minor premise because it is the predicate of an affirmative

proposition, but it is *distributed* in the conclusion because it has there become the subject of a universal proposition.

In looking for the three fallacies—undistributed middle, illicit major and illicit minor—the student will find it helpful to write above each term the abbreviation *dis.* for distributed and *undis.* for undistributed. Once the distribution of each term in each of its two locations in the syllogism is carefully determined, the question of the validity of the syllogism is easily answered. The student should also draw circles to illustrate the fallacies of illicit major and illicit minor, as was done above for undistributed middle. This will make the meaning of these fallacies clearer.

5. *No valid conclusion can be drawn from two negative premises.* The fallacy here is called the fallacy of two negative premises. The reason for this rule is that there is really no universal to connect *S* and *P* together, since, if both premises are negative, both *S* and *P* are denied to belong to the middle term. But if each is entirely outside the middle term, no judgment can be drawn as to their relation to each other, even though they happen to be related. The middle term in this case is a system which excludes both *S* and *P* in such a way as to give no hint as to their relations to each other. Take this example:

> No cows are used for dairy purposes in China.
> No cows are used for meat in Norway.
> Nothing used for meat in Norway is used for dairy purposes in China.

It is obvious that the conclusion is meaningless. The student should represent this fallacy by circles.

6. *If one premise is negative the conclusion must be negative.* In the case in question, of the two terms *S* and *P*, one is asserted to belong to the middle term, and one is denied to belong to it. This being true the only con-

clusion which can be drawn is one in which they are denied to have any relation to each other. The fallacy here is called *drawing an affirmative conclusion from a negative premise.* For example:

> All Indians are semibarbarians.
> No white people are Indians.
> All white people are semibarbarians.

It is evident here that the conclusion should be: "No white people are semibarbarians." But even when I thus eliminate the fallacy of an affirmative conclusion from a negative premise, note that I here get a fallacy of illicit major.

7. *No conclusion can be drawn from two particular premises.*

8. *If one premise is particular the conclusion must be particular.*

Since a violation of these rules always gives either undistributed middle, illicit major or minor, two negative premises, or an affirmative conclusion from a negative premise, these two rules are said to be corollaries of the third, fourth, fifth, and sixth rules. The possible combinations of premises are:

Rule 7:	Rule 8:
I, I, O, O	*A, I, E, I, A, O, E, O*
I, O, I, O	*I, A, I, E, O, A, O, E*

The student should construct a syllogism in each figure with each of these sets of premises, and determine the distribution and the fallacy in each case. For the tediousness of the task full compensation will be received in a surer grip on the contents of this whole chapter. I know of no better way of reaching a complete understanding of the reason for and a complete mastery of the various rules.

We can best summarize these rules by arranging them in a classificatory scheme:

I. Rules of Structure

1. A syllogism must contain three, and only three, propositions.
2. A syllogism must contain three, and only three, terms. Fallacy: four terms (ambiguous middle).

II. Rules of Quantity

3. The middle term must be distributed in at least one premise. Fallacy: undistributed middle.
4. No term can be distributed in the conclusion which was undistributed in its premise. Fallacies: (*a*) illicit major; (*b*) illicit minor.

III. Rules of Quality

5. A valid conclusion cannot be drawn from two negative premises. Fallacy: two negative premises.
6. If one premise is negative the conclusion must be negative, and *vice versa.* Fallacies: (*a*) an affirmative conclusion from a negative premise; (*b*) two affirmative premises with a negative conclusion.

IV. Corollaries

7. No conclusion can be drawn from two particular premises. Fallacies: (*a*) undistributed middle; (*b*) illicit major.
8. If one premise is particular the conclusion must be particular. Fallacies: (*a*) undistributed middle; (*b*) illicit major; (*c*) illicit minor; (*d*) two negative premises.[1]

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[1] The formulation of the rules of the syllogism is the work of the mediæval schoolmen. In his *Logic and Mental Philosophy*, p. 27, Father Charles Coppens puts the rules into the following rhyme. Note that he uses the words *consequent* and *conclusion* as synonymous.

The terms are only three, to this attend,
Nor let the consequent a term extend;
Conclusions ne'er the middle term admit,
At least one premise must distribute it.
Two negatives no consequent can show;
From affirmatives no negations flow.
A universal premise you'll provide,
And let conclusions take the weaker side.

Exercise VII

1. The following syllogisms are in logical form and they are valid. Using the symbols *S*, *P*, and *M* and the abbreviations *dis.* and *undis.*, mark each term each place it appears and indicate its distribution. Indicate also the mood and figure of each syllogism.

(*a*) No isotopes are isotypes.
All isotopes are members of the table of elements.
Some members of the table of elements are not isotypes.

(*b*) All isotypes are pictographs.
No isotopes are pictographs.
No isotopes are isotypes.

(*c*) All radarscopes are parts of radars.
All parts of radars are new inventions.
Some new inventions are radarscopes.

(*d*) All Russians are mongoloids.
All bolsheviks are Russians.
All bolsheviks are mongoloids.

(*e*) No Frenchmen are mongoloids.
All French Canadians are Frenchmen.
No French Canadians are mongoloids.

(*f*) All Canadians are citizens of the British Empire.
Some French people are Canadians.
Some French people are citizens of the British Empire.

(*g*) No atomic bombs are antiquated.
All fissionable explosives are atomic bombs.
No fissionable explosives are antiquated.

(*h*) All radiosondes are delicate mechanisms.
All radiosondes are devices for weather reporting.
Some devices for weather reporting are delicate mechanisms.

(*i*) No altimeters are automobile equipment.
All hydramatic devices are automobile equipment.
No hydramatic devices are altimeters.

(*j*) No airplane carriers are privately owned.
Some yachts are privately owned.
Some yachts are not airplane carriers.

(*k*) All dirigibles are lighter than air.
No airplanes are lighter than air.
No airplanes are dirigibles.

(*l*) All toy elephants are attractive.
Some sleds are not attractive.
Some sleds are not toy elephants.

(*m*) Some bees are honey makers.
All bees are stinging insects.
Some stinging insects are honey makers.

(*n*) All horse races are inciters to gambling.
Some horse races are amateur sports events.
Some amateur sports events are inciters to gambling.

(*o*) Some merchant sailors are not seamen.
All seamen are trained to man ships.
Some persons trained to man ships are not merchant sailors.

2. The following syllogisms are not all in logical form and each commits a fallacy. Put each in logical form which needs to be, mark the terms of each with symbols, and indicate the distribution of each term, and the mood, figure, and fallacy of each syllogism.

(*a*) All G.I. students are subsidized by the Government.
Some persons subsidized by the Government are worthy students.
Some worthy students are G.I. students.

(*b*) All museums are archives for the preservation of treasures.
All libraries are archives for the preservation of treasures.
All libraries are museums.

(*c*) Some areas of the earth are overpopulated.
The Sahara Desert is an area of the earth.
The Sahara Desert is overpopulated.

(*d*) No drones are working bees, and some airplanes are drones, therefore some airplanes are not working bees.

(*e*) Some holographs are wills and since no typed papers are holographs, it follows that some typed papers are not wills.

(*f*) All religions are mixed with superstitions and since all things mixed with superstitions are magical in origin it follows that all things that are magical in origin are religious.

(*g*) All weeping willows are weepers, and some women are weepers, hence some women are weeping willows.

(*h*) All members of the table of elements are subject matter of nuclear physics, and all isotopes are subject matter of nuclear physics, hence all isotopes are members of the table of elements.

(*i*) All male citizens within the age limit are subject to the draft, and all male citizens within the age limit are eligible to vote, and consequently all who are subject to the draft are eligible to vote.

(*j*) All submarines are of relatively small tonnage, all ships of relatively small tonnage are inexpensive, hence all inexpensive ships are submarines.

3. Construct syllogisms to illustrate ambiguous middle, illicit major and illicit minor, indicating mood and figure.

4. Construct a syllogism in the moods, *EAE, AII, AEE, IAI,* and *AOO,* indicating the figure and fallacy (if any).

CHAPTER X

THE VALID MOODS OF THE SYLLOGISM

The Elimination of the Sets of Premises Which Violate the Eight Rules of the Syllogism

We learned above that there are a total of sixty-four different moods, but that many of these are not valid. Our main problem in this chapter is to work out the valid moods for each of the four figures. But before taking up the special rules for each figure, it will simplify matters if we eliminate all of the moods which do not conform to the eight rules. For we can do this without regard to figure.

Since these rules can be applied to the two premises alone, without taking the conclusion into consideration, instead of dealing with the sixty-four moods, our problem will be greatly simplified by taking just the sixteen possible sets of premises set forth in Table IV.

TABLE IV

AA	*EA*	*IA*	*OA*
AE	*EE*	*IE*	*OE*
AI	*EI*	*II*	*OI*
AO	*EO*	*IO*	*OO*

Eliminating from this table all of the combinations which are in conflict with any of the rules of the syllogism, we find that *EE, EO, OE* and *OO* all violate rule five, that no conclusion can be drawn from two negative premises. Likewise *II, IO, OI,* and *OO* violate rule seven, that no conclusion can be drawn from two particular premises. Note that *OO* violates both rules. All the other sets of premises in Table IV conform to the rules of the syllogism except *IE*. It violates rule four in that it always gives a fallacy of illicit major. For no matter whether the major term stands as

the subject or as the predicate it is sure to be undistributed in the major premise, since both the subject and the predicate of an *I* proposition are undistributed. On the other hand, the fact that the minor premise is negative will make the conclusion negative, according to rule six, and the major term, being its predicate, will always be distributed to conform to the rule that negative propositions distribute their predicates. Hence it is impossible to avoid the fallacy of illicit major when the premises are *IE*. The student should work this out by constructing a syllogism in each figure in the mood *IEO*, which is what *IE* premises would always yield, inasmuch as the conclusion would have to be both negative and particular to conform to rules six and eight. We are now left with eight sets of premises as exhibited in Table V. To determine the list of valid moods these eight must now be tested in each figure.

TABLE V

AA	*EA*	*IA*	*OA*
AE	—	—	—
AI	*EI*	—	—
AO	—	—	—

The Special Rules of the Four Figures

The special rules are really only corollaries of the eight rules, or, to be more specific, of the third, fourth, and fifth rules of the syllogism, since their violation yields one or the other of the four fallacies which we considered in stating those rules. Consequently it is not necessary for the student who has mastered the general rules to commit these to memory. Indeed, their chief value is in making the general rules clearer.

1. *The Rules for the First Figure*

M is P
S is M
$\therefore S$ is P

(a) *The minor premise must be affirmative, otherwise either illicit major or two negative premises will result.* We can best test this rule by violating it. Thus:

All soldiers are paid for their services.
No volunteer firemen are soldiers.
No volunteer firemen are paid for their services.

Here the major term is undistributed in its premise and distributed in the conclusion, which is a fallacy of illicit major. The only way it could be avoided would be to make the major premise negative, since the major term is predicate in its premise in Figure I. But that would give a fallacy of two negative premises. By making the minor premise affirmative we avoid both of these fallacies.

(b) *The major premise must be universal in order to avoid the fallacy of undistributed middle.* Here is an example violating this rule:

Some marine animals are edible.
All oysters are marine animals.
Some oysters are edible.

The fact that the previous rule makes the minor premise affirmative, leaves the middle term undistributed in that premise. Now since the middle term is always the subject of the major premise in this figure, an undistributed middle is unavoidable when the major premise is particular.

When we apply these two rules to the eight sets of premises, we find that only four yield valid conclusions in Figure I, namely, *AAA, EAE, AII,* and *EIO.* However, the moods *AAI* and *EAO* would also be valid, since subaltern propositions are always true when the universal under which they are subsumed is true. *AAI* is said to be a *weakened form* of the mood *AAA* and *EAO* a weakened form of *EAE* in the first figure.

2. *The Rules for the Second Figure*

P is M
S is M
S is P

(a) *One premise must be negative to avoid undistributed middle.* This is evident from the fact that M is the predicate in both premises in the second figure, and the rule that only negative premises distribute their predicates.

(b) *The major premise must be universal to avoid illicit major.* Now the previous rule necessitates a negative conclusion and the major term being its predicate, will be distributed. And since in the second figure the major term is the subject of its premise, a fallacy of illicit major is bound to result unless the major premise is made universal. The following example will make this clear:

Some elephants are not wild.
All tigers are wild.
Some tigers are not elephants.

Note that this fallacy of illicit major disappears when I say:

No elephants are wild.
All tigers are wild.
No tigers are elephants,

inasmuch as making the major universal, distributes the term elephant.

Applying these two rules to the eight combinations of premises, we find that only four yield valid conclusions in Figure II. The valid moods are *EAE* (and its weakened form *EAO*), *AEE* (and its weakened form *AEO*), *EIO*, and *AOO*. Notice that two of these moods were also valid in Figure I.

3. *The Rules for the Third Figure*

M is P
M is S
S is P

(a) *The minor premise must be affirmative to avoid illicit major, or two negative premises.* Since in the third figure, the major term stands in the same position in its premise as in the first, the proof here is precisely the same as that given there. This rule can, therefore, be generalized to apply to both figures: Whenever the major term is the predicate of its premise, the minor premise must be affirmative to avoid the fallacy of illicit major or that of two negative premises.

(b) *The conclusion must be particular to avoid the fallacy of illicit minor.* The fact that the minor premise must be affirmative necessitates the minor term being undistributed in its premise, where it is always predicate in Figure III. Hence the conclusion must be particular, so that the minor term as its subject may remain undistributed. Take this example:

No religious magazines are profound.
All religious magazines are devotional.
No devotional literature is profound.

Note that the rule holds even when both premises are universal.

Applying these rules to the eight sets of premises we get six valid moods in the third figure, namely: *AAI, IAI, AII, EAO, OAO* and *EIO*. Note that only *EIO* is valid in the first, second and third figures, but the weakened form *EAO* is also valid in all three. It is not ordinarily counted as a valid mood in the first and second figures, but it is counted here because the universal conclusion cannot

here be obtained from the premises *EA*. Likewise, the mood *AAI* is not counted as a valid mood in the first figure, but it is in the third, although it is valid in both.

4. *The Rules for the Fourth Figure*

P is M
M is S
S is P

(a) *If either premise is negative, the major premise must be universal in order to avoid illicit major.* Violating this:

> Some wild flowers are beautiful.
> No beautiful things are perishable.
> Some perishable things are not wild flowers.

Here the negative premise necessitates a negative conclusion, and that distributes P there. Now, since P is the subject of its premise in Figure IV, the fallacy of illicit major is unavoidable whenever the major premise is particular and the conclusion negative.

(b) *If the minor premise is affirmative, the conclusion must be particular in order to avoid illicit minor.* The same argument applies here which was used in proving the second rule for figure three. Hence that rule can be generalized as follows: Whenever the minor term is the predicate of an affirmative proposition in its premise the conclusion must be particular.

(c) *If the major premise is affirmative, the minor must be universal in order to avoid undistributed middle.* As the predicate of the major premise in Figure IV, the middle term will always be undistributed when that premise is affirmative. Hence it must be distributed in the minor, and since it is there the subject, that premise must

be universal. The following example, violating the rule, will make this clear:

All negroes are natives of Africa.
Some natives of Africa are Mohammedans.
Some Mohammedans are negroes.

Applying these rules, we find that the following moods are valid in Figure IV: *AAI, AEE, IAI, EAO,* and *EIO.* The weakened form *AEO* would also be valid. Note that *EIO* and *EAO* are valid in all figures, and that twenty-four moods are valid when the weakened forms are counted, and nineteen when they are not.[1]

Reduction

It was mentioned above that Aristotle regarded the first figure as the only perfect figure. Hence the valid moods in

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[1] A singular syllogism, with all of its propositions and each of its six terms singular, is valid in each of the following moods: *AEE* in Figures I and III, and *AAA* in Figures II, III and IV. Stanley Jevons (see pp. 36-65 of book cited below) calls this *inference with simple identities.* It has also been called *traduction* to distinguish it from deduction and from induction. For examples see p. 145.

The scholastic logician, Petrus Hispanus, who later became Pope John XXI, is said to be the author of the following ingenious set of mnemonic lines, devised to reduce the problems of syllogistic reasoning to a mechanical process of memory:

Barbara, Celarent, Darii, Ferioque prioris:
Cesare, Camestres, Festino, Baroko, secundae;
Tertia, *Darapti, Disamis, Datisi, Felapton,*
Bokardo, Ferison, habet; Quarta insuper addit
Bramantip, Camenes, Dimaris, Fesapo, Fresison.

The mood names are italicized. The other words indicate the figure to which each set of mood names belongs. Logicians sometimes use these mood names in referring to the various moods. For example, when they speak of *Barbara,* the mood *AAA* in Figue I is meant.

It is also interesting to note that Stanley Jevons invented a logical machine which draws correct conclusions. See the picture in his *Principles of Science* (frontispiece, 2d ed. revised, 1877), and the description (pp. 107 *f.*).

the other figures are sometimes called imperfect moods. But arguments can be changed from the imperfect moods of the last three figures to the perfect moods of the first figure. This is what is meant by reduction in syllogistic logic. Now there are two kinds of reduction or two essentially different ways of making the change to the first figure. One is known as *direct reduction.* It is effected by converting one or more of the propositions making up the syllogism, but sometimes it is also necessary to transpose the premises. The other kind is most commonly referred to as *indirect reduction,* but also as *reduction per impossibile* and as *apagogical reduction.* It is accomplished by use of immediate inference by opposition.

To exemplify the direct process of reduction, consider first a mood which involves only one conversion. Such are the moods *EAE* and *EIO* in the second figure and *AAI, AII, EAO* and *EIO* in the third figure. An example of *EAE* in Figure II is:

> No wild men are educated.
> All college graduates are educated.
> No college graduates are wild men,

which becomes *EAE* in Figure I, by the simple process of converting the major premise, thus:

> No educated people are wild men.
> All college graduates are educated.
> No college graduates are wild men.

A more difficult mood to reduce by the direct process is *AEE* in Figure II. Here is an example:

> All salmon are fish which spawn in fresh water.
> No eels are fish which spawn in fresh water.
> No eels are salmon.

To reduce this it is necessary to transpose the premises because the major would convert by limitation into *I,* and

this would give the invalid mood *IEO* (illicit major). The minor becomes the major by transposition of the premises and is converted by simple conversion. But transposition of the premises also makes it necessary to convert the conclusion so that the major term will be in the predicate. Hence, three separate processes are involved—transposition of the premises, conversion of the new major resulting from this transposition and conversion of the conclusion. Thus it becomes:

> No fish which spawn in fresh water are eels.
> All salmon are fish which spawn in fresh water.
> No salmon are eels.

There are only two moods which reduce by the indirect method, namely: *AOO* in Figure II and *OAO* in Figure III, the reasons being that *O* cannot be converted and the conversion of *A* by limitation would give two particular premises. Here is an example of *AOO*:

> All submarines are commerce destroyers.
> Some superdreadnaughts are not commerce destroyers.
> Some superdreadnaughts are not submarines.

To reduce this, we construct a syllogism in the mood *AAA* of Figure I, showing that falsity of the conclusion in the mood *AOO* in Figure II is inconsistent with the truth of its premises. That is, we assume that if this conclusion, "Some superdreadnaughts are not submarines," is false, its contradictory, "All superdreadnaughts are submarines," is true (by immediate inference by opposition). This is used as a minor to the original major, thus:

> All submarines are commerce destroyers.
> All superdreadnaughts are submarines.
> All superdreadnaughts are commerce destroyers.

Now, this conclusion is the contradictory of the original minor. Granting, then, the truth of our original premises,

the conclusion we drew from them is true. Hence, the original syllogism is shown to be formally valid. What we have here done is to prove, with a syllogism in Figure I, that the mood *AOO* in the second figure is formally valid.

The Basic Principle of Syllogistic Inference

The motive behind the reduction of the moods of the second, third and fourth figures to those of the first is to make it possible to formulate a single principle of syllogistic inference. And since, in the opinion of some logicians, syllogistic inference is one with inference, this means making it possible to formulate a single principle of all inference whatsoever. If such a principle could be reached, all of the rules of the syllogism could be shown to follow from this basic principle, just as we have seen that all of the special rules for the separate figures are corollaries of the four most central general rules. This would satisfy the modern demand, known as the *principle of economy,* or *law of parsimony,*[2] that the ultimate principles or primary axioms of a science should be reduced to as few as possible, and which is, in turn, identical with the famous declaration of the great schoolman, William of Occam, known as *Occam's razor*, that rules should not be multiplied beyond what is necessary.

The classic formulation of a single principle of syllogistic inference is Aristotle's *dictum de omne et nullo.* The form usually given is that of the schoolmen: *Whatever is predicated affirmatively or negatively of a whole class must be predicated affirmatively or negatively of everything in that class.* Some logicians hold that there are really two principles here: (1) the *dictum de omne*—whatever is true of all is true of each, and (2) the *dictum de nullo*—what-

[2] See *Introduction to Reflective Thinking*, pp. 53, 340. Compare also R. H. Dotterer, *Beginners' Logic*, pp. 154 *f.*, and 313.

ever is true of none is false of each. Now, in the first figure, we do appear to proceed on this principle or on these two principles. For when we argue that all M is P, and all S is M, and therefore that all S is P, we do seem to use the *dictum de omne* as the underlying principle of inference. And when we argue that no M is P, and all S is M, and therefore that no S is P, we seem to use the *dictum de nullo* as an underlying principle. Now these two arguments are the moods *AAA* and *EAE* of the first figure. The *dictum de omne et nullo* can be interpreted as the basic principle behind the rules of distribution, and these are certainly the central ones among the general rules of the syllogism, as well as the foundation for the special rules for the separate figures.

It has been argued that the real major premise in every argument is this basic rule or principle. It is supposed to be axiomatic in the sense of self-evident, and syllogistic inference is sometimes said to be from this rule to the conclusion. This caused John Stuart Mill to launch his famous attack on syllogistic reasoning. He claimed that all syllogisms argue in a circle, or beg the question [3] so that the conclusion contains no new knowledge. If we must know something about all before we can affirm or deny that something, whatever it is, of the separate members of that all, we really do not advance in knowledge when we draw the conclusion. For our knowledge of the truth of the major premise carries with it a knowledge of the truth of the conclusion. Hence Mill argued that all fruitful reasoning is inductive, and that the experimental methods of science should be substituted for syllogistic inference. It is interesting to note that Mill here states the same criticism of the syllogism which Creighton and others make of the immediate inferences. Both hold that there is no new

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[3] See the account of this fallacy below, p. 195.

knowledge involved, and, consequently, that there is no genuine inference.

Mill's criticism undoubtedly indicates a real weakness in the underlying conception of inference behind the *dictum de omne et nullo*. For that conception is the *linear theory*, which was dealt with in our discussion of the nature of inference, and according to which the separate terms in an argument remain unmodified in the conclusion, the real justification for the inference being placed in a sweeping general principle, reached by intuition, rather than in a genuine implicative system actually present in the facts. What Mill overlooked was that the real nerve of inference is not in a major premise, stating a mere collective fact, but in systematic connection within an order system. But he was certainly right in attacking the linear theory of syllogistic inference, which was not only widely current in his day, but is still in considerable vogue. As Bosanquet put it:

On the traditional theory of the syllogism, and according to any or all of the maxims which have been suggested as its ground, its terms are marks or properties which affect each other, so far as the technical purpose of the reasoning is concerned, only as indications of each other's presence or absence. . . . The rules forbid you to regard the argument as the construction of a system in which by their combination the terms throw a new light upon one another. . . . Each term must preserve its independent being, as if inclosed in a bracket. . . . In Thackeray's well-known story of the Abbé and the nobleman, the Abbé, talking among friends has just said, "Do you know, ladies, my first penitent was a murderer," and a nobleman of the neighborhood, entering the room at the moment, exclaims, "You there, Abbé? Why, ladies, I was the Abbé's first penitent, and I promise you my confession astonished him." In such arguments, which are of the type that *A*, which has *x*, has also *y*, you get, technically recognized, no bearing of the import of one term upon another at all. They are, so to speak, in capsules, and all you can do with them is to note which lie in the same drawer, and which refuse to do so. . . .

It is the extremity of such a doctrine to suppose that the principle of the syllogism is its ultimate premise, and that a train of reasoning derives its force from having at its head an axiom about a class or rule. When this extreme irrationality is rejected, and it is observed that the necessity of every syllogism is intrinsic to its form, and not borrowed from a rule under which it is subsumed, the way is paved for reconsidering its whole nature.[4]

It is for this reason that the traditional theory of the syllogism, based as it is upon the linear theory of inference, must give way to the modern theory of systematic connection within an order system. That the syllogism can be interpreted in terms of the implicative system has already been made clear. But entirely aside from that issue, there is really no justification for the covert assumption of many of the advocates of the syllogism that all valid inference must be expressible in syllogistic form, nor for the additional assumption that all syllogistic inference should take the form of the first figure. It is a false idea of economy which, for the sake of simplification, blurs distinctions that are really vital. There are interesting varieties of thought which cannot be reduced to the stereotyped form of the syllogism. Modern logicians refer to these as *non-syllogistic* or *asyllogistic types of inference*. Nor should we make the mistake of supposing that all asyllogistic reasoning must be inductive. For this would make it necessary to treat mathematical reasoning as syllogistic, whereas it is asyllogistic but deductive. A good example is a proportional argument such as the following: "A ton of coal costs sixteen dollars. This is half a ton. Therefore, this cost eight dollars." Although this argument is perfectly valid, it cannot be put into syllogistic form without wrenching the reasoning out of shape. The irregular arguments, dealt with at the end of the next chapter, are other

[4] *Loco citato,* pp. 25 *f.* The illustration from Thackeray is in a note, and the sentence following it is slightly modified.

good examples of asyllogistic arguments which are valid. And still other examples could be given from inductive as well as from mathematical reasoning. But an implicative system is always operative in these asyllogistic types of inference just as in the syllogistic type. Hence the basic principle of inference cannot be such a rule as the *dictum de omne et nullo.* On the contrary, it is the concrete implicative system which is the nerve of all inference, syllogistic and asyllogistic alike.

Exercise VIII

1. Prove that *IEO* is invalid in every figure.

2. Prove that whenever the major term is the predicate in its premise the minor premise must be affirmative. In what figures does this hold?

3. Prove that whenever the minor term is the predicate of an affirmative premise, the conclusion must be particular. In what figures may this condition exist?

4. In the following examples name the mood and figure and reduce to the first figure.

(*a*) All radioactive elements are sources of radiant energy. All sources of radiant energy are complicated structures. Some complicated structures are radioactive elements.

(*b*) All inflations are burdensome, and some inflations are due to Government spending, hence some things due to Government spending are burdensome.

(*c*) Some houses are expensive, and all expensive things are luxurious, hence some luxuries are houses.

(*d*) Some technologies are obsolete, and all technologies are applications of science to practical life, therefore, some applications of science to practical life are obsolete.

(*e*) No naval officers are omniscent, and some omniscient beings are supramundane beings, hence some supramundane beings are not naval officers.

(*f*) No cyclotrons are inexpensive, and some optical instruments are inexpensive, hence some optical instruments are not cyclotrons.

(*g*) No fluoroscopes are radarscopes, but some fluoroscopes are equipped with screens, hence some things equipped with screens are not radarscopes.

(*h*) No radarscope maps are printed on paper, and all maps printed on paper are geographical maps, hence some geographical maps are not radarscope maps.

(*i*) No atomic bombs are privately owned. All atomic bombs are stored in Government arsenals, hence some things stored in Government arsenals are not privately owned.

(*j*) No automobiles are capable of shielding atomic power plants, and all battleships are capable of shielding atomic power plants, hence no battleships are automobiles.

(*k*) All atoms are composed of protons and neutrons. No things composed of protons and neutrons are simple in structure. No things simple in structure are atoms.

(*l*) All one-celled animals are microscopic. All one-celled animals are living creatures, hence some living creatures are microscopic.

(*m*) All vitamins are foods, and some molluscs are not foods, hence some molluscs are not vitamins.

(*n*) Some radioactive elements are not used as tracers, but all radioactive elements have a half-life, hence some things that have a half-life are not used as tracers.

5. The following are singular syllogisms. Indicate mood, figure, and fallacy (if any) of each syllogism.

(*a*) Iwo Jima is the largest of the Volcano Islands. Maui is not Iwo Jima, therefore Maui is not the largest of the Volcano Islands.

(*b*) The U.S.S. *Missouri* is our largest naval vessel. The U.S.S. *Missouri* is not a carrier, hence a carrier is not our largest naval vessel

(*c*) The leader who lost the war for Japan is Premier Tojo and the leader who ordered the attack on Pearl Harbor is Premier Tojo. Therefore the leader who ordered the attack on Pearl Harbor is the leader who lost the war for Japan.

(*d*) Joseph Conrad is the greatest novelist of seafaring life, and Joseph Conrad was a Polish sea captain serving on British ships, hence a Polish sea captain serving on British ships is the greatest novelist of seafaring life.

(*e*) Franklin Delano Roosevelt inaugurated the New Deal and the President who inaugurated the New Deal led the nation in World War II, hence the President who led the nation in World War II is Franklin Delano Roosevelt.

6. Construct a valid syllogism in the second, third, and fourth figures and reduce each to the first figure. Indicate the mood before and after reduction, and explain each step you have to go through in making the reduction.

CHAPTER XI

ENTHYMEMES, SORITES AND IRREGULAR ARGUMENTS

Enthymemes

An enthymeme is a syllogism with one of the premises or the conclusion suppressed. When the major is suppressed it is called an enthymeme of the *first order,* when the minor is suppressed an enthymeme of the *second order,* and when the conclusion is suppressed an enthymeme of the *third order.* It is more apt to be the major or minor which is suppressed, but sometimes the conclusion is left unexpressed for rhetorical effect. "No man is free, for every man is a slave either to money or to fortune." "You, as you are old and reverend, should be wise." "I pray you, father, being weak, seem so." These are examples of enthymemes. The first, put into a complete syllogism, would be:

None who are slaves either to money or to fortune are free.
All men are slaves either to money or to fortune.
No man is free. Mood *EAE,* Figure I. First order enthymeme.

The other two are also enthymemes of the first order. The student should formulate the major premise of each. The following is an example of an enthymeme of the second order: "The Thirty Years' War was long and bitter, for all religious wars are so." Expressing this in the form of a syllogism gives this:

All religious wars are long and bitter.
The Thirty Years' War was a religious war.
The Thirty Years' War was long and bitter.

Another example of a second order enthymeme is: "Why be ashamed of a mistake? All men are fallible." Mellone expresses this thus:

> What all men are liable to is not a thing to be ashamed of.
> A mistake is what all men are liable to.
> *Therefore*, no mistakes are things to be ashamed of.

And the following quite manifestly exemplifies the third order: "None but candid men are good reasoners, and few infidels are candid."

Many of the arguments we meet with in everyday life are based on principles or generalizations which are covertly assumed, and rarely, if ever, expressed as premises. Only when some one denies our premises, and thereby refuses to accept our argument, are we made conscious that it was really based on those premises. Suppose, for example, that I argue that the United States should cancel the allied debts, because the fighting of the allies in the late war was beneficial to the United States. Then my suppressed premise is that all countries which benefit from the waging of a war should pay the participants for such benefits. Or suppose a pacifist argues that the United States should abolish her navy, because having a navy breeds war. His suppressed premise is that all war is an evil which no nation should tolerate. This is a very doubtful principle which men can easily affirm in the abstract, but which few are ready to affirm in the face of an overt aggression such as the sinking of the *Lusitania* and its consequent destruction of American lives.

Many arguments, which sound very good, are really based on underlying and unexpressed premises that few people would be willing to accept without qualification. What the partisan propagandist most hates, is having his suppressed premises brought before the bar of reason. It

is for this reason that the use of the enthymeme type of argument is one of the favorite artifices of the propagandist.

Chains of Reasoning

Many arguments are compound, consisting of a series of syllogisms in which the conclusion of one syllogism becomes a premise for the next. This type of argument is usually called a *chain of reasoning*. The first syllogism in such a chain is called a prosyllogism in reference to the syllogism which follows, and this is an episyllogism in relation to the first. If the series is a long one in which there are more than two syllogisms, a syllogism in the middle of the series could be both a prosyllogism and an episyllogism—a prosyllogism in relation to the one which follows it, and an episyllogism in relation to the one which precedes it. Now a chain of reasoning, as well as a single syllogism, may take the form of an enthymeme, that is, some of its propositions may be suppressed. In such a case it is known as an *epicheirema*.

In passing it should be pointed out that chains of reasoning are highly typical examples of the *linear theory of syllogistic inference,* in that the terms and premises appear strung along in them like a string of beads, rather than as members of a genuine implicative system which reciprocally modify each other. However, even in such inferences, the conclusion would not be valid if such an implicative system were not operative.

I take the following interesting example of a chain of reasoning, which is also an epicheirema, from Joseph's *Introduction to Logic* (2nd ed.), p. 353:

Those who have no occupation have nothing to interest themselves in, and therefore are unhappy; for men with nothing in which to interest themselves are always unhappy, since happiness depends on the success with which we advance the objects in

which we are interested; and so wealth is no guarantee of happiness.

The core of this argument can be expressed in this syllogism:

> All men with nothing in which to interest themselves are unhappy.
> All of those who have no occupation are men with nothing in which to interest themselves.
> *Therefore*, all of those who have no occupation are unhappy.

The major of this central syllogism is established by another syllogism, which is a prosyllogism to the one just stated:

> All happy men are those who are successful in advancing the objects in which they are interested.
> No one who has no objects in which to interest himself is one who succeeds in advancing objects in which he is interested.
> *Therefore*, no men with nothing in which to interest themselves are happy.

There is also the following episyllogism involved in this chain of reasoning:

> All who have no occupation are unhappy.
> Some rich men are persons who have no occupation.
> Some rich men are unhappy.

Here is another shorter but interesting example of an epicheirema:

> All sin is dangerous.
> Covetousness is sin, for it is transgression of the law.
> Covetousness is dangerous.

Here the proposition, "Covetousness is a transgression of the law," is a premise in the prosyllogism which confirms the proposition: "Covetousness is sin."

The example of an enthymeme given above could also be expressed as an epicheirema, thus:

> All war is evil.
> All navies are evil, for they breed war.
> *Therefore,* all navies should be abolished.

When an argument advances from the prosyllogism to the episyllogism, or in a forward direction the chain of reasoning is said to be *progressive or synthetic.* Such an argument builds up its results as it advances, using the conclusion of the preceding syllogism for a premise in the next. The reverse process is sometimes found. Starting with the conclusion, we go back to establish the premises, thereby exhibiting the foundation upon which the conclusion is based. This kind of a chain of reasoning is called *regressive, or analytic.*

The Sorites

The *sorites* is a special kind of chain of reasoning. It consists of a series of syllogisms in the first figure in which only the last conclusion is expressed, and in which the intermediate suppressed conclusions are also suppressed premises of the syllogisms. In a sorites the first two expressed premises constitute the first or prosyllogism. The unexpressed conclusion from these two premises is taken either as the major or the minor premise of the next syllogism, and the third expressed premise is its other premise. According to whether the conclusions are taken as major or minor premises we get two types of sorites. The type in which the conclusion is a minor premise is known as the progressive or *Aristotelian sorites.* The term Aristotelian, however, is a misnomer. For, as Sir William Hamilton says, "Aristotle, though certainly not ignorant of the process of reasoning now called sorites, does not enter upon its consideration." It dates from the middle of the

fifteenth century. The type in which the suppressed conclusions are major premises is known as the regressive or *Goclenian sorites,* after its discoverer, the German logician, Rudolph Goclenius (1547-1628).

In breaking up a sorites into syllogisms it will always be found that there are *as many syllogisms as there are expressed premises, less one.* The conclusions must not be counted but only the premises. Now let us consider some examples of each type of sorites, break them up into syllogisms and deal briefly with the rules which govern each type.

The Aristotelian Sorites

First, let us represent it in symbols.

$$\begin{array}{l} \text{All } A \text{ is } B \\ \text{All } B \text{ is } C \\ \text{All } C \text{ is } D \\ \text{All } D \text{ is } E \\ \text{All } E \text{ is } F \\ \hline \text{All } A \text{ is } F \end{array}$$

This breaks up into the following four syllogisms:

$$\begin{array}{rl} & \text{All } B \text{ is } C \\ & \text{All } A \text{ is } B \\ \cline{2-2} 1. & \text{All } A \text{ is } C \end{array} \qquad \begin{array}{rl} & \text{All } C \text{ is } D \\ & \text{All } A \text{ is } C \\ \cline{2-2} 2. & \text{All } A \text{ is } D \end{array}$$

$$\begin{array}{rl} & \text{All } D \text{ is } E \\ & \text{All } A \text{ is } D \\ \cline{2-2} 3. & \text{All } A \text{ is } E \end{array} \qquad \begin{array}{rl} & \text{All } E \text{ is } F \\ & \text{All } A \text{ is } E \\ \cline{2-2} 4. & \text{All } A \text{ is } F \end{array}$$

Note that the minor premise stands first, and that the suppressed conclusion is the minor premise for each syllogism after the first. We know that the minor premise stands first, because the conclusion shows A to be the minor term.

Consider, now, concrete examples:

He who is prudent is temperate.
He who is temperate is constant.
He who is constant is unperturbed.
He who is unperturbed is without sorrow.
He who is without sorrow is happy.

Therefore, he who is prudent is happy.

Again we get four syllogisms when we break this up:

He who is temperate is constant.
He who is prudent is temperate.

---

1. He who is prudent is constant.

He who is constant is unperturbed.
He who is prudent is constant.

---

2. He who is prudent is unperturbed.

He who is unperturbed is without sorrow.
He who is prudent is unperturbed.

---

3. He who is prudent is without sorrow.

He who is without sorrow is happy.
He who is prudent is without sorrow.

---

4. He who is prudent is happy.

Another famous example is frequently given:

The child of Themistocles governs its mother.
She governs her husband.
He governs Athens.
Athens governs Greece.
Greece governs the world.

The child governs the world.

Let the student break this up into syllogisms.

There are two rules which control this type of sorites, the violation of which leads to the fallacies of undistributed middle and illicit major. (1) *The first premise may be particular, but all the rest must be universal, to avoid undistributed middle.* This is really the same as the special rule for the first figure which says that the major premise must be universal, since every expressed premise except the first is the major premise of a syllogism in the first figure. The first premise, being the minor of the syllogism, may be particular. It should also be noted that making any other premise except the first particular would involve one or more of the syllogisms into which the sorites is resolved having two particular premises. But this is equivalent to undistributed middle. (2) *The last premise may be negative, but all the rest must be affirmative, to avoid illicit major or two negative premises.* This is an application of the other special rule of the first figure to the sorites, since making any other premise but the last negative would result in one of the suppressed conclusions being negative, according to general rule six, and since this conclusion is the minor premise of the following syllogism it would violate the rule that in the first figure the minor must be affirmative. The last premise, being the major premise of the last syllogism, can be negative without violating this rule. It only makes the last or expressed conclusion negative, and that does not involve a fallacy of illicit major, because the major term is distributed when the last premise is made negative, it being the predicate of that premise. Making any premise except the last negative would also involve one or more of the syllogisms into which the sorites is resolved having two negative premises. Using the examples already given, let the student violate each of these rules, and note what fallacies result when the sorites is broken up into separate syllogisms.

The Goclenian Sorites

It is exemplified by symbols as follows:

All *A* is *B*
All *C* is *A*
All *D* is *C*
All *E* is *D*
All *F* is *E*
———
All *F* is *B*

This breaks up thus:

All *A* is *B*
All *C* is *A*
———
1. All *C* is *B*

All *C* is *B*
All *D* is *C*
———
2. All *D* is *B*

All *D* is *B*
All *E* is *D*
———
3. All *E* is *B*

All *E* is *B*
All *F* is *E*
———
4. All *F* is *B*

A concrete example may also be given:

An animal is a substance.
A quadruped is an animal.
A horse is a quadruped.
Dan Patch is a horse.
———
Dan Patch is a substance.

This readily breaks up into three syllogisms:

An animal is a substance.
A quadruped is an animal.
———
1. A quadruped is a substance.

A quadruped is a substance.
A horse is a quadruped.
———
2. A horse is a substance.

A horse is a substance.
Dan Patch is a horse.
———
3. Dan Patch is a substance.

The rules of the Aristotelian sorites must be reversed for the Goclenian. *Only the first premise can be negative and only the last particular.* Observe that in both types of sorites there can be only one negative and one particular premise, but in the Aristotelian the first alone can be particular and the last negative, whereas in the Goclenian the first alone can be negative and the last particular.

Irregular Arguments

As we learned at the end of the last chapter, there is a very extensive class of inferences which operate with categorical propositions, but which cannot really be reduced to syllogistic form without their character being fundamentally changed. For this reason they were there called *asyllogistic or non-syllogistic arguments.* Now among these, are some which have long been recognized by logicians under the name of irregular arguments. Indeed, attempts have been made to reduce them to the form of categorical syllogism by those logicians who make the assumption that an inference must be syllogistic to be logical. We have already shown that this is an unwarranted assumption. Hence the traditional irregular arguments are to be regarded as genuine asyllogistic arguments. They fall into two groups: (1) *inferences dealing with spatial, temporal and quantitative relations,* and (2) *a fortiori* (pronounced *ā for′-shî-ō-ri*) arguments.

1. *Inferences Dealing with Relations.* In dealing with *relations,* in the discussion of kinds of terms, we saw how fundamental such entities are for logic. Indeed there is a tendency to-day to restrict logic to relations, to the entire disregard of the concrete objects which stand in, or are constituted out of, these relations. Now among these logical relations *space, time* and *quantity* are most important. There are many arguments in everyday life which are based upon such relations. An example of each

will show how much they are used, and why they are asyllogistic.

(*a*) *Space:*

New York City is situated to the east of Buffalo.
Buffalo is situated to the east of Chicago.
New York City is situated to the east of Chicago.

Here we have four terms, namely: (1) *New York City,* (2) *situated to the east of Buffalo,* (3) *Buffalo,* (4) *situated to the east of Chicago.* But in spite of the fact that there is no middle term this argument is valid, since we know that whatever is east of Buffalo is also east of whatever Buffalo is east. This knowledge is simply a deduction from our knowledge of the nature of the abstract relation of space. Consequently, the proposition just stated has been used as a major premise in reducing such an argument to a categorical syllogism, thus:

Whatever is east of Buffalo is east of whatever Buffalo is east.
New York is that which is east of Buffalo.
New York is east of whatever Buffalo is east, namely, Chicago.

(*b*) A similar kind of argument in which the relation of *time* is employed is the following:

The Battle of the Marne was fought after the Battle of Waterloo.
The Battle of Waterloo was fought after the Huns sacked Rome.
The Battle of the Marne was fought after the Huns sacked Rome.

(*c*) And here is one which uses the relation of *quantity:*

A bushel of corn is a larger heap than a peck of corn.
A peck of corn is a larger heap than a quart of corn.
A bushel of corn is a larger heap than a quart of corn.

Let the student reduce the last two arguments to a categorical syllogism by formulating a major premise similar to the one given about space.

2. *The a fortiori Argument.* Literally this means that something is true for a far stronger reason, or that it is much more true than something else which is admitted to be true. There is a close similarity between this type of argument and the ones just mentioned in that it also uses the same principle, that what is known about the relation of one thing to another and of that other to a third can be used as a basis for an inference about the relation of the first thing to the third, only the *a fortiori* argument uses the relation of *causality,* instead of space, time and quantity. For this reason it is not only applicable to more kinds of subject matter, but it is set apart by a special value in disputation from the arguments just considered.

The *a fortiori* argument may be stated in two ways and therefore may be said to have two forms: (*a*) It may take the form of concluding that a certain event cannot happen because the causes which are alone capable of producing it operate still more strongly in another similar case without producing that effect at all, or so as just barely to produce it. For example, take the Biblical passage: "If the righteous scarcely be saved, where shall the ungodly and the sinner appear?" Here the event in question is salvation. In the case of the righteous the causes are assumed to be barely strong enough to bring about salvation. How then can any one suppose the causes would be strong enough to produce the effect in cases where men are ungodly and sinful? This example can be reduced to three propositions somewhat as follows:

It is admitted that it is easier for a righteous person to be saved than it is for an ungodly and sinful person.
But Scripture teaches that the righteous are just barely saved.
No sinful and ungodly person can be saved.

There is really no way of making this argument conform to the rules of the syllogism.

(*b*) The *a fortiori* argument may also take the form of concluding that something must be true, or some event must take place or be admitted possible, because in similar cases causes which are much weaker than in the case in question have succeeded in producing it. The saying of Jesus: "If ye, then, being evil, know how to give good gifts to your children, how much more shall your heavenly Father give good gifts to them that ask him?" exemplifies very well this type of the *a fortiori* argument. Let the student state it as three propositions as was done above with the example of the first form of this argument.

The *a fortiori* argument is quite common. In essays, orations, religious apologetics, and in argumentative literature in general it is frequently encountered. The ability to use it effectively is often very helpful in disputation, and an understanding of it adds to the delight which comes from general reading.

In reference to all of these irregular arguments it should be especially emphasized again that, whereas they have no real middle term and are not conformable to the rules of the syllogism, nevertheless their validity does depend on whether they correctly exhibit an actual implicative system or inferential whole. Unless such a real system is assumed, as it is in the Biblical arguments quoted above, as well as in the arguments used to exemplify space, time and quantity, the argument falls to the ground. The validity of an irregular argument depends just as much on the nature of its implicative system as does the validity of any other argument. The difference is that the system behind the irregular arguments is not, as in the syllogistic form of argument, expressible as a middle term.

Exercise IX

1. Indicate the order of each of the following enthymemes. Supply the missing proposition; indicate mood, figure, and fallacy (if any).

(*a*) No Russians are occidentals, and no Russians are capitalists.
(*b*) Some cartels are monopolies, because all cartels are organizations to control sales of commodities.
(*c*) All electron microscopes are lighted by electrons, and no ordinary microscopes are lighted by electrons.
(*d*) Being wise in your own conceits, you are lacking in humility.
(*e*) All political principles are humanitarian in purpose and all principles that are humanitarian in purpose should be supported.
(*f*) Since the hymn of hate was written against the British people, it must have been written by a German.
(*g*) We must conclude that all who are forbidden to vote are minors, because all non-registered citizens are forbidden to vote.
(*h*) All cyclotrons are atom-smashers, hence some betatrons are atom-smashers.

2. Construct an enthymeme of each of the three orders.

3. Restate each of the following irregular arguments with three propositions.

(*a*) The Battle of Okinawa was fought after the Battle of Iwo Jima because the Battle of Iwo Jima was not the last battle of the War in the Pacific.
(*b*) An electron microscope has higher magnifying power than an ordinary microscope, because the light rays which light its field are much shorter than those of natural light, and magnifying power is regulated by the length of the rays lighting the field.
(*c*) A jet propelled airplane is faster than a Diesel engine driven truck, but a Diesel engine driven truck is faster than a motor launch, hence a jet propelled airplane is faster than a motor launch.

4. State the following *a fortiori* arguments in three propositions and discuss the validity of each with respect to the truth of the basic assumption.

(*a*) Since it was easier for the League of Nations to solve the post-war problems of World War I than for the United Nations to solve the post-war problems of World War II, we know that the United Nations cannot succeed because the League of Nations failed.

(*b*) Since it is more difficult for the Big Three Nations to make a satisfactory peace treaty following World War II, than for the Big Five Nations to write the Versailles Treaty, we know that no treaty made by the Big Three will endure because the Versailles Treaty proved to be a scrap of paper.

(*c*) Since a nation divided against itself is in danger of falling, how much more danger is there that a divided civilized world is in danger of collapsing.

5. Tell whether the following sorites are Aristotelian or Goclenian. Supply missing premises and make syllogisms for each. Indicate mood, figure, and fallacy (if any) of each syllogism.

(*a*) No radiosondes antedate World War II.
Some things that antedate World War II are radars.
Some radars are used in airplanes.
Some things used in airplanes are used for weather reports.
Some things used for weather reports are maps.
Some radiosondes are not maps.

(*b*) Some globes are in galactic systems.
All spheres are globes.
All occupied areas are spheres.
All conquered territories are occupied areas.
Some occupied areas are in galactic systems.

(*c*) On ignorance depends karma; on karma depends consciousness; on consciousness depend name and form; on name and form depend the six organs of sense; on the six organs of sense depends contact; on contact depends sensation; on sensation depends desire; on desire depends attachment; on attachment depends existence; on existence depends birth; on birth depend old age and death, sorrow, lamentation, misery, grief, and despair. Therefore on ignorance depend old age and death, sorrow, lamentation, misery, grief, and despair.

(*d*) On the complete fading out and cessation of ignorance ceases karma; on the cessation of karma ceases consciousness; on the cessation of consciousness cease name and form; on the cessation of name and form cease the six organs of sense; on the cessation of the six organs of sense ceases contact; on the cessation of contact ceases sensation; on the cessation of sensation ceases desire; on the cessation of desire ceases attachment; on the cessation of attachment ceases existence; on the cessation of existence ceases birth; on the cessation of birth cease old age and death, sorrow, lamentation, misery, grief, and despair. Therefore on the cessation of birth cease old age, and death, sorrow, lamentation, misery, grief, and despair. Items (*c*) and (*d*) are adapted from the *Mahā-Vaga* (Buddhist Scripture). Quoted from *The Harvard Classics*, Vol. 45, p. 639. P. F. Collier and Sons Company.

SECTION IV

OTHER TRADITIONAL FORMS OF INFERENCE AND FALLACIES

CHART SHOWING DIFFERENCES BETWEEN HYPOTHETICAL SYLLOGISMS, DISJUNCTIVE SYLLOGISMS, AND DILEMMAS

	Hypothetical Syllogism	Disjunctive Syllogism	Dilemma
Form of Major Premise	If Einstein's theory of relativity is true, then Newton's laws of motion must be modified.	A. Either there is an ether drift or Miller's experiments are wrong. B. Either Hoover or the Senate is right on the tariff. C. A panic is due, either to mob fear or to economic conditions.	1. If he fails in the classics, he will lose his degree; and if he fails in mathematics, he will lose his degree. 2. If he obtains his degree, either he must pass in the classics or in mathematics. 3. If he obtains his degree, then he must pass in the classics, and he must pass in mathematics. 4. If he fails in the classics, he will lose the A.B. degree; and if he fails in mathematics, he will lose the B.S. degree. 5. If he is to obtain the A.B. degree, he must pass in the classics; and if he is to obtain the B.S. degree he must pass in mathematics.
Form of Minor Premise	1. Einstein's theory of relativity is true. 2. Einstein's theory of relativity is not true. 3. Newton's laws of motion must be modified. 4. Newton's laws of motion need not be modified.	A { 1. There is an ether drift. 2. Miller's experiments are wrong. 3. There is no ether drift. 4. Miller's experiments are not wrong. } (B and C will each yield the same four forms of minor premise.)	1. Either he will fail in classics, or he will fail in mathematics. 2. But he cannot pass in the classics, and he cannot pass in mathematics. 3. Either he cannot pass in the classics, or he cannot pass in mathematics. 4. Either he will fail in the classics, or he will fail in mathematics. 5. Either he cannot pass in the classics, or he cannot pass in mathematics.
Form of Conclusion	1. Newton's laws of motion must be modified. 2. Newton's laws of motion need not be modified. 3. Einstein's theory of relativity is true. 4. Einstein's theory of relativity is not true.	A { 1. Miller's experiments are not wrong. 2. There is no ether drift. 3. Miller's experiments are wrong. 4. There is an ether drift. } (B and C will yield the same four forms of conclusion.)	1. He will lose his degree. 2. He cannot obtain his degree. 3. He cannot obtain his degree. 4. Either he will lose the A.B. degree, or he will lose the B.S. degree. 5. Either he cannot obtain the A.B. degree, or he cannot obtain the B.S. degree.

Note that the minor premise and the conclusion both either affirm or deny an element of the major in the hypothetical syllogism and in the dilemma, whereas when one affirms the other denies, and when one denies the other affirms in the disjunctive syllogism. Note also that only the dilemma has a compound major premise, disjunctive minors, and conclusions.

CHAPTER XII

HYPOTHETICAL AND DISJUNCTIVE INFERENCE

In classifying propositions, at the beginning of Section II, we mentioned the fact that we would be dealing only with categorical propositions through Sections II and III, but that we would return in Section IV to a consideration of hypothetical and disjunctive propositions. We are now ready to make good that promise by showing how such propositions may be used in the construction of arguments which are really quite different from the categorical syllogism. We shall take up first the hypothetical syllogism, then the disjunctive syllogism, and then the dilemma, a complex form of inference involving both hypothetical and disjunctive propositions. (See chart on page 162.) If the student has forgotten the definitions of hypothetical and disjunctive propositions which were given in Section II, he should reread them before proceeding with this chapter.

The Four Forms of the Hypothetical Syllogism

The hypothetical syllogism is one whose major premise is a hypothetical proposition, whose minor premise is a categorical proposition which affirms or denies either the antecedent or the consequent of that major premise, and whose conclusion is a categorical proposition which affirms or denies that part of the major premise not affirmed or denied in the minor. Since both the antecedent and the consequent of the major may either be affirmed or denied in the minor premise, there are four possible forms, called moods. These are: (1) *affirming the antecedent,* (2) *deny-*

ing the antecedent, (3) *affirming the consequent,* and (4) *denying the consequent.* An example of each is here given:

1. Affirming the antecedent (Valid):

If the League of Nations is to be fully effective, the United States must join it.
But its friends will see that it becomes fully effective.
Its friends will force the United States to join it.

2. Denying the antecedent (Invalid):

If France is to receive reparation payments from Germany then she must revise the requirements of the Versailles Treaty.
France will not receive reparation payments.
France will not revise the requirements of the Versailles Treaty.

3. Affirming the consequent (Invalid):

If Bolshevik Russia is to survive, then the Soviet Government must abandon the principle of communism.
The Soviet Government has abolished the principle of communism.
Bolshevist Russia will survive.

4. Denying the consequent (Valid):

If peace in Europe is to be achieved, then the nations must cease seeking only their own advantage.
The nations will not cease seeking only their own advantage.
Peace in Europe will not be achieved.

The moods which affirm are called *constructive,* and the moods which deny are called *destructive.* Since an hypothetical proposition is really made up of two categorical propositions, each of which may be either affirmative or negative, the proposition itself does not have *quality.* Now to affirm one of the categorical propositions embodied in the hypothetical in the minor premise is to leave it as it is in the hypothetical major premise. But to deny is to make it negative in the minor if it is affirmative in the

major, or affirmative in the minor if it is negative in the major. Consequently a destructive hypothetical syllogism may be affirmative in form, in the sense that both the minor premise and the conclusion may be categorical propositions having an affirmative quality. The question of whether an hypothetical syllogism is destructive or constructive is always determined by the relation of the minor premise to the antecedent or consequent in the hypothetical major of which it is the affirmation or denial, and is never determined by the form of the minor premise or conclusion without regard to the major premise. Thus the following example appears to be destructive, when only the quality of the minor premise and the conclusion is taken into consideration, but it is really constructive, because these negative propositions are precisely like the antecedent and the consequent of the major premise:

> If war is not abolished, then happiness is not possible.
> War is not abolished.
> Happiness is not possible.

There may also be cases in which the antecedent is negative and the consequent affirmative and *vice versa*. Hence, there are several possible types of both destructive and constructive hypothetical syllogisms, but the student will have no difficulty determining the mood, if he will look at the relation between the minor premise and that part of the hypothetical major which this represents.

The Rule and the Fallacies of the Hypothetical Syllogism

Not all of the four forms of the hypothetical syllogism are valid. Indeed most logicians hold that there are only two moods because they count only the valid moods. Of the two moods in which the antecedent forms the minor premise only the constructive is valid. It is called the *modus ponens*. Of the two moods in which the consequent

forms the minor premise only the destructive is valid. It is called the *modus tollens*. Hence the rule for the hypothetical syllogism is: *Either affirm the antecedent or deny the consequent.* Of the other two moods, one denies the antecedent and the other affirms the consequent, and these are the fallacies of hypothetical reasoning.

It is a fallacy to deny the antecedent because we do not know that the expressed antecedent is the only condition on which the consequent will follow. There may be other antecedents connected with this same consequent. Hence the denial of one, gives no logical basis for denying the consequent. To reach a valid conclusion by denying the antecedent it would be necessary (1) to know what all of the antecedents to that consequent are, and (2) to deny all of them. Then, and only then, would we be logically justified in denying the consequent in the conclusion. Take this example:

<table>
<tr><td>If the Darwinian theory of natural selection is true,
If a true missing link between man and the anthropoid apes has been discovered,
If the Biblical account of creation is false,</td><td>then the theory of evolution is true.</td></tr>
</table>

Suppose, now, that any of these three antecedents could be absolutely affirmed true, and suppose that the truth of each does actually carry with it the truth of the theory of evolution, then the affirmation of any one would justify the affirmation of the theory. But suppose, on the other hand, that the first antecedent could be proven to be false, we would not then be justified in holding the theory of evolution to be false, since the truth of one of the other antecedents might establish its truth. Assuming a necessary connection between the antecedent and the consequent, we know that the affirmation of that antecedent carries with it an affirmation of the consequent, but we also know that

a denial of that antecedent does not carry with it a denial of the consequent, since the consequent may possibly follow from some other unexpressed antecedent which is just as necessarily connected with the consequent as is the one which is expressed.

On the other hand, it is a fallacy to affirm the consequent, because it may result from other antecedents as well as from the one which is expressed. To affirm the existence of a consequent does not enable me to say that any one antecedent is true to the exclusion of another, or to determine which among them is true. One could only do this in the case where he knew that there was and could be only one antecedent, and one rarely, if ever, knows this. On logical grounds we must assume that there are more antecedents than one. Hence, to revert to our example, to affirm that the theory of evolution is true would not justify affirming that the Darwinian theory of natural selection is also true. But if we know that the theory of evolution is false, we are justified in denying the Darwinian theory of natural selection.

The Relation of the Categorical to the Hypothetical Syllogism

It is easy to reduce an hypothetical to a categorical syllogism by simply stating the hypothetical major as a categorical proposition. To do this it is necessary to remember that there is a categorical element in every hypothetical proposition, or, in other words, that every hypothetical proposition makes a definite assertion of a relation of necessary connection between its antecedent and its consequent. Hence it can be put into categorical form by using the whole of the antecedent for the subject term, and the whole of the consequent for the predicate term. Since the connection between them in the hypothetical is always understood to be universal and to hold of all possible cases, and since

no hypothetical proposition can be negative in quality, the negation always being in the antecedent or consequent or both, the resulting categorical is always a universal affirmative, an *A* proposition. Take this as an example of the reduction of an hypothetical syllogism to a categorical:

> If President Wilson had taken the Senate into his confidence, the peace treaty would have been ratified
> The peace treaty was not ratified.
> President Wilson did not take the Senate into his confidence.

In categorical form this is Figure II, mood *AEE* and valid.

> The case of President Wilson taking the Senate into his confidence is the case of the peace treaty being ratified.
> The actual case is not the case of the peace treaty being ratified.
> The actual case is not the case of President Wilson taking the Senate into his confidence.

Here the major is stated as a singular *A*, but singular propositions are genuine universals.

When a fallacious hypothetical syllogism is reduced to categorical form, the fallacies of denying the antecedent and affirming the consequent show up as the syllogistic fallacies of illicit major and undistributed middle. Thus:

> If Moses wrote the Pentateuch, the theory that it is a compilation from several sources is false.
> Moses did not write the Pentateuch.
> The theory that it is a compilation from several sources is not false.

In categorical form this becomes Figure I, mood *AEE*, illicit major.

> All cases of Moses writing the Pentateuch are cases of the theory that it is a compilation from several sources being false.

The actual case is not the case of Moses writing the Pentateuch.
The actual case is not the case of the theory that it is a compilation from several sources being false.

Note that the major term is undistributed in the premise as the predicate of an *A* proposition, and distributed in the conclusion as the predicate of an *E* proposition.

Take this example of affirming the consequent:

If crystals grow, the inorganic world is really living.
The inorganic world is really living.
Crystals grow.

In categorical form this becomes Figure II, mood *AAA*, undistributed middle.

All cases of crystals growing are cases of the inorganic world really living.
The actual case is the case of the inorganic world really living.
The actual case is the case of crystals growing.

The value of reducing hypothetical to categorical syllogisms is that it shows that the distinction between the two is purely relative. Every proposition has in it a categorical element. It asserts something. On the other hand, it also has in it an hypothetical element. Whatever is asserted is always understood to be subject to some condition or limitation. Suppose I assert that, "All men are mortal." Here I appear to have a pure categorical assertion. Is there an hypothetical element in it? Certainly, as is shown by the attempt to prove it. Suppose some one were to argue that there is a race of men on some distant planet concerning whom Shakespeare's statement: "All that lives must die, passing through nature to eternity," is false. How could we prove to him that all men are mortal? Only by stating it hypothetically. If all men are like men

on the earth then they are mortal. But if the word man is to be extended to cover races of beings on distant planets concerning whom we know nothing, our proposition is false. It is only true when it is asserted subject to the reservation that it applies only to that universe of discourse defined by men born upon this planet. If man actually is what we take him to be, as a part of this system of earth-born creatures, then he is mortal. If a man, in our sense of the word man, then a mortal. Every judgment whatsoever has in it this hypothetical element along with a categorical element. The two aspects cannot be separated. The real value of reducing one type of judgment and of inference to the other is that it brings this undeniable fact into clear relief.

There also lurks here a danger, and that is that we should think that all genuine inference is categorical, hypothetical reasoning being unimportant because reducible to categorical form. This is apt to be the view of those who try to make all reasoning conform to the rules of the categorical syllogism. But the reverse is nearer the truth. In many cases hypothetical reasoning is far more important than categorical reasoning. The former is nearer the temper of science, whereas the latter tends to dogmatism. The method of hypothesis is a striking expression of the extent to which hypothetical reasoning enters into the procedure of scientific research, but we must postpone consideration of it until we take up the explanatory methods.

The Disjunctive Syllogism

The disjunctive syllogism is one in which the major premise is a disjunctive proposition, the minor a categorical proposition affirming or denying one of the alternatives of the major premise, and the conclusion a categorical proposition either denying (when the minor affirms) or affirming (when the minor denies) the other alternative of the

J. C. S.

major premise. There are, therefore, only two forms of the disjunctive syllogism, but owing to the fact that the major premise may be stated in three different ways, each one of these forms may be stated in three ways. The two moods are: (1) the *modus ponendo tollens*—the mood which denies by affirming, and (2) the *modus tollendo ponens*—the mood which affirms by denying.

The three ways of stating the mood which denies in the conclusion by affirming in the minor premise are as follows:

A is either B or C. 1. A is B. A is not C.	Dowie was either a fanatic or a fool. He was a fanatic. He was not a fool.
Either A is B or C is D. 2. A is B. C is not D.	Either the Soviet Government must give up its policy of repudiating the debts incurred by the Czar, or France will lose the money she loaned Russia. The Soviet Government must give up its policy of repudiating the debts incurred by the Czar. France will not lose the money she loaned Russia.
Either A or B is C. 3. A is C. B is not C.	Either Clemenceau or Lloyd George was responsible for the terms of the Versailles Treaty. Clemenceau was responsible. Lloyd George was not responsible.

The three ways of stating the mood which affirms in the conclusion by denying in the minor premise are as follows:

A is either B or C. 1. A is not B. A is C.	The earth's orbit is either a circle or an ellipse. It is not a circle. It is an ellipse.

Either A is B or C is D. 2. A is not B. C is D.	Either the state universities must be greatly enlarged, or private institutions must handle more students. The state universities will not be greatly enlarged. Private institutions must handle more students.
Either A or B is C. 3. A is not C. B is C.	Either Plato or Aristotle discovered the syllogism. Plato did not discover it. Aristotle discovered it.

The disjunctive syllogism is subject to the fallacies of the disjunctive proposition. One of these is that the alternatives may not be *mutually exclusive*, and the other is that they may not be *exhaustive*. A disjunctive proposition which fails to meet either of these requirements is said to be an imperfect disjunction. It is a very difficult matter to find disjunctives which meet both of these requirements. In fact, the whole problem in disjunctive reasoning is to reduce the subject matter to mutually exclusive and exhaustive alternatives. The student should examine each of the examples just given with a view to discovering cases in which the two alternatives overlap or where there is a third possibility. Although disjunctive propositions are often used in everyday life most of them will be found to be defective in one or the other of these respects.

Hypothetical and Disjunctive Chains of Reasoning

Just as we have interesting chains of reasoning made up of a series of categorical syllogisms, so we may have them made up of hypothetical and disjunctive syllogisms. Some curious strings of this sort gave amusement and, perhaps, comfort to some of the soldiers in the late war. Here is one

which has been widely published under the title, "The Philosophy of an Airman":

If you fly well there is nothing to worry about.
If you should spin, then one of two things may happen: Either you crash or you don't crash.
If you don't crash there is nothing to worry about.
If you do crash one of two things may happen: Either you are hurt or you are not hurt.
If you are not hurt there is nothing to worry about.
If you are hurt one of two things may happen: Either you are badly hurt or you are not badly hurt.
If you are not badly hurt there is nothing to worry about.
If you are badly hurt, then one of two things may happen: Either you recover or you don't recover.
If you do recover, then there is nothing to worry about.
If you don't recover you can't worry.

The disjunctive propositions in this string are excellent illustrations of imperfect disjunctions. Although the alternatives are exhaustive and mutually exclusive, they are obtained by negation, and hence the disjunctive propositions which they form contain only two elements, whereas they should contain either three or four.

There are also hypothetical sorites, which correspond to the Aristotelian form of the categorical sorites. Here is an example:

If the non-partisan agitation for the entrance of the United States into the League of Nations continues under the able leadership of former Justice Clark, a strong public opinion favoring our entrance will be created.
If a strong public opinion favoring our entrance is created, the politicians who have blocked our entrance will be forced to yield to popular demand.
If the politicians who have blocked our entrance are forced to yield to popular demand, there will be no opposition to our entrance and the United States will become a member of the League.
Therefore, if the non-partisan agitation for the entrance of the United States into the League of Nations continues

under the able leadership of former Justice Clark, there will be no opposition to our entrance and the United States will become a member of the League.

Note that the conclusion of this sorites unites the antecedent of the first premise with the consequent of the last. There are hypothetical sorites which have the Goclenian form, that is, the conclusion unites the antecedent of the last premise with the consequent of the first. Hypothetical syllogisms in which all the propositions are hypothetical are called *pure,* to distinguish them from the much more common *mixed* type dealt with above.

Exercise X

1. Indicate the mood and name the fallacy (if any) of each of the following hypothetical syllogisms. Reduce each to a categorical syllogism, and give the syllogistic mood, figure, and fallacy (if any).

(*a*) If civil war continues in China, communism will ultimately triumph. Civil war will not continue in China, and hence Communism will not ultimately triumph.

(*b*) If atomic bombs are used in future wars, battleships will be useless. Battleships will be useless, and therefore atomic bombs will be used.

(*c*) If the British statesmen continue to antagonize Russia, World War III is inevitable. The British statesmen will continue to antagonize Russia, therefore World War III is inevitable.

(*d*) If rockets are equipped with atomic bombs, great cities will be destroyed in the next war. But great cities will not be destroyed in the next war, therefore rockets will not be equipped with atomic bombs.

2. Use each of the following as the major premise of an hypothetical syllogism and supply the minor premise and conclusion. Indicate the mood and fallacy (if any) of each syllogism.

(*a*) If Mussolini had not allied himself with Hitler, he could have saved Italy from destruction.

(*b*) If the Japanese had not bombed Pearl Harbor, they would not have been bombed with atomic bombs.

(*c*) If uranium is a radioactive element, its atoms are fissionable.

(*d*) If plutonium atoms are capable of chain reactions, they can be used for the manufacture of high explosives.

(*e*) If radar pips are symbols, they are not signals.

(*f*) If radar pips have meaning, they are not physical entities.
(*g*) If a sound peace treaty is ratified, it can be made effective by the United Nations.
(*h*) If intelligence tests are not properly scored, they have no educational value.

3. Construct disjunctive syllogisms to illustrate each of the two moods, and each of the three ways of stating a disjunctive proposition.

4. Use the rules to test the validity of the following disjunctive syllogisms, and tell why you regard each as valid or invalid. Indicate the mood of each.

(*a*) Either the Labor Union or the United States Government will surrender, but the Government will not, therefore the Labor Union will surrender.
(*b*) Either war must be abolished, or civilization will be destroyed. Civilization will not be destroyed. War will be abolished.
(*c*) The superdreadnaught type of battleship has been made obsolete either by atomic bombs or by drones equipped with rocket bombs. It has not been made obsolete by drones equipped with rocket bombs. It has been made obsolete by atomic bombs.

5. Break up the following pure hypothetical sorites into pure hypothetical syllogisms.

(*a*) If the Japanese had not attacked Pearl Harbor, the United States would not have declared war,
If the United States had not declared war, the Japanese would have kept their Empire,
If the Japanese had kept their Empire, they would have held back Russian agression in the Far East,
If they had held back Russian agression in the Far East, the United States would not have been called upon to settle factional disputes in China,
If the Japanese had not attacked Pearl Harbor, the United States would not have been called upon to settle factional disputes in China.
(*b*) If the terms of the British loan are faithfully carried out, prosperity will return to England,
If prosperity returns to England, the economic conditions in the United States will improve,
If economic conditions in the United States improve, World trade will greatly increase,
Therefore, if the terms of the British loan are faithfully carried out, World trade will greatly increase.

CHAPTER XIII

THE DILEMMA

Definition of the Dilemma

The dilemma came into logic from rhetoric. It began to take its present form during the revival of learning in the Middle Ages, and there is still a great deal of diversity among logicians in their treatment of it. It is true that the Stoics recognized and insisted upon the importance of this type of reasoning. Moreover, it is really a development of the dialectical type of arguing used by Zeno, the Sophists and Socrates, before the development of Aristotelian logic. But in the form in which we know it to-day it goes back to the medieval rhetoricians.

Perhaps its most characteristic mark is that it puts an antagonist in a situation where he must accept one or the other of two alternatives, either of which is bad. Hence it has been defined as a form of argument "in which it is shown that whoever maintains a certain proposition must accept one or the other of two alternative conclusions, and that each of these involves the denial of the proposition in question" (Century Dictionary). The two alternatives are called the *horns* of the dilemma and an adversary is said to be *gored* or *impaled* on the horns. For this reason the dilemma is sometimes referred to as a *horned syllogism.* Another good brief definition is that of Joseph: "The dilemma is a hypothetical argument, offering alternatives and proving something against an opponent in either case."

While such definitions as the two just quoted give a good general idea of the nature of a dilemma they do not make its structure sufficiently clear. For this purpose a longer

definition, bringing out the nature and the mutual relations of each of the three propositions, is essential. I suggest this. The dilemma is a complex argument, consisting of a compound hypothetical or of a compound hypothetical and disjunctive major premise; and of a minor premise which either affirms the two antecedents or denies the two consequents of the major disjunctively, or, in the case where the disjunction is in the major, of a minor premise which denies categorically the two alternatives of the major; and of a conclusion which either affirms or denies, categorically or disjunctively, the other element or elements of the major premise. When the conclusion is a categorical proposition the dilemma is *simple*, and when it is a disjunctive proposition it is *complex*. When the minor affirms the antecedents the dilemma is *constructive*, and when the minor denies the consequents it is *destructive*. Hence there are four forms: (1) the *simple constructive*, (2) the *simple destructive*, (3) the *complex constructive*, and (4) the *complex destructive*. The definition just given is necessarily somewhat complicated. After carefully examining the following examples of each of the four types the student should return to it. It will then be found much clearer and will give a more adequate conception of the structure of the dilemma. While the definition cannot be fully understood apart from the examples, neither can they be fully comprehended apart from the definition.

Examples of the Four Forms

1. *The simple constructive*

> If A is B, E is F, and if C is D, E is F.
> But either A is B or C is D.
> *Therefore*, E is F.

Note that the compound hypothetical major here has two antecedents, each of which has the same consequent. This

makes it possible for the conclusion to be categorical, since it affirms this common consequent. The minor affirms disjunctively the two antecedents. Consider, now, this concrete illustration of this form of the dilemma:

If a man continues to live, he cannot collect his life insurance; and if a man dies, he cannot collect his life insurance.
But either a man will continue to live or he will die.
He cannot collect his life insurance.

This is a simple constructive dilemma, even though the conclusion is negative. For it is the relation of the minor to the corresponding parts of the major premise which determines whether a dilemma is constructive or destructive.

2. *The Simple Destructive*

If A is B, either C is D or E is F.
But C is not D and E is not F.
Therefore, A is not B.

Here the major is a compound hypothetical and disjunctive proposition, in which the two consequents are alternatives to a common antecedent. The minor is a compound categorical proposition, denying both alternatives, and the conclusion denies the common antecedent. The fact that each alternative has the same antecedent makes it possible for the conclusion to be categorical again. This is the only form of the dilemma in which the disjunction appears in the major premise. In all the other forms it appears in the minor. Notice the way the definition given above provides for this case.

The famous argument of the Greek philosopher, Zeno, proving the inconceivability of motion, exemplifies this form very well:

If a body moves, either it must move in the place where it is or in the place where it is not.

But it cannot move in the place where it is, and it cannot move in the place where it is not.
Therefore, it cannot move.

Some logicians hold that this form of the simple destructive dilemma is not really a dilemma, but a hypothetical syllogism with a disjunctive consequent. Among these writers some hold that there are only three forms of the dilemma, and others hold that the simple destructive should be formulated as follows:

If A is B, C is D and E is F.
But either C is not D or E is not F.
Therefore, A is not B.

Mellone gives this example:

If he goes to town he must pay for his railway ticket and his hotel bill.
But either he is unable to pay his hotel bill, or he is unable to pay for his railway ticket.
Therefore, he cannot go to town.

Still other logicians hold that both forms of the simple destructive dilemma are valid. "So far from there being no such thing as a simple destructive dilemma, there are two forms of it, against only one form of simple constructive dilemma." I agree with this position of Joseph.

3. *The Complex Constructive*

If A is B, E is F; and if C is D, G is H.
But either A is B or C is D.
Therefore, either E is F or G is H.

In this form the compound hypothetical major has two different antecedents, so that the conclusion has to be a disjunctive proposition as well as the minor. It is this fact that makes it complex. It is constructive because the minor affirms disjunctively the two antecedents of the

major premise. This is a very common form. Here is a concrete example:

> If the price of food increases, the poor will starve; and if the price of fuel increases, the poor will freeze.
> But either the price of food will increase or the price of fuel will increase.
> *Therefore*, either the poor will starve or the poor will freeze.

4. *The Complex Destructive*

> If A is B, E is F, and if C is D, G is H.
> But either E is not F or G is not H.
> *Therefore*, either A is not B or C is not D.

Here, again, the major premise has two consequents and two antecedents which necessitates a disjunctive conclusion. The minor denies disjunctively the consequents of the major premise, making the dilemma destructive.

> If the Soviet Government lives up to its ideals, it must retain communism, and if it gets financial aid from foreign governments, it must restore the property of their citizens living in Russia.
> But either it will not retain communism, or it will not restore the property of foreigners.
> Either it will not live up to its ideals, or it will not get financial aid from foreign governments.

How to Meet a Dilemma

The most vulnerable spot in a dilemma is usually in the disjunction. We have already seen how difficult it is to get alternatives which are mutually exclusive and exhaustive. When the alternatives offered in a dilemma are not exhaustive, the best way to meet it is to bring forward a third alternative which does not carry with it the untoward consequences. Since the alternatives are called horns this way of meeting a dilemma is usually spoken of as *escaping between the horns*. For example, some of

Zeno's critics attempted to escape between the horns by maintaining that a body can move *in between* the place where it is and the place where it is not. This would be a third alternative which would make motion possible, unless one replies that this is really equivalent to the place where the body is not, since it would be another place from where the body is.

A second way of meeting a dilemma is *to take it by the horns*. However, this usually means taking it by one horn rather than by both. This involves accepting one or the other of the alternatives, and showing that the supposedly bad consequences in the conclusion are not really bad or do not follow from the acceptance of this particular alternative. Escaping between the horns attacks the disjunctive premise, but taking it by the horns attacks the hypothetical premise in that it denies that there is any necessary connection between the alternative or alternatives and the antecedents or consequents with which they are connected in the hypothetical major. Thus Plato has suggested an answer to Zeno which is really equivalent to taking this dilemma by one of its horns. For he instances the top, which moves in the place where it is. Thus he might argue that Zeno's alternatives are all right, but that the denial that a body can move in the place where it is must be regarded as false, since a spinning top moves in the place where it is.

A third way of meeting a dilemma is to *rebut* it, which means to construct a *counter dilemma* leading to a conclusion exactly opposite to the conclusion of the original dilemma. One way to do this is to take the contradictory of the consequent of the second antecedent for the consequent of the first antecedent, and the contradictory of the consequent of the first antecedent for the consequent of the second antecedent, thus making a new major premise the reverse of the first. An example of this method of

constructing a counter dilemma is to be found in Euathlus' answer to Protagoras, in the famous dilemma known as the *Litigiosus*. The latter agreed to teach the former rhetoric for a fee, part of which was to be paid after Euathlus won his first suit. But Protagoras brought suit in order to force payment of the rest of the fee, and used the following dilemmatic argument before the jury.

> If Euathlus loses this case, he must pay by the judgment of the court, and if he wins it he ought to pay by his own agreement.
> But either he will win or lose this case.
> He ought to pay.

To rebut this Euathlus constructed the following counter dilemma, using the method stated above:

> If I lose this case I ought not to pay by my own agreement, and if I win this case, I ought not to pay by the judgment of the court.
> Either I must lose it or win it.
> *Therefore,* I ought not to pay.

There are also other ways in which a dilemma can be rebutted. In fact, any method which will establish a conclusion contradictory to the conclusion of the original dilemma, whether it is reached by using the same or different premises, is regarded as a satisfactory rebuttal. But a valid dilemma cannot be rebutted. Counter dilemmas can only be constructed when the original dilemma is defective. Let us, then, take up the defects to which dilemmas are liable.

The Validity of the Dilemma

Logicians differ in their estimates of the dilemma as a form of inference. Some hold that its defects are so numerous as to make it practically worthless. Thus Creigh-

ton writes: "Dilemmas, like all controversial arguments, are more often fallacious than valid." Jevons says the same: "Dilemmatic arguments are more often fallacious than not." Now it must be admitted that this is true. The staunchest friend of the dilemma must acknowledge that this type of argument is subject to a great many defects.

In the first place, in the cases where it is possible to construct a counter dilemma, there are usually two standards of reference involved, and appeal is made to each alternately in such a way as to cause a shifting of the argument. "There is no attempt to define terms, or to bring the different standards into relation; the argument moves and has its being in the mere limbo of undefined phrases where it seems possible to prove anything, just because it is possible to prove nothing" (Creighton). Thus in the *Litigiosus,* each man appeals to the judgment of the court and to the terms of the agreement, whereas the question could only be decided by appealing to one of these standards. Sometimes one standard is an underlying assumption, as in the famous sophism called the *Liar*. Epimenides, the Cretan, said that all Cretans were liars. Was he lying or telling the truth? This could be expressed as a dilemma, somewhat as follows:

> If Epimenides' statement is true, he is a liar, and if it is false he is a liar.
> Either it is true or it is false.
> He is a liar.

The only way to meet such arguments is to insist that only one standard of reference be used. Either Epimenides must be proven a liar without appealing to the judgment, "All Cretans are liars," or that judgment must be proven true without reference to whether it was uttered by a Cretan. And in either case, the double standard will

be eliminated. Many seemingly good dilemmas derive their apparent validity from this shifting from one standard to another.

Other defects to which dilemmas are liable, are the fallacies of the hypothetical syllogism—affirming the consequent, and denying the antecedent, and imperfect disjunction. As we saw above, it is very difficult, indeed, to find mutually exclusive and exhaustive alternatives. That is why it is so often possible to escape between the horns of a dilemma.

Although recognizing all these defects I cannot agree with those who belittle this type of argument. What if it is often fallacious? It is also sometimes valid. And when it is valid it is an absolutely invincible form of argument. It is doubtful whether any other form of argument has the strength of a valid dilemma. A thorough knowledge of it is of real practical value. It was by using this form of reasoning that Lincoln won his famous debates with Douglas. Many an intercollegiate debate has been won by the effective use of the dilemma. Lawyers and orators use it repeatedly. Indeed, logicians seem to shy away from it because it is an effective rhetorical device.

Exercise XI

1. The arguments against placing restriction on scientists' doing research in nuclear physics can be put in the form of a dilemma, the major premise of which is: "If the atomic bomb is a war weapon, it should not be under exclusive military control because of the international furor and disturbance that is created by the knowledge that one nation alone possesses such a weapon, and if it is not a war weapon, it should not be under exclusive military control because military authorities would then be encroaching upon civilian activities." Complete the dilemma by supplying the minor premise and conclusion, and tell which of the forms of the dilemma it exemplifies.

2. Obviously an opponent could take the dilemma suggested in question 1 by the first horn. This involves admitting that the

atomic bomb is a war weapon, and arguing that for security reasons we had better safeguard our national interests, and ignore all international repercussions. Could the dilemma also be taken by the second horn? If so, how? Try to construct a counter-dilemma to meet the dilemma in question 1.

3. Consider this alternative as a rebuttal to taking the dilemma in question 1 by the first horn as in question 2. If we safeguard our knowledge of atomic energy by strict military censorship, we will so restrict the free exchange of ideas among scientists as to prevent improvements in the bomb. Add this to the major premise in question 1 in such a way as to form the major premise for a trilemma, and then formulate the minor premise, and the conclusion for the trilemma.

4. In his *Story of the Political Philosophers* (McGraw-Hill Book Co., 1939) George Catlin criticises the theory of the state of Bernard Bosanquet, absolute idealist. Dr. Catlin writes: "A pitiful, hopscotching theory. To be pierced by either horn of the dilemma would be painful enough: Bosanquet contrives to be pierced by both, the God-State and Wilson's League as Summum Bonum." (p. 522) Using the following major premise (or one of your own formulation), construct a dilemma on both horns of which Bosanquet might be pierced, and then explain how a counter-dilemma could be constructed to escape from this situation. If Bosanquet asserts that the state is a God-State capable of settling all disputes, continued suicidal wars between powerful states are inevitable, and if he asserts that the League of Nations is a world community morally superior to any existing state, then actual states, being morally subordinate to a world community, are not God-States.

5. Make a dilemma out of each of the following major premises. Tell what kind of a dilemma results, and how it could best be attacked.

(*a*) If the pursuit of truth by scientists is an intrinsic cultural value, it is unethical to restrict such pursuit, and if freedom of scientists is essential to scientific inquiry, it is unethical to restrict such freedom.

(*b*) If atomic energy is a military weapon, civilians should not be allowed to control it, and if atomic energy is directed toward peaceful purposes, military men should not be allowed to control it.

(c) If the destruction of civilization is to be averted, either the atomic bomb must be controlled by an international commission, or its use must be outlawed by an International Superstate.

(d) If President Woodrow Wilson had accepted the Senate's Treaty of Peace, he would have lost the League of Nations, but if he rejected the modifications of the Senate, he had to leave the country in a state of war.

(e) If industrial strife continues on a large scale, prices will rise to inflationary heights, and if large increases in wages are granted workers, prices will rise to inflationary heights.

(f) If ex-service personnel continue as students in colleges and universities, either the Government must increase subsistence allowances, or permit students to earn more money.

(g) If uranium fission is discovered by Russian scientists, our great cities may be destroyed by atomic bombs, and if the Russians obtain the secret by espionage, our great cities are likely to be destroyed.

(h) If our military forces are kept in Europe, many of our citizens will be dissatisfied, but if our forces are withdrawn from Europe, communism will spread.

CHAPTER XIV

FALLACIES

The Classification of Fallacies

It has always been customary to close the exposition of traditional Aristotelian logic with an account of the chief fallacies. This is justifiable, if for no other reason, in order to explain the nomenclature of fallacies so prevalent in general literature.

The word *fallacy* is ambiguous. In *Fallacies,* page 172, Alfred Sidgwick distinguishes four meanings of the term. He writes: "A *fallacy* is used to mean: (1) a piece of false reasoning in the narrower sense; either an invalid immediate inference, or an invalid syllogism; a supposed equivalent form which is not equivalent, or a syllogism which breaks one of the rules; (2) a piece of false reasoning in the wider sense; whereby from true facts a false conclusion is inferred; (3) a false belief, whether due to correct reasoning from untrue premises (reasons or sources) or to incorrect reasoning from true ones; (4) any mental confusion whatever." All of these meanings are important. When the one who commits the fallacy does it for the express purpose of deceiving it is often called a *sophism,* and when it is due to obscure thinking or is unintentional it is sometimes called a *paralogism.* But frequently these terms are both treated as practically synonymous with the term fallacy.

It is exceedingly difficult to make a satisfactory classification of the fallacies to which thinking is heir. As the great logician, De Morgan, put it: "There *is* no such thing as a classification of the ways in which men may arrive at error; it is much to be doubted whether there ever can be." And to this should be added a passage from Joseph:

"If we are satisfied that logic should treat of fallacies, it is very difficult to be satisfied with any treatment of them. Truth may have its norms, but error is infinite in its aberrations, and they cannot be digested in any classification. The same inconclusive argument may often be referred at will to this or that head of fallacies."[1] And he goes on to explain that this is due (1) to the fact that there are many foolish and inconsequential arguments which can hardly be characterized, and (2) to the fact that the detection of some fallacies depends more on acquaintance with a particular scientific subject matter than on general logical training. However, it is essential that we have as good a classification as possible.

The earliest discussion of fallacies that pretends to give a classification is Aristotle's treatise, *On Sophistical Difficulties*. Here he enumerates most of the controversial tricks used by the sophists of his day. He uses the method of dichotomy to arrange these tricks of the sophist's trade into a kind of system, basing the division on the principle of language. That is to say, he had two main divisions, one consisted of the fallacies that are due to language (*in dictione*), and the other of those which are not (*extra dictionem*). Under these two heads he deals with thirteen different fallacies, six under the former and seven under the latter. We shall take up most of these later. The chief defect in Aristotle's classification is that, being based on the method of dichotomy, it does not lay down a positive characterization of the second group.

For this reason it seems best to adopt, with some modifications, the classification of modern logicians. It is set forth in Table VI.

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[1] *Loco citato*, p. 569. The sentence from De Morgan is quoted by Joseph on the same page (note).

TABLE VI. CLASSIFICATION OF FALLACIES

I. *Verbal*:
  1. Amphiboly
  2. Accent and special pleading

II. *Logical*:
  A. Formal:
    3. Illogical conversion and obversion
    4. Four terms (ambiguous middle)
    5. Undistributed middle
    6. Illicit major
    7. Illicit minor
    8. Negative premises
    9. Denying antecedent
    10. Affirming consequent
    11. Imperfect disjunction
      (*a*) alternatives not exhaustive
      (*b*) alternatives not mutually exclusive
    12. Dilemmatic fallacy (Shifting use of standards)
  B. Material:
    (I) Equivocation:
      (*a*) Whole and Part:
        13. Composition
        14. Division
      (*b*) Essence and accident:
        15. Accident
        16. Converse fallacy of accident
    (II) Presumption:
      (*a*) Circularity:
        17. Begging the question
        18. Complex question
      (*b*) Conclusion:
        19. Irrelevant conclusion
        20. Non-sequitur

This arrangement gives twenty fallacies. Many of these have already been sufficiently dealt with above, so that I shall only take up here those which have not already been explained. These come under three headings: (1) verbal fallacies, (2) fallacies of equivocation, and (3) fallacies of presumption.

Verbal Fallacies

1. *Amphiboly*. This is the fallacy which arises when a proposition is stated so that it is capable of two conflicting interpretations. This may be due to the grammatical construction or it may be due to figures of speech, especially metaphors. It is not as common in English as in languages like Latin and Greek. An interesting example is:

> The Duke yet lives that Henry shall depose.
> —*Henry VI*, Pt. II, Act I, Sc. IV.

which may mean that the Duke will depose Henry or that Henry will depose the Duke. The oracle is said to have answered Crœsus' inquiry as to how his military expedition against the Persians would fare, with the saying: "If Crœsus should wage war against the Persians he would destroy a mighty empire." No matter which of the two empires happened to be destroyed, defenders of the oracle could claim truth for the oracular response given to Crœsus. Most responses of soothsayers and prophets are put so that they are capable of two interpretations. This is the trick which modern mediums frequently employ in reporting supposed sayings of "spirits." Note that amphiboly is an ambiguous sentence and not an ambiguous term, such as we have in the fallacy of ambiguous middle, and with which we have already dealt as one of the formal fallacies of the syllogism.

2. *Accent and Special Pleading*. According to Aristotle, the fallacy of accent is a fallacy in the meaning of a word when differently accented. But modern logicians, follow-

ing the lead of De Morgan, extend it to cover cases where a writer italicizes a word, phrase or sentence in a passage quoted from another in such a way as to change the emphasis and mislead the reader. Similarly a quotation may omit qualifying words or phrases and give an erroneous impression of the author's meaning. But the fallacy of accent has also been extended to take in the fallacy known as *special pleading*—emphasizing the parts of a subject matter or those arguments for or against a theory which are favorable to your own position, and omitting the parts which are unfavorable to it. Recently, I heard a well-known socialist lecturer deliver an address on the Puritans in which he emphasized to the extreme their *vices,* and omitted altogether their *virtues.* This is vicious special pleading. It is a characteristic device of the demagogue, the propagandist, the religious sectarian and the partisan politician. Perhaps the best definition of special pleading would be: Telling that part of the truth which is favorable to your own position and that part which is unfavorable to your opponent, and keeping as mum as an oyster about that part of the truth which is unfavorable to your own position and that part which is favorable to your opponent. This wily method of lying is the modernized form of the fallacy of accent. But it must be admitted that special pleading is rather a material than a verbal fallacy.

Fallacies of Equivocation

These fallacies are included by Aristotle with the ones just considered as fallacies *in dictione* or verbal fallacies, but they seem to involve the content of thought rather than the mere verbal expression, and for that reason modern logicians treat them as material fallacies. The word *equivocation* is sometimes used in the broad sense to mean *ambiguity* of any sort, but it is here used in the narrow

sense in which the two fundamental categories or relations of *whole and part* and *essence and accident* lead to confusion of thought when an object is interpreted by means of them. A misapplication of the first of these relations gives rise to the two fallacies—*composition* and *division*, and a false use of the second to the two fallacies—*accident* and the *converse fallacy of accident*.

1. *Composition and Division.* These fallacies are reciprocally converse. The former means assuming something to be true of a whole which in reality is only true of some or all of the parts taken separately. For instance, it might be argued that a certain army is stronger than it really is on the ground that each unit is known to be very strong taken by itself. For it frequently happens that units that are strong when left to themselves, become weaker or even demoralized when thrown with others without proper coördination and integration. Consider also the false assumption that a football team must be strong because most of the individual players are known to be star players. Here the value of the indefinable teamwork is wholly ignored. No matter how good the individual players may be the team as a whole is certain to be weak without teamwork. The fallacy of composition usually results from overlooking that element in the whole which is not present in any of the parts taken separately. For that general character of the whole may be, and usually is, quite different from the individual characteristics of the members of the whole which is in question.

For the same reason, to infer that a general character of the whole must also be a character of every member in that whole is to commit the fallacy of division. The fact that a given football team is an excellent team does not justify the inference that every man on that team is an excellent player. Many excellent football teams contain some mediocre players.

The whole has characteristics of its own and so have the individual members of it. Mixing the two up is the fallacy of whole and part either in the form of composition or in the form of division. Logically we are not justified in concluding that individual characteristics are also characteristics of the composite whole. To do so would be to commit the fallacy of composition. Nor can we conclude that characteristics of the whole must necessarily be possessed by every member of the whole. To do so would be to commit the fallacy of division. Let the student show what the conclusion in each case would be in the following: Psychologists are generally agreed that *group judgments* are more likely to be true than *individual judgments*. By "a group judgment" is meant a collective or combined verdict reached by several individuals working together as in a jury, and by "an individual judgment" is meant the verdict of a single individual in such a group.

2. *Accident and the Converse Fallacy of Accident.* The best way to understand the two fallacies of accident is to think of them as arising from a confusion of essential and accidential features in an object or situation. The best definition of them is *treating accidental as essential features.* Essential features of an object or situation constitute it, and are, therefore, always present in every object or situation of that kind. But accidental features may or may not be present.

Some accidental features, however, are almost always present. Only occasionally do special conditions enter in to prevent their being present. Now to assert that one of these *general* accidental features will always be present, even in cases where there are special modifying conditions, is to commit the fallacy of accident. Thus your liking a good joke does not prove that you enjoy one on yourself. Whenever something is said about an object or situation without any qualification whatever, whereas the assertion

in question requires qualification, this fallacy is committed. Thus the argument that because children should be required to practice music each day a sick child should be required so to do would be an example of this fallacy. Or the argument that a particular blind and crippled horse must be valuable because horses are valuable domestic animals would also exemplify it.

The converse form of this fallacy argues that those accidental features of an object or situation which only apply under very special conditions are true under all circumstances whatsoever, regardless of whether the limiting conditions are present. Statements whose validity depends on special conditions cannot be held true in cases where such restrictions are absent. An example of this fallacy would be the argument that narcotics are valuable for individuals in normal health, because they are sometimes used as medicine for the sick. Or it might be argued that, because card playing is an interesting and harmless way to pass time on a voyage or in a social gathering, therefore no one should object to people spending all of their leisure hours at cards. It is a sign of moral debility to be such a slave to the use of narcotics as to have to justify their use by such an argument as the one just given. And it is a mark of extreme superficiality not to be able to enjoy leisure in any way except at cards. The fallacy of accident in one or the other of its forms usually vitiates the arguments we use to justify acts we know to be morally wrong.

Fallacies of Presumption

Presumption means taking something for granted without attempting to prove it, or without even being conscious of or acknowledging that it has been presumed. Sometimes the conclusion is really assumed in the premises. This is called circularity, or arguing in a circle. Sometimes the conclusion is assumed to be the cogent conclusion

which the disputant wants whereas it is really something else. There are two forms of each kind of presumptive fallacy. The two fallacies of circularity are *begging the question* or *petitio principii* and *complex question*, and the two fallacies of the conclusion are *irrelevant conclusion* or *ignoratio elenchi* and *non-sequitur*.

1. *Begging the Question*. The usual interpretation of this fallacy defines it as the taking of something for granted which is just as uncertain or unproved as the proposition it is used to establish. In a controversy, using as a premise what your opponent will not admit, or has not been forced to admit, is considered a commission of this fallacy. If anything is assumed which involves what is to be proved the fallacy of begging the question results. Thus one might argue that Shakespeare had a classical education from the internal evidence of his works. Another example is the following: "To maintain, as Sir Wyville Thomson does, that thirty-two degrees is the temperature of the floor on which the Antarctic ice sheet rests, is virtually to beg the question" (J. Croll).

A special form of *petitio principii* is known as *vicious circle* or *argument in a circle—circulus in probando*. Here two or more equally unproved assumptions or their equivalents are used to prove each other. When there are few intervening propositions between the two unproved assumptions the circle is *narrow*, and the fallacy is obvious, but where there are many intermediate steps the circle is *wide* and the fallacy is exceedingly difficult to detect. "For instance, if any one argues that you ought to submit to the guidance of himself, or his leader, or his party, etc., because these maintain what is right, and then argues that what is so maintained is right, because it is maintained by persons whom you ought to submit to; and that these are himself and his party; or again, if any one maintains that so and so must be a thing morally wrong, because it is

prohibited in the moral portion of the Mosaic law, and then, that the prohibition of it does form a part of the moral portion of that law, because it is a thing morally wrong—either of these would be too narrow a circle to escape detection, unless several intermediate steps were interposed.''[2]

2. *Complex Question.* This is *petitio principii* in the form of a question which takes something for granted. Titles of magazine articles or books sometimes take this form. Thus two recently published books have the titles: *Can the Church Survive the Changing Order?* and, *What and Where Is God?* The former assumes that there is a catastrophic changing order of society in which the church is likely to be swept away. The latter assumes that God is a *thing* which is in some *place*. The question, *''Who Is God?''* assumes God to be a person instead of a thing.

Lawyers sometimes make effective use of complex questions in cross-examining witnesses. Amusing examples of this fallacy are such popular pleasantries as, ''Have you lost your horns''?; ''Have you given up drinking moonshine?''; and, ''Have you stopped beating your wife?''

Now let us take up the two presumptive fallacies which have to do with the conclusion.

1. *Irrelevant Conclusion.* This fallacy occurs when, in disputation, one proves a conclusion which is not relevant to the argument. It may be valid in the sense that it really follows from the premises, but it is not germane, and does not touch the real point at issue in the controversy. It is a device resorted to by debaters to mislead an opponent and draw him away from his really strong points, thus causing him to waste his time. Any one who does this is said to be guilty of *befogging the issue.* An argument which befogs the issue is best rebutted by briefly pointing out that it is beside the point. Such an argument need

[2] Whately, *Elements* (9th ed.), p. 222.

not be met on its merits until its relevancy to the problem under consideration is established.

Several forms of the fallacy of irrelevant conclusion require separate brief discussion. The terminology used to name each type is best understood by its relation to the expression *argumentum ad rem,* which means an argument that is to the point.

(*a*) The *argumentum ad hominem* is an argument directed against the person advancing a thesis instead of against the thesis itself. Thus lawyers who are called to defend a rascal, knowing that they have no real case, frequently resort to the practice of abusing before the jury the attorney who is prosecuting their client. Politicians sometimes attack an opponent's character instead of the principles or policies for which he stands.

(*b*) The *argumentum ad populum* and the *argumentum ad misericordiam* are appeals to popular passions, feelings and prejudices or to popular sympathy instead of using cold reason to defend a position. The speech of Mark Antony over the dead body of Cæsar, in Shakespeare's *Julius Cæsar,* is an excellent example. That this trick is a very ancient one is shown by Socrates' statement in his famous defense before the Athenian judges, as given in Plato's *Apology:* "Perhaps there may be some one who is offended at me, when he calls to mind how he, himself, on a similar or even a less serious occasion, *prayed and entreated the judges with many tears, and how he produced his children in court,* which was a moving spectacle, together with a host of relations and friends; whereas I, who am probably in danger of my life, will do none of these things."

(*c*) The *argumentum ad ignorantiam* is an attempt to prove a position negatively by showing that its opposite is an absurdity or an impossibility or very undesirable. Some of the arguments in defense of the League of Na-

tions, for example, consist in pointing out the absurdity and the stupidity of war.

(*d*) The *argumentum ad verecundiam* is an argument which claims a proposition or belief should be accepted because it has been held by some highly revered person or institution. Thus all propagandists like to show that their views are *Christian.* Pacifist arguments are a conspicuous example. Sectarians or partisans often argue that people should join their particular sect or party because Garfield or Lincoln or Washington belonged or advocated the views in question. Some attacks on the League of Nations took the form of pointing out a conflict between our entering the League and Washington's famous policy of isolation. This fallacy is the vice which permeates the method of proving things by the results of a *questionnaire.* Thus it has recently been argued that the belief in immortality and the belief in a personal God are dying out because a large per cent of the answers from a group of scientists, who were asked whether they held these beliefs, were negative. There is practically no value in such arguments. In nearly every case they are pure propaganda.

2. *Non-sequitur.* This fallacy is an argument in which the conclusion is not only irrelevant to the point at issue, but is also invalid in the sense that it does not even follow from the premises. The most vicious form of *non-sequitur* is to infer that a certain proposition or theory has been refuted because some argument in favor of it has been exploded. Thus critics of theism or the belief in a personal God have argued that because no single argument proves the existence of God, that therefore there is no God. The different arguments may mutually support each other whether any one by itself is conclusive or not. And even though none of the arguments prove the existence of God, to refute the arguments is not a sufficient proof that there is no God. There may possibly be one even though no

absolute proof can be found. The same is true of attacks like those of Mr. Bryan on such a scientific theory as organic evolution. What if it is possible to show that some of the arguments in support of it are unreliable or even absurd? It remains true that others are very cogent. Mr. Bryan attacked the weak, and passed over the strong arguments, and imagined that he had refuted the theory of organic evolution; thereby giving us an excellent illustration of the logical fallacy of *non-sequitur*.

Exercise XII

Name the verbal or material fallacy in each of the following examples, and briefly state your reason for thinking it to be the fallacy you say it is:

1. Meaning and existence being entirely separate, it follows that metaphysical statements are absolutely meaningless, because they are primarily concerned with existence and no meaningful statement can be made about an existence which is entirely separate from meaning.
2. Politics is the science which answers the question: Who gets what, when, and how?
3. Since this proof half-dollar is an uncirculated half-dollar and is worth a premium as a collector's item, it follows that all uncirculated half-dollars have a premium value.
4. Inasmuch as proof coins usually have a premium value it follows that this dented proof coin also has a premium value.
5. Since official delegations assigned to the United Nation's Assembly are usually made up of statesmen who are willing to compromise on controversial issues, it follows that the Russian delegates will be in a compromising mood at the next session of the Assembly.
6. Since one Justice of the Supreme Court has issued a public statement attaking one of his colleagues, it follows that the Supreme Court is composed of indiscreet Justices whose decisions are merely individual opinions.
7. We know that atomic bombs are the most powerful explosives ever invented by man, and we do not know that men could not use them to destroy civilization, therefore they are in all probability capable of destroying human existence upon the earth, and undoubtedly will if another World War should break out.
8. There is no possibility of our having friendly relations with the Russian people, for Stalin is known by everybody to be a

ruthless dictator and he dominates the Russian people completely, and will not permit them to have friendly relations with the people of the United States.

9. Since our national debt has now reached astronomical proportions, our Government can no longer finance the social security pension system that has been established by social security legislation.
10. A judge insisted that a one-year-old baby boy remain in the courtroom on the ground that he was the plaintiff in a paternity suit. The judge said: "The only thing I could do would be to caution the plaintiff and cite him for contempt, if he became noisy." Assuming that he did cite the baby for contempt when he became noisy; what fallacy would the judge commit?
11. They travelled from Chicago to Seattle via Transcontinental Air Lines, and then flew to Sitka, Alaska.
12. Since social security is especially favorable to the laboring class, all laborers should vote to increase the social security tax.
13. A recent advertisement for cigarettes read: "Modern psychology says, 'If you deny yourself a cigarette when you want one, you jar your nerves.'"
14. Why is hatred of Germans by Frenchmen mixed with respect and grudging admiration?
15. Since this physician is a competent surgeon, he should perform a major operation on himself rather than trust another surgeon to perform it.
16. If the phrase "we know not to be true" means (1) "we do not know to be true" and (2) "we know to be untrue," what fallacy is committed in the following statement: "By faith we may believe that those who are known to be dead are not dead, for faith is a way of believing that which we know not to be true." Adapted from A. Wolf: *Textbook of Logic*, p. 284. The Macmillan Co.
17. The imposition of a gross income tax is a gross imposition on the taxpayer.
18. Since you believe in tolerance in all things, you have no right to be so critical of this man's ungentlemanly conduct.

PART II

INDUCTION OR SCIENTIFIC METHODOLOGY

SECTION V

THE RELATION BETWEEN INDUCTIVE LOGIC AND SCIENCE

CHAPTER XV

THE GENERAL NATURE OF INDUCTION

Definition of Terms

The terms induction and scientific method or methodology are both highly ambiguous. For instance, the term scientific method means to the animal psychologist the method of trial and error, to the sociologist, the economist and the educational psychologist the statistical method, while to the physicist and the chemist it means experimentation. And, as we shall presently see, there is a corresponding difference among logicians in their conception of induction. Then, too, a like diversity of opinion exists in interpreting the relation between scientific method and induction. Some logicians treat them as synonymous terms, while others would differentiate them. In this textbook they are used as synonymous, and the whole of Part II will be devoted to an explanation of what they mean. This chapter is an introduction to what follows. It will take the form of an exposition of the three more or less conflicting theories of the general nature of induction or methodology. While the term induction was used by Aristotle, and a certain form of inductive procedure was recognized by the medieval schoolmen, inductive logic in the modern sense of the word really began as an explanation of the thought processes employed in making the discoveries of the exact experimental sciences.[1]

In order to make the discussion of the three theories of induction intelligible a few technical terms must first be

[1] This development of methodology is sketched below in Ch. XXV, p. 338. It is recommended that the account there given be read before proceeding with this chapter.

defined. The word *phenomenon* (plural, *phenomena*), is quite generally used by logicians to mean any fact whatever, be it *event, thing, attribute, property, principle,* or *law,* providing it can be investigated scientifically or used as an element in a complete theory. Frequently the general problem under investigation in a particular scientific inquiry is referred to as the phenomenon. Then any unit used in the study of the problem is referred to as an *instance* of the phenomenon, and instances may be either *positive* or *negative,* according to whether the phenomenon is present or absent. Sometimes instances of a given phenomenon are themselves spoken of as phenomena. Then they are the phenomena of the more general phenomenon. The context in each case will make clear the sense in which the word phenomenon is used. To illustrate, suppose a scientist is investigating the cause of a certain color, for example, mother-of-pearl. This color or the cause of the color is the phenomenon under investigation. Now any object having the color is said to be a *positive instance,* and the same object without the color a *negative instance* of the phenomenon. Both positive and negative instances are frequently bunched together as phenomena or *data.* Then they are said to be the phenomena or data of the phenomenon, *i. e.*, color of mother-of-pearl. The term *data* is preferable because it avoids a double use of the term phenomenon, but both expressions are current.

Every inductive inquiry starts with concrete facts—with instances of some definite phenomenon—and seeks to discover and formulate a general principle or law to interpret these facts. All logicians agree in holding that induction is the reverse of deduction. In deduction, with which we were dealing throughout Part I, the logician is concerned with the relation between a conclusion and its premises. The truth of the generalization or universal proposition which is used as a premise is not in question,

but is simply assumed. The premises are taken for granted. Deductive logic is interested primarily in what it can get out of these assumed premises, by applying the rules of logic and drawing conclusions which are consistent with those rules. The rules themselves go back to the laws of thought, and constitute with these laws the set of postulates on which traditional logic is based. But every logician agrees in treating inductive logic as the reverse process of getting valid generalizations or universal propositions which can serve as premises. Starting with concrete facts —data, phenomena—induction asks: "How can a general law, universally valid, be obtained from these facts?" From a few instances of falling bodies, how can we reach the proposition, "All material bodies gravitate"?

This gap between *a few* instances of a phenomenon and *all* instances seems to be bridged in an inductive generalization. But how is it bridged? Or, to change slightly the figure, what justifies the so-called *"inductive leap"?* If induction is the process of drawing a general or universal conclusion from particular cases, or relatively few instances, what is the justification for making this inductive leap? Here is the real problem of induction, and according to the way it is answered we get entirely different theories as to the nature of induction.

However, before taking up the three most important theories, a warning must be given against being misled by the statement that induction is the reverse of deduction. *The two ways of proceeding are not antagonistic, but supplementary.* Deduction is really present in all inductive inferences in the form of some covert assumption as to the nature of the real world. To be sure, we must be on our guard against restricting deduction to syllogistic reasoning. But it is essential that the student keep in mind throughout the study of inductive inference that it is really only another aspect of deduction taken in the broad-

est sense of the word. Or, to express the matter more exactly: *Every thought process is really both deductive and inductive, and can be exhibited either as a deduction or as an induction according to the point of view used in interpreting it.* In the next chapter we shall deal with the basic assumption of induction. Now let us consider the three chief theories.

The Enumerative Theory of Induction

One form of this theory of induction is called *induction by simple enumeration.* It is the process of formulating a generalization on the basis of an enumeration of several or at least of a relatively large number of positive instances. It was the type of induction recognized by the scholastic logicians and justly condemned by Bacon as "puerile, precarious, and exposed to danger from contradictory instances." But although Bacon and his contemporaries effectively disposed of the enumerative type of induction as represented by the scholastic logicians, another form of this theory is still in very good repute among logicians. This form takes account of negative or contradictory instances, thus evading Bacon's criticism. It holds that the justification of the inductive leap is the enumeration of many cases in which the generalization holds, *plus the failure to find cases in which it does not hold.* The inclusion of negative instances greatly strengthens the enumerative theory of induction.

~~Since it takes account of negative instances, this form~~ of enumerative induction may be called *induction by complex enumeration* to distinguish it from induction by simple enumeration. It was advocated by the distinguished logician, Stanley Jevons, whose voluminous work entitled *The Principles of Science* is one of the most valuable contributions ever made to scientific methodology. He distinguishes two kinds of induction, both of which are enum-

erative. "An induction is called *perfect* when all the possible cases or instances to which the conclusion can refer have been examined and enumerated in the premises. If, as usually happens, it is impossible to examine all cases, since they may occur at future times, or in distant parts of the earth, or other regions of the universe, the induction is called *imperfect.*" And he adds: "It must be carefully remembered that *no imperfect induction can give a certain conclusion.* It may be highly probable or nearly certain that the cases unexamined will resemble those which have been examined, but it can never be certain." The great defect in this theory is brought out by this last sentence. For it makes *empirical generalizations,* such as, "All the books in my library are bound," more certain than *real scientific laws,* such as "All material bodies gravitate." Indeed, it is impossible to avoid interpreting this theory to mean that the certainty of a scientific generalization is proportional to the number of cases examined, and that any future instance may turn out to be negative. But we know that mere repetition of instances of material bodies gravitating does not add in any way to the certainty of the law. Nor do we fear that a negative instance will be discovered. Moreover, in establishing such a law, *variety* of instances is far more important than mere *number,* and one of each variety is enough, if it be typical.

In spite of this weakness in the enumerative theory it has recently been revived by Peirce, and other logicians. As revised, it may be spoken of as the *sampling type of enumerative induction,* since these logicians refer to the instances as *samples.* Thus Peirce defines induction as, "the inference from the character of a sample to that of the whole lot sampled." And he illustrates it thus: "If one draws a handful of coffee from a bag, and, finding every bean of the handful to be a fine one, concludes that all the beans in the bag are fine, he makes an induction." But

he repudiates what Jevons calls perfect induction. "Induction, as above defined, is called philosophical or real induction, in contradistinction to formal or logical induction, which rests upon a complete enumeration of cases and is thus induction only in form." Royce has developed this sampling type of enumerative induction into a valuable scientific method, with which we shall deal more in detail below, but he recognizes that the sampling method is really only a stage or starting point for "the organized combination of theory and experience" which is the real meaning of induction. Here Royce is nearer to our third theory of the nature of induction, but he never succeeded in freeing himself from this erroneous conception of induction.

In order to bring this error into clear relief, let me point out that Peirce really confuses two problems in the illustration just given. (1) One problem is that of determining the nature of an assumed homogeneous collection by determining the nature of a few samples from the collection. (2) The other problem is that of determining what constitutes a grain of coffee a fine grain. The last is the real inductive problem, and it cannot be solved by mere enumeration of grains of coffee. Various kinds of grains must be collected, compared and analyzed in order to formulate a general principle of grading, before a given grain can be referred to as fine. And, however important enumeration may be in such a process, taken by itself it is worthless. It must be supplemented by a careful analysis. After we have determined what a fine grain of coffee is, we are only justified in assuming that all the grains in a bag are fine grains by a careful use of the method of sampling. The importance of this method in scientific investigations must not be minimized, but the principle which it involves cannot justly be erected into a general theory of induction.

While rejecting, then, the enumerative theory of induction in all of its three forms, it is admitted that enumeration plays a significant rôle in scientific investigations. Jevons is right when he says "that the power of expressing a great number of particular facts in a very brief space is essential to the progress of science." Enumerating instances or collecting data is the first step in any experimental inquiry. But many great discoveries have been made by a careful study of two or three instances. This fact alone is a sufficient refutation of the enumerative theory of induction.

The Eliminative Theory of Induction

This theory of the general nature of induction holds that the justification of a scientific law is to be found in the fact that no other law offers a satisfactory explanation of the facts. According to this view, the scientist starts an investigation with a number of possible and plausible tentative hypotheses, each one of which is supposed to be a good explanation of the facts. Then each is put to the test of experience, and the one which stands after every means of testing has been employed is regarded as the true law, or theory. The tentative hypotheses might even be formulated as alternatives in a disjunctive proposition, thus: *The phenomenon X is either explicable by hypothesis A, or B, or C, or D.* Putting *D* to the test we find it defective, and so with *C* and *B*. Having eliminated all except *A*, we are justified in concluding that the true explanation of *X* is the hypothesis *A*. The hypothesis which is thus verified becomes a theory.

Joseph is an advocate of the eliminative theory of induction. He gives a simple and clear illustration of the process:

> Let a novice notice that his bicycle makes an unpleasant noise in running, and try to ascertain the cause. . . . In this problem

the determination of the alternatives among which the cause is to be sought is tolerably simple; for the noise must originate in one or other (or it may be several) of the non-rigid parts. Say that these are, on the machine in question, the axle bearings of either wheel and of the cranks, the bearings of the head, the pedal bearing, the clutch, the back-pedaling break, and the saddle springs. All that the rider has to do is ascertain which of these parts may be at rest while the noise occurs, and which may be in motion without the noise. . . . It is instructive to observe that the same process of elimination among the members of a disjunction is employed here, as if one were establishing a general conclusion. The novice has to fall back upon ascertaining the origin of the noise by showing that, among the possible origins to which it can be ascribed, there is none but one to which the facts permit him to ascribe it consistently with the principles of causation.[2]

The chief defect in this interesting theory is that it overemphasizes the negative aspect of induction. It rests entirely upon the principle that the failure to eliminate any hypothesis justifies treating it as valid. But suppose that there are a number of alternatives which cannot be eliminated. Then this method is powerless to determine the true one. This Joseph admits: "If a competing hypothesis enabled us equally well to find systematic connection in the same set of facts, I do not see how we could decide between them." Nor could any hypothesis ever be established, "unless we could show that all possible competing hypotheses had been overthrown." This admission of its chief advocate is a sufficient condemnation of the eliminative theory of induction as a general theory, however valuable it may be as a subsidiary process. Moreover, Joseph's own suggestion in the above quotation, that the aim of elimination is the finding of *systematic connection in a set of facts,* shows that induction by scientific

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[2] H. W. B. Joseph, *An Introduction to Logic*, pp. 447 *f*. The last sentence is condensed.

analysis really underlies, and is far more important than eliminative induction.[3]

However, the eliminative theory has the great advantage of emphasizing the overwhelming importance of the work of the theorizing activity of the human mind in reaching valid inductive generalizations.

Induction by Scientific Analysis

The true solution of the problem of the inductive leap is to deny that there is any gap or chasm between the particular instances, and the general scientific law which explains those cases. The scientist does not pass from the facts to the law. *He finds the law, or, better, the systematic connection in the instances.* This he is able to do by subjecting a variety of instances to careful analysis or even to experimentation, and by comparing and studying the results *until the underlying system, of which the instances and their elements are all fragments, is finally brought to light and formulated as a law.* This is what Royce meant, by speaking of induction as a combination of theory and experience. The facts are experienced, and when they are analyzed their elements are experienced—if not with the naked eye, then by the help of high-powered microscopes and other instruments of observation. But mere experience, mere enumeration, mere observation of the data resulting from analysis and experimentation is not enough. The mind must see as well as the eye. The underlying system of connections, in which each separate

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[3] See Bernard Bosanquet, *Implication and Linear Inference,* Ch. IV, for a full discussion of the difference between eliminative induction and induction by scientific analysis, especially oriented to Joseph's theory. The example of Harvey's theory of the circulation of the blood used below is borrowed from that chapter, although I have put it in my own words. Creighton has made the mistake of identifying eliminative induction and induction by scientific analysis. See his *Introductory Logic,* p. 198.

fact and datum lives and moves, must be brought to light by the intellectual activity of the investigator. This is theory. When theorizing brings the system to light the law is discovered. Now Joseph is right in insisting upon the value of eliminating various hypotheses. But there is more to a scientific induction than this negative work. There is the positive work of bringing to light the actual structure of the system involved. This is being done even in the process of elimination. Now the positive aspect of an inductive inquiry may be regarded as a thorough-going analysis of the data, plus a keen insight into the interrelations of the numerous elements resulting from such an analysis. This is the reason for referring to this as *induction by scientific analysis.*[4]

To clarify this interpretation of inductive inference, let

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[4] The failure to distinguish induction by scientific analysis from enumerative induction vitiates the, in many respects, valuable recent book entitled *Introduction to Reflective Thinking,* to which we have already referred. The authors write: "The tremendous advance from collected data to an hypothesis about these facts has itself sometimes been referred to as the 'inductive leap,' and it is indeed an enormous 'jump.' . . . The uniformity of nature, the conviction that things will continue to occur in the same manner as they have hitherto, is undoubtedly the best founded generalization in the whole range of human experience" (pp. 74, 93). Here the enumerative theory is plainly implied, and it is also erroneously assumed that its principle is established by experience, whereas, as Russell has said, it can neither be proved nor disproved by experience (See *Problems of Philosophy,* p. 106). But the theory of induction by scientific analysis is implicit in the following: "Our life is one long course of discovery . . . of fixed patterns in events. . . . They are, in fact, precisely that feature of our universe that makes it an ordered cosmos rather than a mere chaos. . . . These patterns which force themselves upon our attention and to the analysis of which the scientist devotes his life, are causal relations, uniform correlations between causes and effects, such that, unless some other cause has prevented it, when one part of the pattern is discovered the rest is sure to follow" (p. 92). This conception of fixed patterns is synonymous with our conception of the implicative system, and it is incompatible with the enumerative theory of induction.

us illustrate it by Harvey's great discovery of the circulation of the blood. He had before him certain facts. He knew there were veins and arteries, a heart and lungs. By the phenomenon known as the pulse he knew the blood flowed. He studied the action of the heart, analyzing it to determine its structure. Reflecting on his results he reached certain conclusions: such as, that there is no passage for blood in the wall which separates the chambers of the heart, that the blood passes from the right to the left ventricle of the heart through the lungs, and that the heart contracts and forces the blood through the arteries. This gave him enough facts about the circulatory system to enable him to surmount the theory current in his day that the blood in the arteries is entirely different from the blood in the veins. As a result of his analysis and careful reflection upon the facts, he *induced* the true nature of the circulatory system, *that the arteries and the veins are connected* so that the blood in the left ventricle of the heart is driven through the arteries into the veins by the force created by the heart's contraction, and is then carried back to the right ventricle, from which it goes to the left ventricle again through the lungs, where it is purified. The missing link in the circulatory system Harvey never saw by actual observation. His mind inferred its existence. That missing link was the connection between the arteries and the veins. In 1661, years after Harvey's theory was put forth (1628), Malpighi, using the newly invented compound microscope, discovered by actual observation the *"capillary channels,"* connecting the arteries and the veins, the existence of which Harvey had induced from his knowledge of the rest of the system.

Harvey's circulatory theory is not a generalization reached by jumping from certain facts over to it. It is not an inductive leap. Rather it is an accurate formulation of the actual system constituted by the heart, lungs, arteries,

veins and capillary channels. The discovery of the table of elements in chemistry is another excellent illustration of induction, and numerous others might be mentioned. Scientific analysis discovers the actual nature of a complex system of facts, expressing it as a universal law or theory. The law thus found is not something tacked on to the facts. It is rather the expression in language of the actual connections which bind the facts into a system. The nature of this system is always present in each fact entering into its constitution, only it cannot be found until enough, and a sufficient variety of facts are known to bring it to light. Now that we know of the existence of the circulatory system, we inevitably think of each member of it in terms of the whole system.

Our account of the general nature of induction has thus brought us to the same point which we reached in our exposition of the nature of inference in Chapter VIII, namely, to the conception that the essence of induction is the discovery of concrete order systems or inferential wholes. And it is entirely consistent with our general theory of knowledge that we should be led to this point. If all inference and all knowledge are in their essence the intellectual discovery of the actual orderliness in a definite set or region of facts, then our general theory of induction should emphasize the overwhelming importance of order systems. The counting of instances and the eliminating of competing hypotheses are valuable subsidiary phases of inductive procedure, but its essence is that process of scientific analysis and theorizing which brings to light the implicative systems in the nature of things. Royce is right in treating induction as a combination of theory and of experience.[5] For it involves enumeration and sampling, observation and

[5] See his article in Ruge's *Encyclopedia of the Philosophical Sciences*, Vol. I, pp. 78-92. See also the article "Induction" in the new (14th) edition of the *Encyclopedia Britannica*.

experimentation, but in all genuine scientific research each one of these subsidiary processes is really swallowed up in a creative theoretical activity of the mind, which leads the investigator straight into the secret recesses of the order system of which his data are fragments.[6]

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[6] In a discussion of the problem of the "inductive leap" entitled *The Relation Between Induction and Probability* (see *Mind*, N. S., Vol. XXVII, pp. 389-404 and Vol. XXIX, pp. 11-45), C. D. Broad subjects all three of the theories discussed above to a searching criticism. He rejects all three, but he admits that the scientific analysis theory is, in some important respects, superior to the other two. He summarizes his own theory as follows:

"All particular inductive arguments depend on probability and only lead to probable conclusions, *whatever* we may assume about nature. But *unless* we assume something about nature they give no finite probability to any law (*a*) because an indefinite number of alternative hypotheses which are not laws are as probable antecedently as the suggested law, and (*b*) because we are not equally likely to have met with any instance of the class under discussion, since it is quite certain that if there be instances remote in space or time they *could* not have fallen into the selection which we observed. What we actually assume is that nature consists of a comparatively few kinds of permanent substances, that their changes are all subject to laws, and that the variety of nature is due to varying combinations of the few elementary substances. These assumptions are neither self-evident nor mutually independent nor are they capable of complete proof or disproof by experience." (Vol. XXIX, p. 42.)

Experience only suggests "a simple ground-plan for the natural world to us." But "it is reasonable to suppose that this plan extends beyond what we have actually experienced." Yet we have no "knock-down" proof for this. "The kind of evidence is that this plan is suggested to us in a rough form by crude experience, and that, as we investigate nature more and more thoroughly, experience itself *suggests* ways in which we can state this plan with greater and greater definiteness and rigour, and, at the same time, nature is found to *accord* with the more rigorous and definite plan far better than it did with the first crude suggestion of a plan." (*Ibid.*, pp. 43 *f.*) *Cf.* W. E. Johnson, *Logic*, Part III.

These passages prove to me that Mr. Broad is much nearer to the scientific analysis theory than he is willing to admit.

CHAPTER XVI

SCIENCE—ITS ASSUMPTION, NATURE AND METHODS

The Inductive Assumption Underlying Science

In the last chapter the fact that all scientific investigations of concrete facts have in them a deductive element was pointed out. This means that every scientific investigator makes a definite assumption as to the nature of the world of facts. This assumption is at the background of all scientific research, even though the particular investigator may not be conscious of its presence in his own thinking. The scientist has his eye on the facts and not on his assumptions. But it is the business of the logician to focus attention upon the underlying conception of the world which is active in the scientist's intellectual work. What is the basic assumption underlying science?

According to the enumerative theory of induction, this assumption is that what is found to be true of the instances of a phenomenon which have been examined will always hold true of all other instances exactly like those, be they actually existent, imminent, or in an indefinite future. The future will resemble the past and new instances will behave like old ones. This is the assumption of the scientist, according to the enumerative theory of induction. *Why* what holds good of instances which have been examined *must* hold good of instances which have not been, and by the very nature of things cannot be examined, this theory does not answer. It cannot answer because it is based on another assumption which makes it impossible to give an answer, namely, that the world is made up of isolated, particular instances or events, and that the laws of nature are not expressions of actual relations in nature but

merely descriptions of the way these separate events appear in human experience. This assumption denies the *continuity* of nature. If the world is made up of isolated particular instances or events science is really impossible, and what we suppose we know about the world itself we only know about our own sense impressions or experiences. *"No account can be given of induction at all if a perception of the necessary relation of things is denied"* (Lossky). Or, as Whitehead has expressed it, such an assumption leads to the *"bifurcation of nature"*—dividing it into two parts, one of which consists of subjective sense impressions and the other of hard atoms or isolated particular facts. Consequently, instead of science being an account of nature as it is as an independent reality, it only tells us what our subjective experiences are. Instead of being a description of *the real order of nature*, it is only a description of *the succession of the scientists' own individual sense impressions.*

The assumption underlying the eliminative and scientific theories of induction is exactly opposite. It is that nature itself is an ordered system. The whole universe is an ordered and systematic whole, not a chaos of isolated facts. The laws of nature express actual necessary relations between natural entities or facts. They are actual bonds which hold the facts together into systems. The scientist discovers the actual integrations constituting the nature of things. This is the basic inductive assumption underlying every scientific inquiry.

John Stuart Mill expressed this assumption as *the law of the uniformity of nature.* The expression has often been criticized and it is undoubtedly open to objections, but when it is properly interpreted it is of real value. The concept of nature must mean more than physical nature. It must include the whole class of mathematical and logical relationships, which are sometimes spoken of as forming a world of *subsistence,* to distinguish them from the world of

existence in space and time. In Mill's mind the term *nature* included only the world of existence. Modern logicians are agreed in recognizing the reality of systems of objects and relations which do not exist in space and time, and are, therefore, said to subsist. If the law of the uniformity of nature is interpreted so as to include in the concept of nature those realities which *subsist,* as well as those which *exist,* it is a very serviceable formula for the basic assumption of science.

But uniformity must not be interpreted to deny the existence of an infinite *variety* and *novelty* in nature. Its real purpose is the assertion of *the absolute and unbroken reign of law.* "That which collectively we call nature is a vast assemblage of substances of divers kinds diversely intermingled: interacting with one another in ways that depend upon their abiding character and their shifting situation. Even what we call single things are highly complex, and their properties and behavior depend upon their composition, and upon the situation in which they are placed relatively to other things; we may believe that whenever one complex thing of precisely the same kind is placed in precisely the same situation as another, it will behave in precisely the same way; nor is more required by the principle of the uniformity of nature; and yet, we may doubt whether such precise repetition ever occurs. Watch the movements of a waterfall, how it breaks into a thousand parts which seem to shift and hang, and pause and hurry, first one, and then another, so that the whole never presents quite the same face twice; yet there is not a particle of water whose path is not absolutely determined by the forces acting upon it in accordance with quite simple mechanical laws. No one would suppose that because these mechanical laws are unchanging, the waterfall must wear a monotonous and unchanging face; and so it is, on a larger scale, with the course of nature" (Joseph, p. 402).

It is such a conception of nature which every scientist assumes and must assume. He cannot prove either by experience or by technical experimentation that the universe is such an ordered system. But he could not prove anything else if he did not assume this. It is the *major premise* in all scientific reasoning. The only demonstration it needs is that unless it is true all our human knowledge is a fairy tale.

The Nature of Science

What, then, is science? There are two ways of answering this question. One is to look at it from the side of the material with which science deals. We have just seen that there is nothing whatever, subsisting or existing, which does not, or may not, form material for science. We have to assume that the whole of nature, in the broadest sense of the term, and in all of its infinite detail, novelty and variety, is the content of science. "The work of the true man of science is a perpetual striving after a better and closer knowledge of the planet on which his lot is cast, and of the universe in the vastness of which that planet is lost" (J. N. Lockyer). From this point of view, then, science may be defined as that more or less integrated and systematized body of human knowledge of the whole universe, in its vastness as well as in its infinitesimal details, which is made up of all of the special fields of knowledge constituting the several courses offered in various technical schools, colleges and universities.

The other way of defining science approaches it from the point of view of its methods. Any body of knowledge is said to be scientific when it has been reached by a careful and painstaking use of *observation, experiment* and *reflection* on the data thus obtained. And the whole body of knowledge which has been so built up is called science. One test of whether a given piece of knowledge is scientific is

whether other trained observers can verify it by using the same observations and experiments, and by going through the same processes of reflection. Not every one is capable of making such a verification because most people lack the necessary disciplinary training and the freedom from personal and environmental prejudices. Another test of whether knowledge is scientific is whether it has been *depersonalized* and *de-emotionalized.* The emotional halo which characterizes so many burning convictions held by unscientific people is "sicklied o'er with the pale cast of thought" whenever a cool-headed scientist puts the belief in question to the test of the laboratory.

The Aim and Mood of Science

The primary aim of science is the discovery of the actual connections in the particular material with which it deals. It tries to give an accurate, clear and consistent account of order systems as they are. That is why it must be depersonalized. The continuity and coherence which constitute the very nature of things is delved after by every scientific investigator. While it is generally admitted that the capstone of the edifice of science is its modifications of the conditions of human life, so as to add to the enrichment of human civilization and "to illumine man's little day," nevertheless the real motive which spurs on the true-hearted scientist is not practical but a burning curiosity to find out the truth. There is nothing so irritating to a true scientist as obscure and opinionated thinking. He longs to see everything as it actually is. To be a scientist, then, one must have (1) a passion for facts; (2) be extremely cautious, even to the point of scepticism, in reaching a conclusion; (3) have a clear mind, which is disgusted with the credulous acceptance of dogmatic assertions, and (4) have the ability to see the relations between phenomena which the average person regards as entirely unconnected.

"Science is born anew in that wonderful world within each man when with deliberate will he succeeds in thinking about the principles of his work in the great world without in a clear, logical and systematic way, and courageously puts his conclusions to the test of experiment; and the so-called sciences are the written record of such thinking, only more extensive because clear, systematic and consistent, and more true to reality because they have been tested by countless experiments and experiences in the race" (Benchara Branford).

Classification of the Sciences

There is an appalling diversity of opinion among educated people as to what kinds of subject matter have really reached that stage of development where they can be referred to as sciences. Some would limit the use of the term to the so-called exact sciences, like chemistry and physics. Others would include as sciences all the *social sciences,* such as history, economics, political science and education. Others would go a step further and include ethics, æsthetics and metaphysics. Indeed, a book has recently appeared by a prominent theologian, D. C. MacIntosh, entitled *Theology as an Empirical Science.* Now there is unquestionably a vast difference between such a body of knowledge as chemistry or physics, and theology, or even political science. This difference has been well expressed by J. A. Thomson: "An exact science is like a solar system, a young science is like a nebula, but we see no reason why the student of dreams may not be as 'scientific' as the student of rocks, provided that he does not allow assertion to outstrip evidence, and understands what he knows." And the logician must concur with him in the "commonsense view that science includes all knowledge, communicable and verifiable, which is reached by methodical observation and admits of concise, consistent and connected for-

mulation."[1] The term science is used in this wide sense in the following classification of the various sciences.

The first group to be distinguished is the abstract and formal sciences, which neglect concrete objects and deal with purely formal relations. Here belong mathematics, in all of its branches, and logic, especially mathematical or symbolic logic, as distinct from inductive logic. These sciences are all highly deductive. They assume certain sets of axioms and "primitive ideas" to be true for the purpose of deducing their detailed consequences.

All the other sciences are much more concrete. Inasmuch as they aim to give an accurate *description* of actual phenomena, they are sometimes spoken of as descriptive sciences. Among these, five *general* and *fundamental* sciences are usually distinguished. They are chemistry, physics, biology, psychology, and sociology. While each has a relatively distinct province they inevitably overlap in many ways.

A group of *derivative sciences* depend upon these general and fundamental concrete sciences. Botany, zoölogy and paleontology are special sciences within the field of biology. Much of astronomy and meteorology comes under physics and a great deal of mineralogy under chemistry. But these special derivative sciences have to be distinguished from *sub-sciences* such as morphology, embryology and physiology.

A special group of the derivative special sciences is

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[1] See his article entitled "Science" in the *Encyclopedia of Religion and Ethics*, from which some of the material presented in this chapter has been taken, with the permission of Charles Scribner's Sons. See also his "Introduction to Science," in the *Home University Library*, and his article entitled "Science and Modern Thought," in the *Outline of Science*, Vol. IV, pp. 1165 *ff*. The outline map of scientific knowledge below is taken, with some modifications, from the latter (p. 1171), with the permission of G. P. Putnam's Sons.

known as the *synoptic sciences.* These are really combinations of parts of other sciences. That is to say, they use the methods and many of the principles of other sciences, but develop such a unique interpretation of their subject matter as to entitle them to separate rank. Geology, for instance, focuses attention upon the earth, but it combines parts of physics, chemistry and paleontology. Geography and anthropology are other examples. Even astronomy, one of the most ancient and sublime of all sciences, is really a mixture of mechanics, thermodynamics, optics and chemistry, with special reference to the constitution and movements of sun, moons, planets, stars, and other celestial phenomena

OUTLINE MAP OF SCIENTIFIC KNOWLEDGE

Abstract, Formal Sciences	*General Descriptive Sciences*	*Special Derivative Sciences*	*Synoptic Sciences*	*Applied Sciences*
METAPHYSICS	SOCIOLOGY	Ethnology	History	Economics
LOGIC	PSYCHOLOGY	Æsthetics	Anthropology	Education and Vocational Psychology
	BIOLOGY	Zöology Botany	Natural History	Medicine
MATHEMATICS	PHYSICS	Astronomy Meteorology	Geology Geography	Engineering Aëronautics
	CHEMISTRY	Mineralogy	History of the Solar system	Metallurgy Agriculture

Finally we have to speak of *applied sciences.* They are either separate departments of the various special sciences, or combinations of principles taken from several of these

with special reference to their bearing on practical life, particularly on the arts and crafts. Examples are engineering, agricultural science, medicine, educational psychology and aëronautics. The very new science of vocational psychology is an applied science within the field of theoretical psychology.

The above "outline map of scientific knowledge" may very well serve graphically to summarize the contents of this paragraph.

Classification of the Scientific Methods

For the purposes of logic, it is necessary to supplement this classification of the sciences with a classification of the more important logical methods employed in science.

The first group to be distinguished may be called *general basic methods,* because they really underlie and are used in the other methods. Three such methods need to be distinguished: (1) *observation,* (2) *enumeration,* or, as it is sometimes called, *simple counting,* and (3) *classification.*[2] We dealt with the first two processes in our account of enumerative induction, as well as in the paragraph on the nature of science. Both of these methods are sometimes grouped together under the heading, *collecting the data.* Classification has already been expounded in Chapter V under logical division.

All other scientific methods, except the special technical methods which are not here considered, fall into three groups. The first group may be called *methods of probability,* because they seek laws which can be applied to great masses or assemblages of facts, but which cannot be applied to the individual members of these collections with anything like the degree of certainty with which they can

[2] Some writers include under general basic methods such processes as *analysis, synthesis, comparison, imagination* and even *experimentation.*

be applied to the whole mass. Two methods belong here, namely, *sampling* and *statistics*.

Another group of methods are those known as Mill's *experimental methods*. These are primarily concerned with the problem of causality and may, therefore, rightly be spoken of as *methods of causality*. There are five of these methods, but, as we shall show later, three of them are really only corollaries of the first two. Mill's methods are: (1) *the method of agreement;* (2) *the method of difference;* (3) *the joint method of agreement and difference;* (4) *the method of concomitant variations,* and (5) *the method of residues.*

The last group of methods may be called *methods of explanation*. They utilize processes which are employed in the other methods, but they represent the means at our disposal of getting at a complete explanation of all of the facts connected with a given problem. Thus they attempt a further synthesis than the methods of probability and causality. Here four methods may be distinguished: (1) *analogy;* (2) *hypothesis;* (3) *the complete method of explanation,* and (4) the *historical method*. The third of these is really the most important of all and may be called the scientific method *par excellence,* since it combines as a single method all of the other methods. It might be thought that analogy should be included among the methods of probability, instead of being put with the methods of explanation, because it never gives anything more than a certain degree of probability. But it has usually been employed to reach a complete synthesis of a large number of facts, and for that reason it is rightly regarded as a method of explanation.

This classification of the methods is really an outline of the succeeding chapters. It may be summarized as follows:

A. General Basic Methods
   1. Observation
   2. Enumeration or Simple Counting
   3. Classification

B. Other Methods
   I. METHODS OF PROBABILITY
      4. Sampling
      5. Statistics

   II. METHODS OF CAUSALITY (Mill's Methods)
      6. The Method of Agreement
      7. The Method of Difference
      8. The Joint Method of Agreement and Difference
      9. The Method of Concomitant Variations
      10. The Method of Residues

   III. METHODS OF EXPLANATION
      11. Analogy
      12. Hypothesis
      13. The Complete Method of Explanation
      14. The Historical Method

This is by no means an exhaustive classification of all the methods used in scientific investigations. Every science has to develop special methods of its own. But for the purposes of an elementary textbook this classification is useful. Mellone calls the complete method of explanation the *Newtonian method,* because it was employed by Newton in his discovery of the law of gravitation. Mill spoke of a method similar to it as the *deductive method of induction.* Dewey calls it a *complete act of thought.* But no matter what it is called, it is undoubtedly extremely important. We shall now take up in separate sections a detailed consideration of each group of methods, except the first, the methods in that group having been sufficiently dealt with already.[3]

[3] The article entitled "Scientific Method" in the new (14th) edition of the *Encyclopedia Britannica* is especially recommended to supplement this chapter.

SECTION VI

PROBABILITY AND THE STATISTICAL METHODS

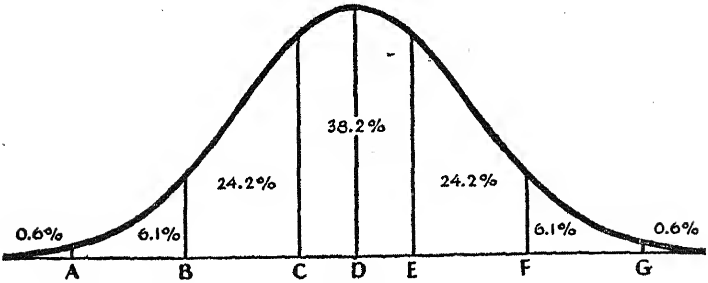

FIG. VIII. THE NORMAL PROBABILITY CURVE

Standard deviation is the square root of the average of the squares of all the individual deviations from the group average or mean. In Fig. VIII D shows the position of the average or mean in a normally distributed group. C and E are at points located ½ of a standard deviation (S.D.) below and above the mean; B and F at 1½ S.D., and A and G at 2½ S.D. from the group mean. The percentages of the total number of cases falling within each of these divisions is shown above.

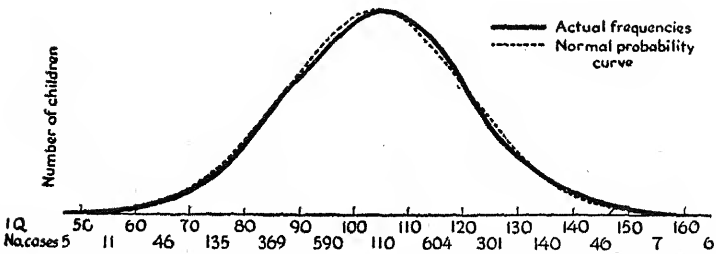

FIG. IX. THE NORMAL PROBABILITY CURVE

Fig. IX shows the distribution of IQ's of 2,970 children tested by the 1937 revision of the Stanford-Binet Scale (original standardization group).

Both figures are from Florence L. Goodenough's *Developmental Psychology*, 2nd Ed., 1945, pp. 369 and 370 respectively. Reprinted by permission of D. Appleton-Century Company, Inc.

CHAPTER XVII

PROBABILITY

"Probability," said Bishop Butler, "is the very guide of life." And if he had lived to see the development of statistical science in our day he would have realized more than he could possibly have realized when he wrote it, how profoundly true this assertion is. Not only are we constantly using such expressions as so-and-so is *quite*, or *very* or *extremely probable*, but in our modern civilization we are constantly solving social, political and economic problems which baffled every attempt at solution at the time Bishop Butler lived. (He died in 1752.) And we are doing this by an application of the methods of probability. However, before we consider these methods it is worth while to deal with the question: What is probability? This is an extremely important logical question which has been answered by a very broad definition and by a much narrower and more definite one. Let us begin with the former.

Probability as Equivalent to the Relativity of Knowledge

For many years a theory known as the *doctrine of the relativity of knowledge* has been in vogue among logicians. According to this theory none of the propositions which make up the content of human knowledge can be held absolutely true. On the contrary, every bit of human knowledge must be treated as only *probable opinion*. And its probability depends on the strength of the evidence in its support. But this never reaches certainty. If human beings possess any certain knowledge at all it is direct knowledge or knowledge by acquaintance, to use the termi-

nology which was explained in the chapter on inference. Nearly all direct knowledge and absolutely all of indirect or inferential knowledge is held by this view to be merely probable. For example, every scientific law lacks certainty but possesses a degree of probability varying with the number of instances which have been examined in formulating it. To summarize this view in a definition: *"Probability is a relation, essentially indefinable, between any proposition and the evidence which supports it"* (Keynes).

This is the conception of probability which dominates the logical literature produced by the new school of realistic logicians, which was founded by Bertrand Russell. In his little book entitled *Problems of Philosophy* Russell has a chapter entitled "Induction" in which he succintly sets forth this view, and it has been developed at great length in J. M. Keynes' study entitled: *A Treatise on Probability* (1921). Russell says: "The greater part of what would commonly pass as knowledge is more or less probable opinion." And again, speaking of such everyday expectations as expecting the sun to rise to-morrow and the bread we shall eat at our next meal not to poison us, he writes: "It is to be observed that all such expectations are only *probable;* thus we have not to seek for a proof that they *must* be fulfilled, but only for some reason in favor of the view that they are *likely* to be fulfilled." He points out that domestic animals expect food when they see the person who usually feeds them. But "the man who has fed the chicken every day throughout its life at last wrings its neck instead, showing that more refined views as to the uniformity of nature would have been useful to the chicken." But the significant point in his theory is that he treats scientific laws such as the law of gravitation and the law of the conservation of energy as only being different in degree from such expectations. And this is even true of the belief in the reign of law which we referred to

as the law of the uniformity of nature. Concerning such scientific laws, he writes:

> It must be conceded to begin with, that the fact that two things have been found often together and never apart does not, by itself, suffice to *prove* demonstratively that they will be found together in the next case we examine. The most we can hope is that the oftener things are found together, the more probable it becomes that they will be found together another time, and that, if they have been found together often enough, the probability will amount almost to certainty. It can never quite reach certainty, because we know that in spite of frequent repetitions there sometimes is a failure at the last, as in the case of the chicken whose neck is wrung. Thus probability is all we ought to seek (pp. 101 *f*.).

This shows clearly how far this school of writers identifies the theory of the relativity of knowledge with probability.

There is much truth in this view. All logicians recognize that the uniformity of nature, in the sense in which it was explained in the last chapter, has to be assumed and cannot be established by any appeal to instances, however numerous the instances may be. But it certainly is highly misleading to treat the belief in the reign of law as being on a par with a chicken's supposed belief that the man who has come to wring its neck has come to feed it. Can any one really think that Newton's law of gravitation exhibits as little knowledge of the actual structure of nature as a chicken has of the system of relations between it and its owner? Or, to put the question in more precise form: Can any one believe that the difference between Newton's law and the chicken's belief is only a matter of *degree of probability?* Such a view is simply the logical consequence of the enumerative theory of induction. If the truth of a law of nature is dependent upon the number of instances of the phenomenon which have been examined, then the theory

that all such laws are only probable becomes the only possible view. But if there is an order of nature into which the scientist is able to enter by careful observation, analysis and reflection, then the element of probability is no longer significant. For instance, what good does it do to insist on the element of probability in Harvey's theory of the circulation of the blood?

However that may be, it is not justifiable to extend the word probability to cover the degree of truth in an inductive generalization or law of nature. This is an entirely different meaning of the word than that ordinarily given in logical discussions. The real logical meaning of probability is much narrower and far more definite.

The Logical Meaning of Probability

There are, in the order of nature, many extremely complex phenomena and numerous relatively homogeneous masses of material, which, precisely because of their complexity and great extent, are beyond the reach of the experimental methods employed in the exact sciences. Embedded in these phenomena are so many and such intricately interwoven factors that the real operative cause cannot be fully understood. Even if it is discovered and its exact nature becomes known, the other factors present with it may either entirely neutralize its effect in some instances or greatly accelerate its activity in others. Given such a complexity, all that can be done is to count the instances when the phenomenon under investigation occurs and the instances when it fails to occur, assuming that the operative cause is present in both sets of cases, along with varying conditions. By means of such an enumeration, and in proportion to its exhaustiveness, we can estimate the probability of the event occurring whenever the operative cause is present. An example of this sort of phenomena is the rise and fall in the stock and money markets. This is

extremely susceptible to variations among highly complicated forces far beyond any one's power to calculate or predict. Similarly certain conditions of the wind, temperature and other atmospheric phenomena will one day bring rain and another fair weather. In such cases counting instances enables the investigator to get at a fairly definite probability.

Here is an entirely different, and a far more precise meaning of probability than that which identifies it with a relation between a conclusion and the evidence which supports it. To bring the difference to sharp expression, it should be especially noted that there is no *universal generalization* here to which the probability is attached as a relation. In such phenomena the probability is attached to the *particular* case, and not to the general law. All we can do is to say that the probability of the weather tomorrow being fair or rainy is such and such, based on the results of observation of previous instances. *The real logical meaning of probability is to be found in this comparison of the number of cases when the event occurs with the total number of cases, both positive and negative.*

Three Aspects of Probability

If, now, we seek a little deeper analysis of probability, three different aspects may be especially noted:

1. One form of probability is based entirely upon observation and enumeration of the number of times a given event has happened in the past, whenever there has been a repetition of a certain set of highly complex circumstances, which sometimes produce it and sometimes do not. Suppose the event in question has happened seven and failed to happen three times out of ten cases studied. Then the probability is said to be 7/10. In this case the *counter probability* is said to be 3/10. The rule for calculating the probability of an event is: "Take as numerator

the number of times when the event has been known to occur, and as denominator the total number observed, both of happening and failure; and the resulting fraction will be the probability." If the probability is unity we get the fraction 1/1 which is equivalent to complete certainty. On the other hand 0/1 is equivalent to complete impossibility. All the fractions between 1/1 and 0/1 express the various degrees of probability. This phase of probability may be referred to as *experiential probability* because it can only be determined by actual observation and enumeration of past occurrences. The illustration of the weather mentioned above exemplifies this aspect of probability.

2. *Theoretical probability* is that which is based on a deduction from the known structure of the phenomena under investigation without regard to the actual observed behavior. Thus we know from the nature of a penny that the probability that a given throw will bring heads or tails is one-half. And we know from the nature of dice that the probability of double sixes is 1/36. It is in dealing with such phenomena that we get what is known as the *calculation of chances*. In such calculations a common fallacy is to assume that the probability becomes greater that the next throw will bring double sixes because, say, thirty-five throws have been made without double sixes appearing. Unless the dice are loaded the probability of each and every throw is 1/36, and this does not change. Even though one hundred throws were made without the appearance of double sixes the probability that the next throw would yield double sixes remains 1/36. For the probability in such cases is not determined by the number of throws, but by the nature of the dice. Theoretical probability is usually restricted to *artificial phenomena* such as the various gambling devices.

3. *Probability as Applied to Aggregates.* We have to distinguish between probability as applied to aggregates

and as applied to the individual instances in an aggregation. For what is only probable as applied to an individual in an aggregation, may be a practical certainty as applied to the aggregation. For example, it is known by statistical investigations of large numbers of cases that the number of children who reach the age of sixteen is three fourths of the number born. And it would be a practical certainty that one fourth of five thousand children born in a given locality would die before the age of sixteen. But we could not say of any child among five thousand that it would die before sixteen years of age. In other words, what is only a probability for the individual is a certainty for the aggregate. This has been expressed as a law: "While in a small number of cases there is irregularity in the observed ratio between the number of times a given event has happened and its failures, still in a large number of instances this ratio tends toward a constant limit."[1]

An excellent illustration of this law is to be found in the attempts of Quételet and Jevons to verify theoretical probability by observation and enumeration of large numbers of instances. Thus, Quételet made 4,096 drawings from an urn containing 20 black and 20 white balls. Theoretically, this should have yielded 2,048 of each color. The actual drawings resulted in 2,066 white and 2,030 black balls. Jevons made 20,480 throws of a penny and the result was 10,353 heads and 10,127 tails. This shows that when a sufficiently large number of instances are taken into account the probable laws approach complete certainty. The results are sure to approximate a definite ratio, and, according to the law stated above, the larger the number of instances the nearer the approximation to

[1] See J. G. Hibben's article, "Probability," in the *Encyclopedia of Religion and Ethics*. I am indebted to this article for some of the facts and illustrations used in this and the preceding paragraph. Used by permission of Charles Scribner's Sons.

such a constant ratio. It is for this reason that statistical investigations are so extremely important in fields where the phenomena are highly complex. Insurance companies, for example, are able to do business because what would be a serious risk as regards the individual, ceases to be such when large numbers are involved. For the American Experience Table of Mortality shows that the death rate approximates a constant ratio. We shall take up more in detail the value of statistics after we have considered the methods of probability.

The student should especially note that probability as applied to large aggregates and masses of facts is really an extension of the experiential type of probability in which relatively few instances are examined, and that such an extension has to be made whenever we attempt to verify to the very limit theoretical probability. Of the three aspects of probability, its application to aggregates is by far the most important. Probability is knowledge of the actual ratio of variability between positive and negative instances of a large aggregate. Such a variability, determined either by extensive enumeration of members of the aggregation or by deduction from the known nature of the collection under consideration, is expressible as a law of the behavior of the separate members. All such laws are spoken of as probable laws to distinguish them from laws of nature such as the law of gravitation, which are invariable in the sense that the events or instances which they govern are all, so far as is known, positive. Since the events or instances with which probable laws deal are known to be both positive and negative, these laws possess certainty for the aggregate or total collection, but only a degree of probability for any particular member of the collection.

CHAPTER XVIII

THE STATISTICAL METHODS

The General Nature of Statistics

The methods by which probable laws are determined are usually spoken of as *statistical methods*, or simply as *statistics*. The word comes from state. As modern states grew in complexity it became necessary to devise methods for collecting, arranging and drawing inferences from large masses of data of various kinds. Consider, for example, such a problem as taking a census of a nation. Now such data gradually came to be known as statistical data. An extension of the meaning of the word to all complex phenomena in which it is impossible to get at the actual causal connections was the next, and a perfectly logical development of statistical theory. Thus the general field which we have described as probability, in the precise logical sense of the word, came to be regarded as the natural subject matter for statistical investigation. Hence the rather vague phrase, statistical methods, has come to mean the very elaborate technique, chiefly mathematical, which has been worked out to deal with large aggregates or masses of phenomena. This technique has been very fruitfully employed in such sciences as psychology, education, economics and sociology, but it has also been used to advantage in solving certain problems in physics, chemistry, mechanics and astronomy. It is not possible in an elementary textbook in logic to give a satisfactory treatment of the detailed methods which constitute statistics. In fact, statistics has become a separate science which requires a whole book for its exposition. All that can be done here is to present the broad outlines of the method.

To do this, it seems best to distinguish two different types of statistical method. One is the *method of sampling* and the other is the *statistical method proper*.

The Method of Sampling

1. *Natural Samples.* The method of sampling was originally restricted to the taking of samples from a large and relatively homogeneous mass of material and using them to judge the quality of the entire mass. In the commercial world a prodigious amount of business is conducted by the use of samples. Let us call this *natural sampling* to distinguish it from the later development of the method. It rests upon the assumption that the whole will be like the samples.

It is obvious that the validity of this inference depends on how the method is applied. Two important requirements must be met. (*a*) The selected instances must be what are known as "fair samples." Hence great care must be used in selecting the samples if this method is to have any logical value. Three devices are employed to guarantee that the samples will be fair. In the first place each sample is drawn *at random* from the mass of material. This means that no subjective prejudice of the investigator should enter in choosing the samples. The aim should be to select samples as nearly representative of the entire mass as possible. In the second place each sample must be chosen *independently* of every other sample. The careful observation of the samples must be delayed until all have been selected to avoid letting what is known about those selected first influence the selection of later samples. The third device is to *sample the samples.* This means mixing up the samples as thoroughly as possible, for example pulverizing the samples of coal taken at random from a carload, and then selecting a second set of samples from the new mass. These may then be subjected to chemical analysis or

to any other form of careful inspection. When these three devices are properly employed fair samples are reasonably certain. (*b*) The validity of the inference from the samples to the entire mass depends also upon the whole set of samples being a *fair proportion* of the total amount of material being sampled. Suppose there are forty tons of coal in a car to be sampled. Then two or three samples would hardly be enough for a fair proportion. Let us say that at least one sample per ton of coal should be drawn in the first sampling. In sampling these samples, fewer would suffice. The larger and the more extensive the mass the more samples needed to justify the inference. It follows that sampling has little value when the mass of material is of unlimited extent. That is why the sampling form of the enumerative theory of induction is unsatisfactory. The total number of events to which each scientific law refers is always so vast that any finite number of samples would be too small a proportion of the total to justify the inference.

To show that the method of natural sampling has been fruitful in scientific investigations, I quote from Huxley's famous essay entitled *A Piece of Chalk*.

It became desirable to ascertain and to indicate the nature of the sea bottom, since this circumstance greatly affects its goodness as holding ground for anchors. . . . Lieutenant Brooke, of the American Navy, some years ago invented a most ingenious machine, by which a considerable portion of the superficial layer of the sea bottom can be scooped out and brought up from any depth to which the lead descends. In 1853, Lieutenant Brooke obtained mud from the bottom of the North Atlantic, between Newfoundland and the Azores, at a depth of more than 10,000 feet, or two miles, by the help of this sounding apparatus. The specimens (samples) were sent for examination to Ehrenberg of Berlin, and to Bailey of West Point, and those able microscopists found that this deep-sea mud was almost entirely composed of the skeletons of living organisms—the greater proportion of these being just like the *Globigerinæ* already known to

occur in the chalk. Thus far, the work had been carried on simply in the interests of science, but Lieutenant Brooke's method of sounding acquired a high commercial value, when the enterprise of laying down the telegraph cable between this country and the United States was undertaken. For it became a matter of immense importance to know, not only the depth of the sea over the whole line along which the cable was to be laid, but the exact nature of the bottom, so as to guard against chances of cutting or fraying the strands of that costly rope. The British Admiralty, therefore, ordered Captain Dayman to ascertain the depth over the whole line of the cable, and to bring back specimens of the bottom. He performed the task assigned to him with great expedition and precision. The result of all these operations is that we know the contours and the nature of the surface soil covered by the North Atlantic for a distance of 1,700 miles from east to west, as well as we know that of any part of the dry land. It is a prodigious plain—one of the widest and most even plains in the world.

Huxley proves here conclusively that we possess almost certain knowledge of the nature of the floor of the Atlantic as a result of the fair samples of the deep-sea mud brought to the surface by Lieutenant Brooke's sounding apparatus. No better illustration could be given of the value of natural sampling in extending human knowledge.

2. *Artificial Samples.* In recent years the method of sampling has been greatly extended by investigators constructing artificial samples and using these as standards of measurement or as the basis for inferences. Information gained by careful observation of numerous instances is used to construct a test, embodying what the investigator regards as a representative sample. This test is then used to rate individuals. But artificial samples need not be tests. Whenever an investigator in any field constructs a sample by controlling the conditions, instead of drawing his sample directly from nature, this method is being used. For example, a biologist grew sample colonies of flies in milk bottles to determine what happens when the number of flies

in the colony becomes so great as to exceed the food supply. And a geologist simulated field conditions in his laboratory by rotating grains of sand in bottles to determine how sand produced by wind action on sandstone differs from sand produced by the action of water.

When employing the artificial sampling method how can an investigator make sure that his artificial samples are really "fair samples"? It is obvious that the difficulty here is greater than it is in the case of natural sampling of a homogeneous material. In fact there are two difficulties involved here. In the first place the constructed sample may not be properly put together, or important items may be omitted. The field conditions may not be correctly simulated but may be oversimplified. In the second place, even though a constructed sample is truly representative, it may not be satisfactory from the standpoint of individual cases.

In mental tests the first of these difficulties is dealt with by using the test on a few cases for the purpose of standardizing it. By this means the value of the test may be determined before it is used to establish definite ratings. And the second difficulty is overcome by giving what is called a *battery of tests,* and securing an average rating by giving various values to the score on each test in the battery. Or the score on the tests may be supplemented with information gained from questionnaires or in various other ways.

By such means educational and vocational tests have become widely recognized as possessing high value in selecting individuals from a mass of applicants, as well as in the placement of individuals after they are selected. Most universities, city school systems, and large industries are using this method with conspicuous success. The following passage contains good examples and indicates the importance of this method in present day research.

Thus a truck driver is measured in trade skill by having him make a sample trip, accompanied by the judge. But the tests of

different men, in different circumstances, and ratings by different judges, are given objective character and definition by their adhering to the general principles of a mental test. A standard outfit, standard trip, specified situations and emergencies, prescribed methods of scoring and of interpreting the score, give results that have a validity far exceeding that of the mere subjective opinion of an inspector. The "performance test" for general blacksmiths neatly illustrates the method of sampling, inasmuch as only one of hundreds of possible tasks is used. But the task is so chosen as to have demonstrated value in differentiating the various skill levels recognizable in the trade. It is thus a significant sample and serves as an index of the total equipment of information and skill possessed by the candidate. In this test, standard equipment, materials and tools are provided, standard instructions formulated, and a standard scoring plan prescribed. The candidate, presented with a blue-print specification for the making of a twisted hook of definite shape and size, carries out to the best of his ability the various processes of preparing, welding, twisting, punching, and bending the materials provided, so as to make a product conforming to the specification.[1]

The Statistical Method Proper

The statistical method proper involves four separate steps. Since the method of sampling is sometimes used in one or another of these steps, it can be regarded as a subsidiary process within statistics. But inasmuch as it is very often used apart from the more involved method of statistics it is justifiable to treat it as a separate method. However, the fact that it is sometimes used as a subordinate phase of the statistical method proper proves that the latter is much more important and also far more complicated.

1. *Classification.* At the outset of a statistical investigation it is necessary to arrange the phenomena under consideration into several different groups known as classes. What the classes are in a given case will be determined primarily by the purpose of the investigation. Indeed the

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[1] H. L. Hollingworth, *Judging Human Character*, pp. 155 *f.*

same mass of phenomena can be classified in various ways, and actually are so classified according to the angle from which different investigators approach them. Sometimes it requires long study before suitable classes can be made. Even the sending out of a questionnaire involves careful study, for here each question is really a class, and the separate answers obtained from different individuals are the data for each class. Asking the right question is not easy. Yet this is usually much easier than classifying properly as a preliminary to an extensive survey, such as that of all the educational institutions of a state.

2. *Enumeration.* After provisional classes are carefully distinguished, the investigator is ready to enumerate instances of the phenomena coming under each class. Such an enumeration should be very extensive. In fact, the more exhaustively the data are enumerated the better, for we saw above that the probable law which is based upon the most extensive enumeration is the most certain. It is this second step in the statistical method which is usually spoken of as *gathering the statistics.* Consider, for example, the way the Departments of Agriculture and Commerce of the United States are continually gathering statistics in crop surveys and reports of imports and exports.

3. *Ordering the Data.* After the statistics are gathered it is necessary to arrange those for each class into a serial order. This is extremely important for it is an indispensable preliminary to the drawing of definite conclusions from the data. That the final results may be really trustworthy the same principle should be used to order each series. But the data may be reordered by some other principle. The principle or principles actually used in ordering a series of statistical data will be determined by the kind of information which is wanted. Thus this step is similar to classification, only it follows, instead of preceding, the gathering of the data.

4. *Correlation of the Separate Series.* The final step in the method of statistics is to correlate the separate series. To borrow an expressive phrase from William James, "this is where the investigator begins to taste the milk in the coconut." For it is only after the correlating is completed that he learns the actual extent of the connection between the various sets of facts. The ratio or percentage thus obtained is the exact expression of the probable law or laws which control the phenomena under consideration.

Karl Pearson's formula for working out correlations is generally applicable and is widely used, although there are other formulæ. It is as follows:

$$r = \frac{\Sigma xy}{n\sigma_x\sigma_y} \left(\sigma_x = \sqrt{\frac{\Sigma x^2}{n}},\ \sigma_y = \sqrt{\frac{\Sigma y^2}{n}}\right)$$

r is the symbol for the *coefficient of correlation,* that is, a pure number indicating the degree of relation between the two variables which are being correlated. It may vary from $+1$ to -1, the former coefficient indicating a perfect *positive* and the latter a perfect *negative* correlation. 0 is the coefficient of correlation when there is absolutely no relation between the two arrays.

x stands for the deviation of any item of the first array from the arithmetic mean of all members of that series, and y stands for the deviation of any item of the second array from the arithmetic mean of all members of that series. Σ stands for summation. Hence the numerator of the formula means the sum total of all the products obtained by multiplying each item of the x series with its corresponding item of the y series.

σ_x and σ_y are the respective standard deviations of the two series, and, as the formulæ indicate, they are found by squaring each of the figures in the x and in the y series, adding these squares, dividing the result by the number of items (n) and taking the square root of that. When σ_x and

σ_y are determined, they are multiplied together and this product is multiplied by the number of items (n). This gives the denominator of the Pearson formula. Dividing the number obtained for the numerator by that obtained for the denominator gives r—the coefficient of correlation.

SIMPLE CORRELATION PROBLEM

Items	Weight	Height	x	y	xy	x^2	y^2
A	152	68	—5	0	0	25	0
B	154	65	—3	—3	9	9	9
C	156	67	—1	—1	1	1	1
D	163	69	6	1	6	36	1
E	160	71	3	3	9	9	9
5	785 / 157	340 / 68			25	80	20

157 = the arithmetic mean for weight.
68 = the arithmetic mean for height.
$-5, -3, -1, +6, +3$ = the deviations of *A, B, C, D, E* respectively from the arithmetic mean for weight.
$0, -3, -1, +1, +3$ = the deviations from the arithmetic mean for height.

$$n = 5,\ \Sigma xy = 25,\ \Sigma x^2 = 80,\ \Sigma y^2 = 20.$$

$$r = \frac{25}{5\sqrt{\frac{80}{5}}\sqrt{\frac{20}{5}}} = \frac{25}{5\sqrt{16}\sqrt{4}} = \frac{25}{5 \times 4 \times 2} = \frac{25}{40} = .625$$

The reliability of this coefficient of correlation may be determined by the *formula for probable error:*

$$P.\ E._r = .675\frac{1-r^2}{\sqrt{n}} .675\frac{1-.3906}{\sqrt{5}} = \frac{.675 \times .6094}{2.236} = \pm .18$$

This would mean that the chances are even that the true correlation for the above simple problem is between .44 and .80. The high probable error in this example is due to the

small number of items and proves the necessity for adequate data.

The conclusions reached by the statistical method are frequently best presented in tables, graphs, or charts. The student should study the graphs of the Normal Curve of Probability above (p. 228), and the isotypes below (p. 249).

The Values and Defects of Statistical Method

Just at present, the statistical method is in high favor among scientists in fields such as psychology, sociology and economics. Older methods have been discarded, and men are applying the method of statistics to fields of study to which no one even thought of applying it two decades ago. Indeed, many investigators use the word statistical as synonymous with scientific method. In certain fields no one is regarded as scientific who is not an adept in the use of statistics.

Now, although recognizing the great benefit that has come to mankind as a result of the discoveries made by the statistical method, every logician is in duty bound to point out that the widespread vogue of statistics to-day is, to a certain extent, a passing fad, and that many so-called scientific discussions based on statistics are arbitrary speculations which are almost the polar opposite of what a scientific discussion should be. "Figures do not lie, but liars figure," is a popular proverb which has a special application to many statistical investigations. There are two treacherous sources of fallacy which must be guarded against. In the first place, the investigator must be sure to take enough instances into consideration to make his conclusions reliable. Many conclusions which are heralded as new discoveries are based on insufficient statistics. Secondly, the statistics are not always fair. They are sometimes gathered to support a preconceived idea. Other statistics, equally reliable, could be gathered to prove the

exact opposite. It is easy to overlook the part of the facts which are against one's own view. No genuine scientist ever makes hasty generalizations from statistics covering only a part of the field, but it must be admitted that pseudo-scientists do.

Trained statisticians are seldom guilty of overestimating the value of statistics. They realize that all such studies are dealing with probability, and are hardly to be compared with the results reached in the exact sciences. The great English statistician, Yule, gives as fair a statement of the real place of the statistical method as could be given. He writes:

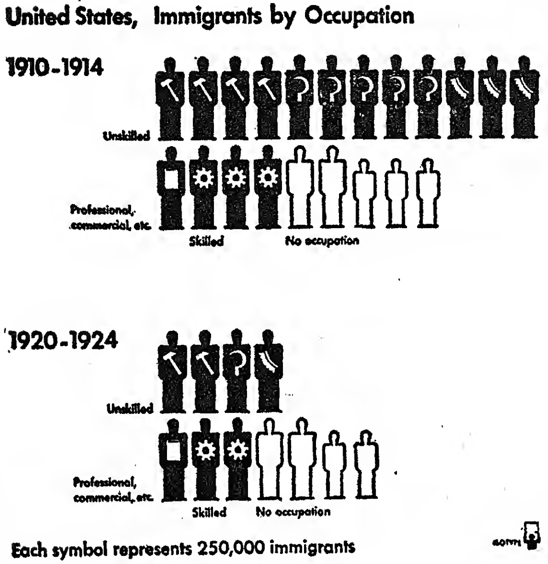
> Statistics should be regarded as ancillary, not essential. They are only essential where the subject of investigation is itself an aggregate, as a swarm of atoms, or a crowd. . . . Statistical methods are only necessary in so far as experiment fails to attain its ideal, the ideal of only permitting one causal circumstance to vary at a time. And it should always be the aim of the experimenter not to revel in statistical methods, but steadily to diminish by continual improvement of his experimental methods, the necessity for their use and the influence they have on his conclusions. Statistical methods are not only ancillary; they are, to the experimenter, a warning of failure.[3]

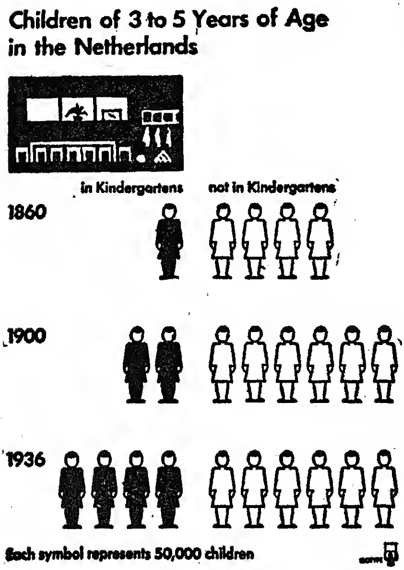
Yet, although admitting that "statistics are very human" and hence may lie, if the human manipulators of them wish to deceive, we must not forget their great value when properly used. "Like dynamite in the hands of a mining engineer, they have a force and power which excite admiration when they are used, as modern business is tending more and more to employ them, to blast away dead rock and strike to the new, rich veins." As this comment suggests, one of the great values of statistics is to give us a comprehensive grasp of actual facts that are so complex

[3] *British Journal of Psychology*, Vol. XII, pp. 106 *f.*

as to be unmanageable by any other method. The knowledge which statistics give us of social and economic conditions is far better than the complete absence of knowledge which exists in the minds of those who have never attempted to understand the problems. And in the second place statistics are guides to action. They make it possible to predict the future with considerable accuracy. As the American statistician, Horace Secrist, has well said: "The old idea was to regard statistics as records, and their analyses as a means of explaining past performances, but not as a means of determining future activity." But during both world wars the Federal Trade Commission, The War Industries Board, the Army and the Navy, used statistics and statisticians "for developing constructive plans and testing the degree to which these plans were realized in actual performance." Statistics are the lanterns by which we light our way through the dense darkness of the future. Then, too, they frequently suggest causal connections which would otherwise remain hidden. The actual presence of these causal connections can then be worked out by experiment. It is this that Yule had in mind in referring to statistics as ancillary. They are valuable to the experimenter because they suggest to him fruitful hypotheses which can be worked out by other methods.

The late Dr. Otto Neurath made a distinctive contribution to science in developing a pictograph language for use in disseminating statistical information. He originated over two thousand pictograph symbols which he called *isotypes*. On the next page two sample isotypes are reprinted from *Modern Man in the Making* by Otto Neurath, by permission of Alfred A. Knopf, Inc. Copyright 1939 by Alfred A. Knopf, Inc.


United States, Immigrants by Occupation
1910-1914
Unskilled
Professional, commercial, etc.
Skilled
No occupation
1920-1924
Unskilled
Professional, commercial, etc.
Skilled
No occupation
Each symbol represents 250,000 immigrants


Children of 3 to 5 Years of Age in the Netherlands
in Kindergartens
not in Kindergartens
1860
1900
1936
Each symbol represents 50,000 children

EXERCISE XIII

1. Criticize the following examples of the method of sampling, with special reference to whether they conform to the rules for a fair sample. Distinguish the examples of artificial sampling from those of natural sampling.

(*a*) Up the Potomac from Washington the Navy Department has for years had a basin in which ships of all weights and shapes have been tested in miniature. In this manner naval architects have reached decisions about the behavior of new types of hulls without going to the expense of building the full-sized ship.

(*b*) The Navy constructed a miniature Bikini atoll in the basin. In it they anchored little ships as nearly as possible of the same proportional weight and balance of their full-sized counterparts. The depth of the water in the lagoon was proportionately the same and it was salt water. Then a charge of explosive—not atomic, of course—of carefully estimated power proportionally equal to that of the atomic bomb was set off and results noted. From these miniature explosions estimates, which proved to be substantially accurate, were made of what would happen.

(*c*) Following the explosion of the atomic bomb at Bikini Lagoon, Navy men used Nansen's sampling device to obtain samples of water ninety feet below the surface to determine the amount of radioactivity at that depth.

(*d*) To determine the best location for opening a lead and zinc mine, a mining prospector obtained samples of ore in the Ozark Mountains.

2. Apply the Pearson formula to find the coefficient of correlation between the two columns in the following table. Then find the probable error of the coefficient of correlation, using the formula for probable error. Students may substitute heights and weights of players on their own college or university team.

TABLE I

HEIGHTS AND WEIGHTS OF THE PLAYERS ON A FOOTBALL SQUAD

Player	Height	Weight	Player	Height	Weight
Center	5′ 8″	190 lbs.	Quarterback	5′ 6″	152 lbs.
Right guard	5′10″	175 lbs.	Left halfback	6′ 2″	179 lbs.
Right tackle	6′ 0″	182 lbs.	Right halfback	6′ 3″	186 lbs.
Right end	5′ 7″	168 lbs.	Full back	6′ 4″	195 lbs.
Left guard	5′11″	183 lbs.	Sub. back	5′ 9″	189 lbs.
Left tackle	6′ 1″	187 lbs.	Sub. line	5′8½″	177 lbs.
Left end	5′11″	172 lbs.	Sub. quarter	5′6½″	160 lbs.

SECTION VII

CAUSALITY AND MILL'S EXPERIMENTAL METHODS

CHAPTER XIX

CAUSALITY

CAUSALITY, as William James has well said, is an altar to an unknown god. For it is a conception which dominates our thinking to an almost unbelievable extent, and yet every attempt to comprehend its inner nature is confronted with overwhelming difficulties. Causality is another one of those peculiar things to which St. Augustine's principle is applicable: "If you ask me what causality is I do not know, but if you do not ask me I know." Yet it is the business of a logician to throw what light he can on such concepts. What, then, is meant by such terms as *cause, causation* or *causality?* What is the actual reality to which such terms refer?

The Law of Causation

The first essential to the understanding of causality is to distinguish carefully the so-called *law of causation* or *principle of causality* from concrete causal relations. According to this law or principle, *every event which happens in the universe is the result of some previous event or events, without which it could not have happened, and with which being present, it must happen.* Many discussions of causality are primarily concerned with this law. How to prove it, or whether it can be established inductively or by experience, is one of the problems. Whether it is a purely man-made principle, or, on the other hand, an expression of the actual nature of things is another much-argued point. These questions were considered somewhat in our discussion of the assumption of induction, and we shall return to them again in our account of the nature of explanation. For it is generally admitted that the law of causation is equivalent

to the law of the uniformity of nature which was interpreted above. In fact, I regard these phrases as strictly synonymous. The law of causation is simply another way of stating the basic assumption behind all science. Its very essence is an insistence upon the universality of the reign of law, and science cannot move an inch without making this assumption.

What we are here concerned with, however, is not this universal principle according to which causal relations are held to be absolutely limitless in their extent, being coextensive with the whole of nature, but with what is meant by any particular *causal nexus*. We are here raising the question of the *nature* of a causal nexus, rather than the question of the *validity* of the law of causation or the principle of causality. Just what is meant by saying that one thing or event causes another or is the effect of another—this is the question with which we are concerned.

Now, it is generally agreed among logicians that causality is a *relation*. It comes under the category of relation. But just what kind of a relation is it? How are we to conceive of its inner nature? This is the real causal problem with which we must wrestle. The problem connected with the law of causation is quite different from this, being one with the problem of the inductive leap or the basic assumption of induction. That question is not here raised because we disposed of it above. What we now are seeking is a definition which will state the essential intensive nature of the unknown god called causality.

A Critical Analysis of the Conception of Causation

Let us begin with the notion of cause held by the "man in the street," the naïve person who has never studied the question but who constantly uses the concept. And to take a specific instance, What does the average person mean when he says that a stone is the cause of a certain broken

window pane? Now he means at least three different things, an account of which will lead us into the very heart of the problem and difficulties of causality. Of course, he may not be conscious of meaning these things every time he uses the word *cause,* but if he is pressed these are the things he will insist upon.

In the first place, he means that there is some power or energy resident in the stone, which is released somehow in the event known as the striking of the glass, and, being released, *produces* the effect of breaking the glass. Thus the moving stone is one event, its contact with the glass being the culmination of that event, and the breaking of the glass is another event. The former is the cause and the latter is the effect. Every cause is assumed by the man in the street to be a certain amount of released energy which *effects* something. Note how this idea is inherent in the very word effect. Perhaps this conception of efficient action is the most prominent element in causality so far as the man in the street is concerned.

And in the second place, this view implies also the idea of a *necessary connection* between the two events, between a moving stone striking a glass with sufficient force and the glass breaking. This necessity means *invariability.* Cause and effect are so related to each other that whenever the cause occurs the event known as the effect must also occur, unless prevented by what are known as counteracting causes. Given an effect and a cause must be lurking near. Given a cause and we may be sure of some effect taking place. One cannot exist without the other. For instance, if some magician in a vaudeville performance hurls what looks to be a stone with great force against what looks to be a pane of window glass, and the glass does *not* break, the man in the street will say that it is a trick. Either it is not a real stone or it is not a real pane of glass, or the stone did not actually strike the glass, or something else

has entered in to prevent the causal operation from taking place. This conception of necessary connection between a cause and its effect is a very essential element in the popular conception of causality.

The third element is *temporal sequence.* The cause *precedes* the effect. The effect *follows* the cause. According to the man in the street the glass will not break until after it is struck by the stone. The causal relation involves a definite and an irreversible temporal sequence such that cause must stand first and effect second, cause must precede and effect follow. Even when the two seem to be simultaneous the man in the street thinks that there is a succession involved, only he would say that the interval of time between the stone striking and the glass breaking is so small that no timepiece can measure it. Behind the temporal sequence idea is the tacit assumption that the world is made up of isolated events, following one another in rapid sequence, but still readily distinguishable from each other and forming a causal series. Hence this theory has been well characterized as the "bead theory of causality," the causal series being like a string of beads. The idea that the cause *must* precede and the effect *must* follow really mixes up the idea of necessary connection with the idea of temporal sequence, but the two are distinct elements in the popular conception of causality.

What, now, is to be thought of this apparently plausible account of the nature of causality which the average naïve person holds? Is it tenable, or does subjecting it to criticism make it fall like a house of cards?

Well, it is obvious that the notion of there being a *power* in the stone which *produces* the effect is a very crude idea. In fact, it is on a level with the idea of the savage known as animism—the belief that everything is alive like himself, and it is simply a survival of that idea. For it undoubtedly comes from our *own experience of exerting ef-*

fort when we produce a certain effect, or from our *feeling of restraint* when something prevents us from doing a thing. Certain it is that we are here explaining one mystery by another, for, as David Hume pointed out, "the connection between the movement of my limbs, and what I regard as the psychical cause thereof, is no more intelligible to me than that between their movement and the movement of a body which they strike." And, in any case, we cannot *see* or *perceive* the power in the stone which is supposed to break the glass. Then how do we know it is there? The human mind is incapable of forming any conception of an "influence" passing from the cause over to produce an effect. And even though it were possible to form a clear conception of it, how would this help us to comprehend the nature of causality? It is a one-sided view which attempts to make the causal relation wholly dependent upon the cause. "So long as the cause is looked upon as what, by its action, exclusively determines the nature of the effect in a purely passive object, efficiency is perhaps impossible as well as inconceivable. The action of *A* on *B* cannot be grounded in *A* alone; the change attributed to *A* as cause must be determined in part by *B* also. For it depends upon the nature of *B* how *B* will behave under *A's* action. The sun which softens wax hardens clay. The popular view of causal action, in grounding the change of *B* entirely in *A,* is therefore one-sided. The action must be reciprocal. In physics this receives expression in Newton's third law of motion: 'Action and reaction are equal and opposite.' Causation, then, is *inter*action; cause and effect are simultaneous; the effect is not contained in the cause; there is not a passive factor."[1] Hence this part of the popular notion of cause is full of obscurities.

[1] F. R. Tennant, in the article entitled "Cause," in *Encyclopedia of Religion and Ethics.* This and the quotation from Tennant below are used with the permission of Charles Scribner's Sons.

In like manner, David Hume and John Stuart Mill tried to eliminate from the conception of causality the idea of necessary connection, but with less success. Hume says that the necessity which we infer to exist between a cause and its effect is entirely due to *custom* or *habit*. Because we have seen one thing follow another many times in the past, we expect it to follow again when the cause recurs. In answer to this, it should be pointed out that we speak of a thing as a cause, in *unique* cases where the particular thing in question was never before experienced. And we do so simply because we recognize the existence of a necessary connection in the situation. Mill follows Hume in stripping from the popular conception of causality the idea of necessary connection, as well as the idea of a power to produce. He defines a cause as an "*invariable* antecedent," which is also "*unconditional.*" But the words invariable and unconditional are really synonyms for necessary connection. Hence his attempt to eliminate from the conception of cause this idea of necessary connection fails. "We do mean something by the words *necessity* and *connection* and know what we mean, even though we are unable to see between what changes in a changing world connection lies. Within the fields of geometry and mathematics, and in philosophical inquiries, we see this necessarily to involve that, and the connections are apprehended with their terms. In that which changes we realize that there must be connection between successive states, without knowing what is connected with what; we understand that a cause produces its effect necessarily, without understanding that it must produce just this or that effect" (Joseph, p. 406). In this sense, then, the idea of necessary connection can and must be retained as one of the essential features of the causal relation.

But what about temporal sequence? According to Mill and Hume this is the very life blood of causal relations.

But in the light of modern knowledge their view is highly questionable. The new idea of time or duration as a *continuous process,* or, more briefly, the idea of *continuity,* makes it impossible to conceive of any separation between cause and effect. The causal relation must be strictly continuous. This makes it impossible to separate the cause from the effect, as beads on a string are separated. "A body, *A,* cannot be said to act causally on another, *B,* if, while *A* is changing, *B* is not. A cause is a cause only in so far as, and at the moment in which, it produces its effect; just as a soldier is a target only when he is being the object of a marksman's aim. If causation is production of change, then cause and effect would seem to be necessarily simultaneous. We speak, indeed, of the swallowing of poison as the cause of a subsequent death; but in thus singling out one event in a series and calling it the cause of a later one, we are using language which may be convenient, but which is certainly arbitrary and inaccurate. Between the act of swallowing and the cessation of life, a physiologist could distinguish many successive events, each of which may equally claim the title of cause of the final effect. Indeed, every event permits of conceptual division into parts, *ad infinitum;* it is really a system of events, and these are again systems of a higher order. Science resolves planets, for instance, into atoms, and these into electrons; a flash of light into waves caused by vibrations" (Tennant). Here is brought out the fact that the idea of cause has to give way to the idea of system—our inferential whole or implicative system. The real meaning of the causal relation is this idea of an underlying system in which the connection is grounded.

Summing up the above discussion, we may say that a causal relation is a continuous connection within a series of events constituting an implicative system, some of which may be singled out as constituting the cause and others as

constituting the effect. The important element is the necessary connection which forms the continuity in the series. This is the modern conception of causality. *Wherever a constant coexistence or a regular sequence in events indicates a necessary connection the events in question are said to exhibit the causal relation.* This is the real meaning of causality in science.

Difficulties in Determining Causal Relations

The actual existence of a causal relation in a given group of phenomena is frequently exceedingly difficult to determine. Mill's experimental methods are formal statements of the methods employed by scientists in determining causal relations, but before explaining them let us consider three difficulties which are often met with in applying them to a concrete problem.

1. In the first place, we sometimes meet with what is known as a *reciprocity of phenomena.* This really means that the causal relation is hard to find when the phenomena or events are coexistent instead of successive. Consider, for example, the phenomenon of a burning candle. It is hard to say whether the melting wax is cause or effect. In fact, the flame is the cause of the melting of the tallow or wax, and the melting of the wax is the cause of the flame. Here we have a case of the same event being both cause and effect.

2. In the second place we often get a *plurality of causes.* When the cause of a death is sought the answer might appear simple, but usually there are numerous causes at work, some of which can hardly be isolated. In fact, Mill was finally led to the view that the cause of every event is the *sum-total of events in the universe which precede it.* Then he made the distinction between the *remote* and *the immediate cause or causes* of an event. Usually a scientist is primarily interested in finding the immediate cause. This

distinction applies also to the effect. There are immediate and remote effects. A single human act frequently carries in its train a veritable maze of consequences or effects. That is why it is so difficult to pass a fair judgment on the conduct of another.

3. And this leads to a third difficulty usually known as the *complexity of phenomena.* In dealing with probability we were really dealing with exceedingly complex phenomena where the causal methods are blocked, and the investigator has to rely upon statistical enumeration. But even in cases where the causal methods are applicable, the phenomena are sometimes so highly complicated that it takes years of experimentation to unravel the various causal strands, thereby effecting a complete solution of the problem. Scientists have to work piecemeal, isolating first one *causal nexus* and then another. It may even fall to the lot of some later thinker to discover the cause of the *residual phenomena,* those which have not been explained in the discovery of such causal relations as have been brought to light by previous investigators.

CHAPTER XX

MILL'S METHODS: AGREEMENT AND DIFFERENCE

Origin and Basic Principles of Mill's Methods

John Stuart Mill's five experimental methods are the most substantial innovation which he introduced into logic, as it was represented in his day by men whom Bosanquet has well characterized as "degenerate representatives of Aristotle." It is for this reason that he ranks as one of the great reformers in the science of logic. But like most other innovations this one was not altogether original. For before Mill there had been two separate lines of development of scientific method. One of these was the purely logical development initiated by Francis Bacon. The other was the scientific development initiated by William Gilbert, but greatly enriched by Sir Isaac Newton and Sir John Herschel, the latter being a contemporary of Mill. The greatness of Mill consists in his bringing together these parallel lines of development, and in his giving a more precise formulation of the methods than had hitherto been given either by the logicians or the scientists. He had an admirable grasp of the methods by which a scientist deals with his concrete problems, and a thorough knowledge of the history of logic, especially of the inductive logic which Bacon founded. This gave him just the necessary equipment for working out in precise logical form the methods used in scientific investigations. In his account of the methods, Mill acknowledges his obligations to Bacon when he says that the principles underlying his methods have "been known, since the time of Bacon, to be the foundation of experimental inquiry." And he also admits his indebtedness to Sir John Herschel's *Discourse on the*

Study of Natural Philosophy, a work, as he says, "replete with admirably selected exemplifications of inductive processes from almost every department of physical science, and in which alone, of all books which I have met with, the methods of induction are recognized, although not characterized and defined nor their correlation shown, so distinctly as has appeared to me desirable."[1]

It is, therefore, a mistake to think that Mill originated these methods. They are all the more important because he took them from one of the great scientists and correlated them with the discussion of method in Francis Bacon's *Novum Organum*. For Bacon's famous tables of (1) *presences*, (2) *absences* and (3) *degrees* are the germs of the methods of *agreement*, *difference* and *concomitant variations*. And since Bacon was also indebted to the great scientist, Sir William Gilbert, for the principles underlying his tables, these methods may be said to be doubly indebted to men of science.[2]

Mill understood full well that his five methods, or as he sometimes called them, *canons*, are all based on just two principles. He wrote: "The simplest and most obvious modes of singling out from among the circumstances which precede or follow a phenomenon, those with which it is really connected by an invariable law, are two in number. (1) One is, by comparing together different instances in which the phenomenon occurs. (2) The other is by comparing instances in which the phenomenon does occur, with instances in other respects similar in which it does not. These two methods may be respectively denominated the method of agreement and the method of difference." Consequently, Mill himself tacitly implies that the joint method of agreement and difference, the method of concomitant variations

1 John Stuart Mill, *Logic*, Bk. III, Ch. IX.
2 See Ch. XXV, p. 338, especially the note.

and the method of residues are all corollaries of these two. Moreover, as we shall soon see, the method of agreement is really only a preliminary stage in the method of difference. Hence the one basic method among the five is the method of difference. It is essentially identical with what the scientist calls the method of experiment. Or, to express it differently, Mill's method of difference is the method of experiment *par excellence*.

The Method of Agreement

Mill's statement of this method is as follows: "If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon." Now, as previously explained, instances in which the phenomenon is present are called *positive instances*. Hence this method operates only with positive instances. But it emphasizes *variety* rather than *number* in that the positive instances should differ in every circumstance save only one. Hence this method can be stated much more simply: *The sole invariable circumstance accompanying a phenomenon is causally connected with the phenomenon.* Or, as Jevons expresses it: "The sole invariable antecedent of a phenomenon is probably its cause, and the sole invariable consequent of a phenomenon is probably its effect." The only difficulty in this way of putting it, is that it emphasizes too much the temporal sequence theory of causality which was considered in the last chapter, and which Mill adopted from Hume.

A fuller statement of the method of agreement may be given by dividing it up into separate stages. In the first place, a variety of positive instances of the phenomenon under investigation must be collected. This involves careful observation and enumeration. In this first stage care must be used to select instances which are different in as many

respects as possible, with the exception of having the phenomenon present in them. In fact, as Mill understood, the method is not effective unless the positive instances differ in every respect save one, in addition to the presence of the phenomenon. But it is seldom possible to find such instances. That is one reason why agreement has to be supplemented by the method of difference. In the second place, these positive instances must be carefully analyzed into their component parts and their several antecedents or consequents must be noted. It is in this way that the *common circumstance* which accompanies the presence of the phenomenon is isolated from other circumstances. After this common feature is once isolated it is read off as the cause or the effect of the phenomenon, as the case may be.

Suppose, now, that we illustrate this method by symbols in order to make it just as simple and as clear as possible. Let the phenomenon be P and the different positive instances be P^1, P^2, P^3, and P^4. Now suppose P^1 analyzes into *abcd*, P^2 into *efch*, P^3 into *jkcl* and P^4 into *cmno*. Since *c* is the only circumstance present in each of the different positive instances, we are led to believe that *c* is causally connected with the phenomenon.

Consider, now, a concrete example of the application of this method. In his well-known essay entitled, *The Scientific Use of the Imagination,* the distinguished scientist, John Tyndall, discusses the blue color resulting from the action of a turbid medium upon light rays. He mentions the following positive instances of turbid media having light rays pass through them, each of which may be roughly analyzed to bring out the differences between them.

Phenomenon—Blueness
Cause—Conjunction of light with fine particles of matter in the turbid media

Instances:

A. A beaker of turbid water of the Visp
  1. Light striking suspended particles of fine matter
  2. Light striking the glass beaker
  3. Composition of water—H_2O

B. Steam issuing from the safety valve of a locomotive
  1. Light striking the suspended particles of fine matter
  2. Steam liberated in the air with no container
  3. Water in the form of steam

C. Brückes precipitate (a solution of mastic and absolute alcohol)
  1. Light striking suspended particles of fine matter
  2. Transparent container for the solution
  3. A liquid different from steam or water

D. Peat-smoke columns from the cabin chimneys of Killarney
  1. Light striking suspended particles of fine matter
  2. Smoke also liberated in the air without a container
  3. Composition of peat-smoke chemically different from water, steam or Brückes precipitate

The fact that the only common circumstance in these four quite different instances is light striking suspended particles of fine matter is revealed by analysis, and we are justified in concluding that this is in all probability the cause of the blue color which is always present when light strikes turbid media. As Tyndall points out, this is also the cause of the blueness of the sky, and this could be treated as a fifth instance or variety of instance.

Another illustration of the method of agreement usually given in the logic books is Sir David Brewster's discovery of the cause of the color known as "mother-of-pearl," which is the name given to a certain shell formation. He took gum arabic, balsam and beeswax, and impressed a piece of shell of this color on each, whereupon he discovered that each took on the brilliant mother-of-pearl coloring.

Analyzing these instances in which the phenomenon is present we get the following:

A. Piece of mother-of-pearl, the inner layer of a shell
  1. Has a definite form, shape or contour
  2. Made of lime and other materials secreted by a living organism
  3. Solid

B. A piece of balsam
  1. Has the same form or shape as shell which has been imprinted upon it
  2. Resinous substance exuded from tree
  3. Oily and aromatic

C. A piece of beeswax
  1. Has the same form or shape as shell which has been impressed upon it
  2. A wax secreted by bees
  3. Having the peculiar odor of beeswax

D. A piece of gum arabic
  1. Has same shape as shell which has been impressed upon it
  2. A secretion from the acacia tree
  3. Soluble in water and having a luster

Now, since each instance where the mother-of-pearl coloring is present agrees in having a certain definite shape, we may infer that the cause of the coloring is this peculiar shape of the object.

These two illustrations bring out clearly what Mill meant by the canon of agreement. And it must be conceded to have a certain value in that it does enable one to eliminate those elements which differ in the various positive instances. And when there is only one element in common it is fair to assume that this is the real cause (or effect) of the phenomenon under investigation. In the actual concrete world of facts, we must admit, such cases very rarely occur. Frequently there are several circumstances that are

common to all of the positive instances. In such cases all that the method of agreement would enable us to do would be to eliminate the features which are not common. Which one, or ones, of those left after this elimination takes place is the real cause, this method does not tell us. We must apply the method of difference to each of these common circumstances, until we find out which one, or ones, is causally connected with the phenomenon. Hence the method of agreement is really only a stage in the method of difference. To complete the investigation the method of difference must be applied to the same set of instances which have been used in the method of agreement.

The Method of Difference

Mill states this method as follows: "If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance save one in common, that one occurring only in the former; the circumstance in which alone the two instances differ, is the effect, or cause, or a necessary part of the cause, of the phenomenon." But this can be stated much more simply, thus: When the subtraction of one of the aspects of a positive instance of a phenomenon takes away the phenomenon, we know that this subtracted aspect is causally connected with the phenomenon. For we then create a negative instance by experiment. And when any aspect which the experimenter is able to eliminate takes the phenomenon away with it, the two are causally connected. Nothing can be the cause of a phenomenon in the absence of which the phenomenon appears, and everything is causally connected with the phenomenon, the elimination of which also eliminates the phenomenon.

The key to understanding this method is to see that it goes a step further than the method of agreement in that it turns a positive instance into a negative instance, by

eliminating what the method of agreement has suggested to be the cause. Keeping every other circumstance the same except the one we assume to be the cause, the experiment of taking this supposed cause away is performed. If the phenomenon disappears with it, we know that it is really causally connected with the phenomenon. Hence the rule is that *only one circumstance can be varied at a time.*

It is worth noting that Mill himself spoke of this as a method of *artificial experiment.* He explains that it is of the very nature of an experiment to introduce a perfectly definite change into a preëxisting state of circumstances. While nature may sometimes perform a kind of "spontaneous experiment," which will give a positive and a negative instance differing only in one circumstance, this is extremely rare. And he also notes that the method of difference is impracticable in some fields of investigation, because of the "impossibility of artificially producing the phenomena." This means that Mill considered the method to be primarily a method of experiment, and to apply to such exact investigations as are under the control of the scientist who is doing the experimenting. Now let us illustrate the method, first, with symbols, and then with concrete examples.

Suppose I have a positive instance of a phenomenon which analyzes into four aspects. This would be represented as follows:

P analyzes into abcx.

Now suppose the method of agreement has led me to assume x to be the aspect in the positive instance which causes the phenomenon, then when x is taken away P should become *not-P* or a negative instance. Thus, *not-P analyzes into abc.* If P turns into *not-P* when x is taken away, the method of difference says that x is the cause of P.

The coin and feather experiment in physics is a good

illustration of this method. It is designed to prove that the cause of a light article falling to the ground more slowly than a relatively heavy one is the resistance of the air. The phenomenon under investigation may be said to be the retardation of the feather, and the cause is assumed to be the resistance of the air. When a coin and a feather are dropped together in the receiver of an air pump in which air is present, the feather is retarded. This is the positive instance. But when the air is pumped out, and the two are dropped together they reach the bottom of the receiver together. This is the negative instance in which the phenomenon does not occur and it proves that air is the cause of the retardation.

Mellone adapts another excellent illustration of this method from Baden-Powell's *History of Natural Philosophy:* "The production of colors by light passing through spherical and prismatic glasses had already been noticed; and Newton proceeded to make it the subject of exact experiment by repeated applications of the method of difference. A beam of the sun's rays admitted through a small hole in an otherwise darkened room, produces on a screen a circular image of the sun (negative instance). But on passing the beam through a prism, the image becomes nearly five times as long as it is broad, and is colored from end to end by a succession of vivid tints (positive instance). Hence, *something in the glass* is the cause of the colors. Newton now proceeded to vary the size of the prism, to vary the quality of the glass, to pass the beam through different parts of the same prism, and to try other minor suppositions; but none of these changes made any difference in the colors. Hence he concluded that the *prismatic shape* of the glass was the real cause. He eliminated this by placing on the original prism a second one of exactly the same angle, but inverted, so that together the two prisms formed a solid with parallel surfaces. The

light, passing through both, came out uncolored, and gave a perfect undistorted image of the sun. Hence the prismatic shape of the glass was proved to be the cause of the colors.''[1]

Although the usual procedure in applying the method of difference is to begin with the positive instance and turn that into a negative instance, it is also ~~possible to~~ reverse the process, beginning with a negative instance and introducing a supposed cause to see whether a supposed phenomenon will result. Logically this amounts to the same thing, only the usual way of stating the method is that given above. To take account of both ways of proceeding the method may be stated: When the subtraction of one of the aspects of a positive instance of a phenomenon takes away the phenomenon, or when the addition of a single new feature to a negative instance produces a given phenomenon, the aspect subtracted or added is considered to be causally connected with the phenomenon. Thus, in proving that light is the cause of the green color in plants one may either take the light away from plants that are green by putting a board over them or by any other blanching process, or one may take plants that have been kept away from the light and bring them under the influence of the light to produce the green color. No matter which procedure is followed, the method of difference is being used.[2]

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[1] *Introductory Textbook of Logic,* pp. 274 *f.* Compare this with the von Laue experiment given below, p. 314.

[2] The exercise on Mill's methods is at the end of the next chapter.

CHAPTER XXI

MILL'S OTHER METHODS

The Joint Method of Agreement and Difference

To deal with complex phenomena where artificial experiment is impossible, Mill formulated the joint method of agreement and difference. He also called it the *double method of agreement*, and it is unfortunate that the former name has been almost universally adopted in the logic textbooks, because it leads students to think that it is stronger than the method of difference. We shall soon see why it is really much weaker than that method. But even though the name *double method of agreement* is preferable to the name *joint method of agreement and difference*, it seems best to conform to usage and hold to the latter designation.

"If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance; the circumstance in which alone the two sets of instances differ, is the effect, or cause, or a necessary part of the cause of the phenomenon."

This rather involved statement of the method, which I have taken from Mill, may be simplified as follows: Where a variety of positive instances have only one circumstance in common, and a variety of negative instances agree only in having this circumstance absent it may be inferred to be causally connected with the phenomenon. An important difference between this and the method of difference is to be found in the fact that this uses a number of positive and negative instances, whereas the method of difference has but one positive instance which is turned into a nega-

tive instance by experimentation. It is also important to note that the negative instances in the joint method differ from the positive instances *in more than one circumstance,* thus violating the rule for the method of difference that only one circumstance can be varied at a time in producing a negative instance. For this reason the negative instances used by the joint method must be carefully distinguished from the true negative instances, obtained by experiment, which characterize the method of difference. It is for this reason that we should think of the joint method as a double method of agreement. Its instances are all found by observation and enumeration, as contrasted with experiment. First, instances exhibiting the phenomenon are collected which have some one common circumstance. Then instances in which the phenomenon is absent are collected in which this common circumstance is absent.

To represent this method by symbols, the following arrangement is necessary:

Positive Instances	*Analysis*
1.	*a b c d e*
2.	*f a g d k*
3.	*l m n a e*
4.	*a w x y z*

Negative Instances	*Analysis*
1.	*b w d c y*
2.	*m f d k e*
3.	*z e n y c*
4.	*l b g f d*

Here, *a* being the only common element among the four positive instances, and the only absent member of the negative instances, we may assume it to be causally connected with the phenomenon.

To take a concrete illustration, suppose an investigation

is made of the cause of the decline of the country church. And let us assume that the cause is the removal of owners from the farms to town or city, and the resultant occupancy of the farms by tenants. Four or five sample communities might be selected in which the church has either died out entirely, or practically lost its hold on the life of the community. Then an equal number of sample communities should be selected in which the church is still very active and the center of the life of the community. The phenomenon being the decline of the country church, the former would be positive and the latter negative instances. Carrying out the illustration, let us roughly analyze each instance:

Positive instances = the church either dead or very inactive.

Community A......Thickly populated, tenant occupants, great interest in Sunday sports, good roads and conveyances.

Community B......Very few young people, tenant occupants, large farms, few extra laborers in the community, poor roads and conveyances.

Community C......Sunday sports indulged in, tenant occupants, illiterate people, poor land.

Community D......Amusement community center, good roads and conveyances, moral conditions excellent, tenant occupants, some one irreligious but public-spirited large landowner.

Negative instances = the church very much alive and active.

Community E......Farms well-kept, owner occupants, good roads and conveyances, amusement center.

Community F......Many young people, owner occupants, strong, efficient pastor, excellent roads and conveyances.

Community G......Illiterate and ignorant people, poor land, bad roads and few conveyances, owner occupants.

Community H......Sparsely settled, no villages near by, no amusement center or playground, many young people, owner occupants.

Here the common feature in the first set of instances is tenant occupancy of the farms, whereas the feature absent when the church is active is tenant occupancy, so that we may assume that the cause of the decline of the country church is tenant occupancy. This illustration brings out the close similarity between the joint method and the statistical method. To solve the problem dealt with in this illustration, it would be necessary to gather extensive statistical data, and to take many more communities into consideration, and from widely separate rural districts. The joint method really shares the weakness of the statistical method, and it is the statistical method in germ.

Mill, commenting upon the joint method, rightly said that it is not equivalent to a proof by the method of difference. This is due to the fact noted above, that the negative instances differ from the positive instances in more than one circumstance in the joint method. It should be clear that the method of difference is really a *joint method* in that it also uses both positive and negative instances. Hence, we must be on our guard against being misled by the term *joint*.[1] For the so-called joint method

[1] That the authors of the *Introduction to Reflective Thinking* have been so misled is evident from their statement: "Mill combined these two methods so that the logical conclusiveness of the method of difference might be combined with the practical availability of the method of agreement. The canon of the joint method seems somewhat alarmingly involved, but to those who understand the methods of agreement and difference, the advantage to be gained by their joint use will become apparent" (p. 81).

is really much weaker than the real experimental method of difference. To be sure, it does have more value than the method of agreement, since it checks up on that method by using such negative instances as can be found ready at hand. The table of absences of Francis Bacon is really a mixture of negative instances of the experimental type and of the purely observational type. Bacon did not recognize the difference between the principle of the method of difference and that of the joint method, and this is one of the most significant improvements which Mill made in the inductive logic of Bacon. To-day, the joint method is practically supplanted by the more elaborate and technical method of statistics, and for that reason it is not as important as it was in Mill's time.

The Method of Concomitant Variations

Mill's formulation of this method cannot be improved: "Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation."

Although the method of concomitant variations introduces no new principle, it is nevertheless quite significant, and it has often been fruitfully employed in scientific research. It really deals with two phenomena rather than with positive and negative instances of the same phenomenon. It argues to the existence of some necessary relation in a larger system of phenomena which vary together. To vary together or concomitantly may mean either that both phenomena increase together, or that both diminish together, or that one increases as the other diminishes, or *vice versa*. In fact, any sort of variation in one phenomenon, accompanying a variation in another, comes within the scope of this method. For the establishment or pointing out of such a concomitance in variation indicates a

deep-lying connection between the two phenomena. Such a concomitance in variation may sometimes be produced by experiment, and it may also be discovered by observation. Consequently this method has a very wide scope. It has been fruitful in astronomy, which is restricted to observation, and it has been fruitful in physics where exact experiment is possible. Frequently, the variations discovered or produced by this method can be exactly expressed by the aid of mathematical calculations, so that the resulting causal laws have a high degree of exactness and certitude. Hence this method also approximates closely to the statistical method, when a determination of the variations has to be reached by statistical data. On the other hand, when it is possible to produce variations experimentally, the method of concomitant variations is really only a corollary of the method of difference. For up to the moment when the supposed cause and the phenomenon entirely disappear we have only a variation, but at that moment we get a real negative instance. As an experimental method, then, the method of concomitant variations is a stage in the method of difference. Yet it is very valuable at times because there are phenomena in which it is impossible to eliminate entirely a causal feature, whereas it is possible to produce a variation of some sort in it. And even though a full negative instance in the sense of the method of difference cannot be obtained, the fact that we are able to produce a definite variation justifies the inference that there is a causal relation involved.

Mill recognized that the method of concomitant variations is really a special form of the method of difference. He writes:

It is scarcely necessary to say, that in order to ascertain the uniform concomitance of variations in the effect with variations in the cause, the same precautions must be used as in any other

case of the determination of an invariable sequence. We must endeavor to retain all the other antecedents unchanged, while that particular one is subjected to the requisite series of variations; or in other words, that we may be warranted in inferring causation from concomitance of variations, the concomitance itself must be proved by the method of difference (Bk. III, Ch. VIII).

Mill's illustration of the method makes its relation to the method of difference even clearer. Take Newton's first law of motion, that all bodies in motion continue to move in a straight line with uniform velocity until acted upon by some new force.

This assertion is in open opposition to first appearances; all terrestrial objects, when in motion, gradually abate their velocity and at last stop. . . . Every moving body, however, encounters various obstacles, as friction, the resistance of the atmosphere, etc., which we know by daily experience to be causes capable of destroying motion. It was suggested that the whole of the retardation might be due to these causes. How was this inquired into? If the obstacles could have been entirely removed, the case would have been amenable to the method of difference. They could not be removed, they could only be diminished, and the case, therefore, admitted only of the method of concomitant variations. This accordingly being employed, it was found that every diminution of the obstacles diminished the retardation of the motion, and inasmuch as in this case the total quantities both of the antecedent and consequent were known, it was practicable to estimate with an approach to accuracy both the amount of the retardation and the amount of the retarding causes or resistances, and to judge how near they both were to being exhausted; and it appeared that the effect dwindled as rapidly as the cause. . . There could therefore be no hesitation in assigning the whole of the retardation of motion to the influence of the obstacles; and since, after subducting this retardation from the total phenomenon, the remainder was a uniform velocity, the result was the proposition known as the first law of motion (*Idem*).

For another illustration, take the experiment of the bell jar which proves that air is a necessary medium for

transmitting sound waves to the ear. When the bell is struck with air in the bell jar, sound is conveyed. But when the air is pumped out sound cannot be heard. This is the method of difference. But if the bell is sounded continuously while the air is being pumped out, the sound will diminish with the air until both finally disappear together. Up to the time when they disappear we have the method of concomitant variations, but after they disappear we have the method of difference. This shows very clearly the relation between the two methods.

We have already emphasized the fact that the method of concomitant variations is frequently fruitful in fields of investigation where experimentation is impossible. As an example of this, the frequently quoted passage from Jevons may be given: "The most extraordinary case of variations consists in the connection which has of late years been shown to exist between the aurora borealis, magnetic storms, and the spots on the sun. It has only in the last thirty or forty years become known that the magnetic compass is subject at intervals to very slight, but curious movements; and that, at the same time, there are usually natural currents of electricity produced in telegraph wires, so as to interfere with the transmission of messages. These disturbances are known as magnetic storms, and are often observed to occur when a fine display of the northern or southern lights is taking place in some part of the earth. Observations during many years have shown that these storms come to their worst at the end of every eleven years. . . . Close observations of the sun during thirty or forty years have shown that the size and number of the dark spots, which are gigantic storms going on upon the sun's surface, increase and decrease exactly at the same periods of time as the magnetic storms upon the earth's surface. No one can doubt, then, that these strange phenomena are connected together, though the mode of the

connection is quite unknown" (*Elements of Logic,* Hill ed., p. 223)

The Method of Residues

In many scientific investigations there are what is known as *residual phenomena.* After certain causal connections are discovered there remain factors, the cause of which are not known and Mill's method of residues was devised to deal with such residual factors. He stated the method as follows: "Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents." Now this is not really a separate method, since the precise nature of the causal connection between the residue of a phenomenon and the residue of the antecedents would have to be determined by some other method. Its greatest value is in the fact that it brings to light causal connections which would not otherwise be suspected to exist: It often informs us of "sequences in which neither the cause nor the effect were sufficiently conspicuous to attract to themselves the attention of observers." Hence there are some remarkable examples of discoveries made by this method.

Perhaps the most remarkable of all is the discovery of the planet Neptune. It might not have been suspected of existing had it not been for certain deviations in the movements of Uranus from the orbit calculated for it by computations based on the gravitational forces bearing on it. According to these gravitational forces of the sun and other planets astronomers calculated the orbit of Uranus to be an ellipse of a certain form. But the actual orbit of Uranus, as calculated from observed positions of the planet, was known to be different from the orbit calculated from the forces bearing upon it. Here was a residual phenomenon in the orbit of Uranus. What causes this slight

deviation? There must be another planet bearing upon it, another gravitational force in addition to those already known to astronomers. Two mathematicians even calculated the approximate position of the planet; J. C. Adams in England, and U. J. J. Leverrier in France. Adams sent his calculations to an astronomer named Challis, who discovered Neptune on August fourth and twelfth of the year 1846, but failed to recognize it as a planet. Leverrier sent his observations to the astronomer, Galle, of Berlin, who discovered the planet and recognized it as a planet on September 23, 1846. This is one of the most remarkable scientific discoveries ever made. Of course the real method used was mathematical calculation and observation, but the probability of there being another planet was suggested by the method of residues.

Another interesting example of the method of residues is the very recent discovery of the function of the spleen. Dr. Joseph Barcroft, on measuring the blood volume from time to time as temperature was gradually increased, found that there was a gradual increase in the amount of hæmoglobin in the circulatory system corresponding to the increase in temperature. This was determined by the method of concomitant variations. But in carrying the investigation further, the method of residues came into play, as is evident from Dr. Barcroft's description of the investigation following the discovery of the greatly increased blood volume.

> Whence came this outpouring of hæmoglobin? It was not credible that the bone-marrow could have provided the body with new corpuscles at the rate required. Moreover, there was no evidence of increase of immature corpuscles in circulation. . . . The question then was forced upon us: Has the body any considerable but hidden stores of hæmoglobin which can be drawn upon in case of emergency? . . . In searching for a locality which might fulfill such a condition, one naturally seeks in the first instance for some place where the red blood

corpuscles are outside the circulatory system—some backwater outside the arteries, capillaries, and veins. There is only one such place of any considerable size in the body—that place is the spleen.[2]

Note that the suggestion reached by the method of residues, that the spleen is the source of the residual supply of blood corpuscles, would have to be proven by the method of difference. This Dr. Barcroft did by bleeding to death two cats—one whose spleen had been removed and one having a spleen. The cat whose spleen had been removed died much sooner. Thus residues is not really a separate method, but it calls attention to a further problem which can be investigated by one of the other methods.

Exercise XIV

In the following examples of Mill's methods tell which of the five methods is exemplified in each, and analyze the example sufficiently to prove that it exemplifies the method you say it does. Analyze each example to bring out the separate steps in the method you think is used in it. Formulate an opinion of your own as to the validity of the proof.

1. Synthetic catalysts, made of activated alumina contained in a silica gel, are highly active and quite rugged. Introduction of such a catalyst into a suitably prepared oil stock causes a reaction to take place, consisting of a rearrangement of organic molecules, and this produces high-octane gasoline.
2. Exactly what happens in this oil cracking process is a problem that has not yet been solved. Using radioactive carbon 14—an isotope of common carbon 12, in that it is heavier, or of different atomic weight—attempts are being made to trace the measurement of migratory carbon atoms from one part of a petroleum molecule to another, and thus discover why a catalyst works so effectively in the cracking process.
3. It has been discovered that little droplets of synthetic substances added to motor oil decrease the wear of bearings making a longer period between oil changes.
4. The number of bacteria per cubic foot of air during rush hours in public places has been determined by a portable electric air sampler to be approximately 27 compared with 15 when no patrons were present.
5. Germicidal lamps are used to destroy bacteria in the Phila-

[2] Joseph Barcroft, *The Lancet*, Feb. 14, 1925, p. 321.

delphia Zoo. Special lamps are used to provide short wave lengths of ultra-violet light that are effective in destroying bacteria in areas that are not exposed to the sun's rays.

6. In radiography, the higher the atomic weight of the material penetrated, the longer the exposure and the higher the voltage. Thus, photographing through the human body takes only a fraction of a second and 70,000 volts, whereas photographing through steel requires several minutes exposure and over 200,000 volts.
7. Sabadilla, a new insect killer extracted from the seeds of a Central American lily, will kill bugs without killing plants, as has been proven by experiments in an agricultural experiment station.
8. A new method of extracting oil from shale by treatment with "heating gases" carries the oil yield to 15 per cent of the weight of the shale.
9. Two sets of individuals suffering from hemophilia, or excessive bleeding differed from each other in that one group of four "bleeders" who were injured seriously all bled to death, but another group of "bleeders," who were equally seriously injured, all got well because they were injected with an antihemophilic substance known as globulin, which is a protein. With globulin, and thrombin, another clotting agent taken from blood, hemophiles can have operations performed and even undergo amputations without danger of bleeding to death.
10. An experimenter, Dr. M. Demerec of the Carnegie Institution, followed the growth of bacteria in various concentrations of penicillin. In a culture treated with only a little penicillin all of the 100,000,000 bacteria used survived. When he doubled the amount of penicilllin only 10,000 survived. Then he increased the amount of penicillin ten times, and only five of the millions survived.
11. Scientists believe that the present table of elements is incomplete. Since nuclear fission has already added plutonium and neptunium and two others, the 92 has now become 96. But seven more, those from 97 through 103, should be discovered. This will be the actinide series and each of these elements will be radioactive with a very short half-life, so short that they will be hard to detect. This much is known from recent nuclear research.
12. Five persons of different ages, conditions of living, different dress, some smokers and some non-smokers were all victims of respiratory infections in a city of excessively great smog. Examination proved that the respiratory organs of all five were definitely allergic to fog, and it was inferred that the cause of the respiratory infections was the smog.
13. Sugar-cane leaves were infected with virus and on examination

it was found that such infected leaves have a lower catalase activity than normal leaves, and even the infected area of a given leaf contained less catalase than a healthy portion of the same leaf.

14. Using two groups of human subjects experimenters found that the injection of nicotinic acid increases the V-factor content of human erythrocytes whereas injection of nicotinamide produced no such increases. Why this is so was not discovered.

15. "The interesting question as to whether pepsin and trypsin can combine only with subtrates, or whether combination can also occur with proteins not hydrolyzed by the enzymes, was considered by Kleczkowski. Previous work by Stanley has shown that tobacco mosaic virus solutions lost their infectivity when treated with trypsin.... In studying the combinations of potato X virus and tobacco mosaic with pepsin and trypsin, Kleczkowski found that pepsin combined with potato X virus, which it also hydrolyzed, but not with tobacco mosaic virus, which resisted digestion. However, when the latter virus was denatured by heat, it combined with, and was digested by pepsin. With trypsin the situation was reversed, in that this enzyme combined more readily with tobacco mosaic virus, which is not a substrate, than with potato X virus, which is digestible." *Annual Review of Biochemistry*, Vol. XIV, 1945, p. 41. James M. Luck, Editor Annual Reviews, Inc., Stanford University. Reprinted by permission.

16. In the study of the digestive processes, Maschmann has observed that the rate of hydrolysis of different peptides by the same enzyme extract may be maximally activated by different metal ions, manganous ion usually being the most potent. "Thus with extracts of kidney, liver, and intestine of rabbit and guinea pig, with chick embryo, and with most sarcoma extracts, it was usually found that manganese was the best activator." Adapted from *Annual Review of Biochemistry*, 1945, p. 51. Reprinted by permission.

17. Experimenters are clearing up the mysteries in the process of photosynthesis in relation to the energy-yielding "photochemical reaction" in plants that exhibit this phenomenon, notably *Chlorella*. Fan, Stauffer, and Umbreit "were able to show that, in the absence of carbon dioxide, the addition of a number of reducible substances to suspensions of *Chlorella* allowed production of oxygen during illumination. Detailed studies were made using benzaldehyde; it was shown that this substance did not produce carbon dioxide and that illumination in the absence of benzaldehyde or other reducing substances did not result in oxygen production." Adapted from *Annual Review of Biochemistry*, 1945, p. 22. Reprinted by permission.

18. "The theory that 'the function of light energy in photosynthesis is the formation of "energy rich" phosphate bonds' was proposed by Emerson, Stauffer, and Umbreit who presented evidence that, in *Chorella*, the distribution of phosphate compounds was markedly different after irradiation either in the presence or absence of carbon dioxide from that in control suspensions kept in the dark." *Annual Review of Biochemistry*, 1945, p. 22. Reprinted by permission.
19. Experiments by Navy scientists show that two weeks' exposure to sunlight for three of four hours daily reduces a person's sensitivity to very dim light by 50 per cent. The ability to detect very dim light rests in certain parts of the retina of the eye, which become insensitive after exposure even to the brightness of a 100-watt bulb. Usually their sensitivity returns after about fifteen minutes in the dark, but men who underwent the Navy tests needed an hour in the dark to reach their peak of night vision. One day's protection with sun glasses that transmitted only 12 per cent of sunlight brought their eyes back to normal night-sensitivity.
20. Dr. Frank Holtman of the University of Tennessee, followed up the observation that mice will survive a polio infection more readily if kept at low temperatures. Since low temperatures speed up the food-burning rate of animals, he tried out the effect of the hormone responsible for the speed-up, thyroxin. He found that polio-infected mice lived longer if treated with this hormone or a similar metabolism stimulant.
21. Dr. Vincent C. Barry of University College, Dublin, Ireland, prepared a new drug from a chemical, diploicin, extracted from a lichen. Small doses of the drug kills TB bacteria in test tubes.
22. Daguerre accidentally left an exposed photographic plate in a closet containing an open dish of mercury. When he returned, he discovered that the plate had been "developed." Eventually he proved that mercury vapors were responsible, and a short time later the daguerreo-type photographic process was established.
23. Perkin, attempting to make quinine from coal tar, casually added alcohol to the black mess of an unsuccessful experiment. A beautiful purple color resulted. Following up this gift of luck, the scientist produced the first of the coal-tar dyes.
24. Becquerel was inspired to begin the investigation of uranium radiation because he accidentally placed a piece of uranium near a photographic plate and later noticed that the developed plate was marked by mysterious streaks and spots.[1]

[1] Numbers 19 to 24 were adapted from *Science Illustrated*, Oct. 1946, Vol. I, No. 7, p. 8 and p. 66, McGraw-Hill Publishing Co. Reprinted by permission.

25. Normal blood contains a blood-clotting substance. This has been proven experimentally by taking a person suffering from the disease known as hemophilia, which disease is due to a lack of the blood-clotting substance in the blood, and introducing a trace of the venom of the snake called Russell's viper, whose venom is known to contain the blood-clotting substance, into the blood stream. Blood drawn from this patient before the venom was injected into the blood stream did not clot for over forty hours but blood drawn after the injection of the venom clotted almost normally (that is, as in a patient not suffering from the malady.)
26. Another kind of bleeder—one who bleeds into his own internal cavities—is not relieved by the venom of Russell's viper. After careful observation of three or four of this kind of bleeder of different age, habits of living and nationality, it was found that all of these patients had weakened capillary walls. So it was concluded that the weakened walls of the capillaries is the cause of such internal bleeding.
27. In seeking a cure for this internal bleeding Dr. Peck, evperimenting with animals, discovered that moccasin-poison contains a substance which, when used in small doses over a period of time, gradually toughens the delicate walls of the blood capillaries to such an extent that internal bleeding was greatly decreased.
28. A number of individuals who differed in other respects each had the misfortune to be bitten by a cobra while on a snake-hunting expedition in India, whereas others in the party escaped being bitten by the dangerous reptile. All of those who were bitten soon lapsed into a state of medical shock and died of paralysis of their breathing apparatus, but those who did not get the venom of the cobra into their bodies quickly recovered their composure after the excitement of the furious battle with the cobra.

SECTION VIII

EXPLANATION AND THE EXPLANATORY METHODS

CHAPTER XXII

EXPLANATION

What Explanation Is

Explanation has to do both with facts and with laws. It differs only in degree from causality and probability. Any definitely established causal connection may be said to explain the facts which are causes and effects within that particular order system. Hence the discovery of causal laws by the use of Mill's methods is a partial explanation of facts. Similarly a law of probability may be said to be an explanation of the numerous data which are studied statistically in reaching it. But the important point is that both the causal and the probable law, established for a definite set of data, are only *partial explanations*. They omit many of the detailed aspects of the implicative system in which they live and move. Now, explanation is the scientist's attempt to correlate or synthesize different laws of both kinds, causal and probable, so as to effect a fuller interpretation of the whole system. Thus explanation deals both with the facts, and with the laws discovered by statistical and causal methods to interpret these facts, but it always seeks a higher *principle of explanation* which will clarify *all* of the interrelations within the ordered system of data and laws under consideration. We do not fully explain a fact until we comprehend the nature of the underlying system sufficiently well to tell just what the fact is in its numerous relations, and *exactly why it is what it is.* Once the deeper nature of this system is discovered, and formulated as a *principle of explanation,* the whole confused, or relatively confused, mass of facts and laws which come under its juris-

diction are brought into an ordered unity. The *why* of each of them is apparent. Consequently the goal of science as a whole, as well as of each particular science, is a complete explanation of all concrete facts and special laws. And within a separate science such a complete explanation is usually spoken of as a *theory* or *explanatory principle* rather than as a law. Yet some very general laws are also explanatory principles. Such, for example, is Newton's law of gravitation. The tremendous significance of the theory of evolution is to be found in the fact that it is an explanatory principle of the greatest range and moment. The nebular theory in astronomy and the theory of relativity for the physical sciences are other examples. Every science tends to culminate in some all-inclusive theory, or explanatory principle of this sort. Such a theory is the scientist's attempt at a full explanation for all his laws and facts, and that is why there is such an upheaval in a science when a new theory supplants an old one. The Copernican theory in astronomy created a fearful disturbance in upsetting the old Ptolemaic theory, and the new theory of relativity seems now to be producing a similar effect in the physical sciences.

Perhaps the nature of explanation can be made still clearer by a very simple illustration. Suppose I find a single piece of a child's pasteboard picture puzzle which consists of twelve pieces in all. Taken by itself, I may ponder over what it is and not be able to understand it at all. But when I am shown the other eleven pieces, and am told that the one I hold in my hand is the twelfth of a set forming a single picture, I begin to understand what the single piece is. When I put the twelve pieces together to form the picture, I get a much better understanding of the single piece, in that I now see its relations to the other pieces and its contribution to the complete picture. When I break up the picture and take this piece apart from

the others, it thereafter has for me the meaning it got from its relations to the other pieces. I have found the complete explanation of that piece in its relations to the whole puzzle. All explanation is like this—only the systems into which facts are fitted in scientific research are the infinitely complex systems which make up nature or the real world. The full explanation of a fact does not come, even to a scientist, until the work of comprehending the system in question is completed. Sometimes this takes years and years, and the coöperation of many highly intelligent and well-trained research workers. Each one builds on what was accomplished by his predecessors, and the final explanation is not reached until the work is completed. A fact or law is explained only when a sufficient knowledge of the system to which it belongs is reached to enable one to interpret the fact or law in terms of that system, and as one of the actual members of that coherent and orderly whole.

The Relation of a Theory to Its Facts

What is the relation between a theory and the facts which it is supposed to explain? Does the explanation of a set of facts take us away from the facts? Is it purely human or manmade, whereas they are natural and independent of man's mind? Or does an explanation express an actual system of things in which the facts live and move? Are the facts retained in the explanation, or are they dropped out and left behind? These questions are among the most significant and searching that can be asked in the whole field of logic. The opposite answers divide logicians into hostile camps to such an extent that it is doubtful whether a better introduction to contemporary logical theories of science could be found than that afforded by a study of this controversy. Here it is only possible to sketch the central theories, but in order to make

the discussion more intelligible it seems best to give a clear illustration of a universally recognized bit of scientific explanation on which to hang it.

No illustration of scientific explanation is more famous or better suited to elucidate the real meaning and the conflicting theories of explanation than the investigations of Tycho Brahe, Kepler, and Newton which led to the abandonment of the old Ptolemaic theory of the planetary motions and the substitution for it of the Newtonian theory.[1] At different times during the year, Tycho Brahe made numerous careful observations of the positions of the planet Mars in the attempt to discover its actual orbit. Kepler used Brahe's tabulations as a starting point for his own investigations and finally, after countless failures—"driving him," as he says, "almost to insanity," he at length had the intense gratification of finding that an elliptical orbit described about the sun in one of the foci agreed accurately with the observed motions of the planet Mars."[2] Extending this to the other planets he formulated his famous first law: *The orbits of the planets are ellipses having the sun at one focus.* Before this law was formulated the actual planetary motions were a baffling mystery to astronomers. Kepler's law was a *partial explanation* of these movements. It brought into an orderly array a prodigious number of facts which had hitherto been in a chaotic state, namely, the various observed positions of Mars and other planets. The principle of the elliptical orbit now became an explanation of these positions. But this was only a partial explanation. It explained only the positions. The problem now arose of explaining the ellipses. "Why do

[1] For a fuller account of this development in astronomical theory, see *Introduction to Reflective Thinking,* Ch. III. On explanation see Ch. VI, *ibid.*, and article in *Encyclopedia Britannica* (14th ed.), by E. Meyerson.

[2] Baden-Powell, *History of Natural Philosophy,* p. 150. Quoted from Mellone's *Introductory Logic,* p. 295.

the planets move in elliptical orbits?" is the question which Newton answered, thereby giving a *complete explanation* of the data. Newton showed Kepler's law to be a special case of a far more general law. His principle of gravitation explains why the planets must move in ellipses, enables us to calculate the exact orbit each planet must follow, and the extent to which the gravitating forces of other planets would create aberrations in the elliptical orbit of any particular planet. This is what led to the discovery of Neptune, which we used to illustrate the method of residues. Newton's law of gravitation is thus an explanatory principle of the greatest range and of the highest value. Its explanation of the planetary movements may be said to be complete. Keeping in mind this wonderful illustration of an explanatory principle, let us return to the question asked above. What is the relation of this or any explanation to the actual facts it is supposed to explain?

1. *The Abstractionists' Theory.* Consider first the answer of logicians who may be called *abstractionists.* According to this view, explanations are purely abstract human constructions or generalizations. They are just our human way of looking at the facts and of reducing them to a conceptual unity. Just as the human being forms universal ideas, like *redness,* by *abstracting* some feature of a single experience away from that particular experience and erecting it into a general abstract idea, so the scientist, or many coöperating scientists, abstract away from *concrete sense data* some common characteristic which is called a law or explanatory principle. We must not think that in reality or the nature of things there is any such law. It is purely the work of the scientist. Indeed the more general it is, the farther removed it is from concrete facts. Science is the pursuit of the widest pos-

sible generalizations, which, when reached are the farthest possible removed from the actual detailed facts of experience.

Thus Karl Pearson, a well-known advocate of this position, writes:

> A scientific law is the *résumé* or *brief expression* of the relationships and sequences of certain groups of perceptions and conceptions, and exists only when formulated by man. . . . The law of gravitation is not so much the discovery by Newton of a rule guiding the motion of the planets as his invention of a method of briefly describing the sequence of sense impressions which we term planetary motion. . . . The statement of this formula was not so much the discovery as the *creation* of the law of gravitation. A natural law is thus seen to be a *résumé* in mental shorthand, which replaces for us a lengthy description of the sequences among our sense impressions. Law, in the scientific sense, is thus essentially a product of the human mind and has no meaning apart from man. It owes its existence to the creative power of his intellect. There is more meaning in the statement that man gives laws to Nature than in its converse that Nature gives laws to man.[3]

It is my opinion that nothing could be falser than this view. To say what is here said is to make nature, as we know it, a construction of the human mind. Can any scientist think that he is only expressing the connections among his own sense impressions when he formulates an explanatory law? Certainly not. He knows full well that his explanatory principle is worthless if it fails to express *the actual order in the nature of things.* The value of Newton's explanation of the planetary movements is to be found in the fact that it is a very exact formulation of the order of nature itself and not simply of his own impressions. But in spite of its falsity this view of explanation is very widespread among contemporary logicians. It is a vicious subjectivism which is really rooted in extreme scepticism. *Positivism* is another name for it.

[3] Karl Pearson, *Grammar of Science* (3d ed.), pp. 82, 86.

2. *The Bergsonian Theory of Explanation.* The French philosopher, Henri Bergson, has put forth the theory that all explanation falsifies the facts. He is the most eminent living representative of a view known as *anti-intellectualism.* Science is for him, too, a construction of the human intellect. It is valuable in practical life to make these scientific constructions known as *explanations,* but they are profoundly untrue to the actual facts. To get at the facts *as they are* we must use *intuition* and not *scientific analysis.* Only an intuitive insight, only a placing of oneself inside of the living facts will enable one to know facts as they are. Explanations give us exterior views, inert and dead constructions, of living facts. Newton's formula is about as far removed from the living movements of the planets as it could possibly be. Those movements are real, but his formula is a dead scientific abstraction.

Indeed, according to Bergson, there are four distinct stages in explanation, each one of which keeps getting farther and farther away from the facts. Facts are directly known by intuition. But we begin to leave these intuitively known facts behind in the very first stage of explanation, which is the distinguishing of common qualities in the facts. For these qualities are not really distinct. But science must have cut and dried classes and it makes them by separating out the various qualities—color, weight, solidity, *et cetera.* The second stage in explanation is to classify the facts by putting them into these man-made classes. Thus I say "Red is a color and coal is heavy." Here we are two degrees removed from fact. We have artificial classes into which we try to fit each fact, but we first got our classes by setting up distinctions which do not exist in the facts. As a result of classification the facts are falsified into *events* and *sense objects.* Now the third stage in explanation Bergson calls induction. This

is the point at which we started in our account of explanation above. Here laws are formulated to express the relation between events and sense objects. But since these things are unreal members of unreal classes, it follows that the relations are also unreal. And the fourth stage takes us far beyond facts. For here we substitute for these objects mathematical symbols such as the formulæ used in physics. Then the relations expressed are relations between symbols, and are no longer even connected with the facts from which we started. Now, the poor deluded analytical scientist makes the mistake of thinking that these dry bones of explanation are the actual facts. Bergson, however, knows by intuition that they are absolutely unconnected with actual facts and are mere constructions of man's insatiable desire to analyze and pigeon-hole nature.[4]

Now the scientist is surely wasting his time if all of his attempts to explain the world of our human experience are carrying him out into an ocean of fantastic dreams and unrealities such as Bergson supposes scientific explanations to be. The only answer that can be made to all such views of the relation of explanations to facts is to point out that they make of modern science a fairy tale. And this is a sufficient answer. We have to assume that scientific explanations are taking us deeper and deeper into the real nature of things. We must believe that theoretical scientists, who are engaged in a disinterested pursuit of truth, are formulating the actual structure of the universe in their explanatory principles. To make any other assumption would be to repudiate the one solid foundation upon which modern civilization is built. Explanation is the goal of science. As science advances, the explanatory principles which it formulates express more and more accurately the actual order systems which constitute the real world. They represent

[4] See Mrs. Karin Stephens, *The Misuse of Mind.* Ch. I.

actual discoveries of the intricate systems in nature, and are not mere inventions of the human mind. The theories of the scientists do not falsify the facts. On the contrary, they actually comprehend the facts by bringing into the light and giving precise expression to the actual underlying system of reality within which these facts are fragments. Thus the inmost essence of explanation is the implicative system.

E. Meyerson refuses to follow either Bergson or the positivists. He admits that the motivating force behind the true scientist is the desire to find a completely rational theory of the real world. But in the end he is always confronted with what Meyerson calls an *irrational.* The contradiction between Carnot's principle in physics and the law of conservation of energy is an example. But there are other examples. Just when we think that a complete explanation has been reached, such an irrational turns up. The history of science proves that there is much truth in this theory. But what are we to do about it? Must we attribute the irrationality to reality itself, as Meyerson does? Let us rather attribute the irrationality to human finitude, and let us think of the past victories that scientists have won over such irrationals. *Such victories rationalize the belief that reality is rational.*

It should be emphasized that the methods of explanation cannot be sharply separated from the methods already considered. For those methods are employed in the early stages of explanation. But in the working out of a complete and comprehensive explanatory theory special methods are used, which are known as explanatory methods just because of their effectiveness in taking the investigator beyond the point reached when he restricts himself to the use of the statistical and experimental methods. For this reason the most important method of this group, namely, the *complete method of explanation,* is also the most important method known to science.

CHAPTER XXIII

THE METHOD OF ANALOGY

Justification for Treating Analogy as a Scientific Method

The term *analogy* is another one of those logical terms which is used in a great variety of ways. Under the name of the *paradeigma* it was recognized as a form of argument by Aristotle. And ever since his day analogical reasoning has been used extensively and deliberately in every department of human knowledge. It has always played an especially significant rôle in theology. Owing to the great antiquity of this form of reasoning it might be thought that it should have been included in Part One, under traditional Aristotelian logic, instead of being reserved for treatment here as one of the inductive methods. Then, too, the fact that it has been one of the pillars by which theology has always been supported is also calculated to deprive it of the right to be treated as a scientific method. Hence, before explaining in detail what analogical reasoning is, and as a preliminary stage to such a setting forth of its nature, it is first necessary to state briefly the reasons for treating analogy as a scientific method and for including it among the explanatory methods.

In the first place, almost all modern textbooks place analogy under inductive rather than under deductive logic. This is because it includes, as Bosanquet has rightly remarked, "an aspect of discovery, as well as the aspect of proof." It is this aspect of discovery, which sets analogical reasoning apart from traditional syllogistic inference, and justifies treating it under the methods of science.

In the second place, analogy is employed quite exten-

sively in every branch of science, from the most exact and abstract, such as physics and chemistry, to the more concrete and complex, such as the social sciences. Botanical classification, the diagnosis of diseases in medical science, historical interpretation, social and economic theories, chemical and physical discoveries, all more or less involve analogical reasoning. This extremely widespread use of analogy in the various sciences also justifies treating it as one of the scientific methods.

Yet, as was suggested above in the classification of the methods, why not regard analogy as one of the methods of probability, instead of treating it here under explanatory theory? For it is usually considered to be a part of probable reasoning. Now it must be conceded that analogy gives only a degree of probability rather than that kind of certitude which results from careful experiment. If the word probability is used in the broad sense to mean the degree of certitude which a method yields, analogy would undoubtedly belong under the methods of probability. But in the last analysis so would all other methods. However that may be, analogy is frequently used to effect a more complete explanation than can be reached by the methods of probability in the sense of sampling and statistics. Moreover, as we have already suggested, it is very often applied in fields of investigation which fall outside of statistical data. Then, too, and from the point of view of our classification of the methods this is very significant, analogy stands in the same relation to the complete method of explanation that agreement stands to the method of difference, or simple enumeration to statistics. Its function is to suggest fruitful explanatory principles, just as it is the function of the method of agreement to suggest causal connections, and of simple enumeration to suggest probable laws. These, then, are the reasons for treating analogy as one of the explanatory methods.

The Dual Meaning of Analogy

There is a narrow and a broad interpretation of analogy, and a discussion of each will help to make clear what is meant by this type of reasoning.

1. *Mathematical Analogy.* According to this theory, analogy should be restricted to *relations* and should never be applied to *things or objects*—the terms of relations. Thus we say that the relation between a captain and his crew is analogous to the relation between the Mikado of Japan and his subjects. There is a *resemblance* or *similarity* between these two relations such that one can argue to unknown details about the relation between the Mikado and his subjects from a knowledge of those details in regard to the relation between a captain and his crew. Or, to put it differently, the fact of a resemblance justifies one in using what is known about one relation to explain what is not known about the other. Because certain things are known to be involved in one of the relations, it is assumed that the fact of resemblance carries with it the same things for the other relation. Now this view that only relations are involved in analogical reasoning, or that analogy consists in an argument from known features of one relation to unknown features of another relation, by virtue of a known resemblance between the two relations, is what is meant by mathematical analogy. And, although this meaning of analogy must be retained, it is far too narrow. Analogy is also concerned with things or objects which resemble. On this point practically all logicians are agreed. Concrete realities, and not abstract relations, are the material with which analogy is primarily concerned.

2. *Logical Analogy.* The view which includes in the definition of analogy objects which resemble, as well as similar relations, may be designated *logical analogy*, to distinguish it from mathematical analogy. Resemblances between relations, then, are only a small part of the re-

semblances with which analogy works. Any similarity between objects may be used as the basis for an inference by analogy. Now, to be sure, the deeper the resemblance is rooted in the nature of the two objects or relations involved, the greater the value of an inference from it. Moreover, the larger the number of resemblances, the more probable the conclusion. Logical analogy, then, means that inference which is drawn about unknown features of a certain object, based upon a known resemblance between it and another object, about which what is inferred in the analogy is already known. Thus, Mars and the earth are both alike in being planets, they are nearly equidistant from the sun, each has an atmosphere, they are alike in conditions of day and night, temperature, seasons, humidity, etc., each has seas and continents. Now all these resemblances being established by various observations, we may infer by analogy, from our knowledge that the earth supports organic life, that the planet Mars is also inhabited. Since they resemble in so many different ways, why not in this matter of supporting life? This is a stock illustration of reasoning by analogy. All that can be claimed for it is a certain degree of probability.

Analysis and Exemplification of the Method of Analogy

Having made clear the general meaning of analogical inference, it is now necessary to give a more detailed analysis to bring out the separate steps in the method.

The first step consists of a careful observation and listing of the points of similarity between the two objects or relations under investigation. If the inference is to be of real scientific value, this should be done with the utmost care and attention. It is especially important to find as many as possible *fundamental resemblances*, as distinct from purely external and superficial likenesses. Hence, each observed similarity must be carefully *weighed or estimated*,

in comparison with the others, so that the fundamental ones may be sorted out from those which are really insignificant. In certain cases this testing of the resemblances might involve experiment.

2 The second step in the method of analogy is the observation and tabulation of all of those aspects which are known about one object, and not about the other. Two lists should be drawn up. The more exhaustively each object is known, over and above the similarities between them, the greater the value of the inference.

3 The third step is to select from among those things which are known about one of the objects, ~~and unknown~~ about the other, those which are most likely to be true of both. These should be carefully formulated, as hypotheses or explanatory principles. That is to say, they should be expressed as definite analogical inferences to the effect that since one object has the characteristics in question, and also resembles the other object in various known respects, therefore the other must also possess these same characteristics.

Now it goes without saying that the method of analogy may be, and frequently is, used without these separate steps being differentiated. But in order to comprehend fully what analogy involves, it is necessary to represent the process as being more formal than it usually is in actual scientific procedure.

A distinguished French historian, E. F. Gautier, has recently put forth an interesting explanation of the great World War in which the method of analogy plays a significant rôle. He enumerates certain resemblances between the history of France and the evolution of species of animals. The data of history, he holds, are very similar to the data of paleontology. There are, for instance, the same laws of growth, maturity and senescence, and evolution into a new and more developed species. And he

goes on to enumerate five chief cycles in the evolution of France. These are: (1) the conquest of France by the Romans; (2) the invasion of the barbarian hordes, which overthrew and utterly destroyed the Roman power in all of Gaul; (3) the invasion of the Normans; (4) the One Hundred Years' War; and (5) the great World War of 1914. He points out that each of these leading events was a great catastrophe, similar to those recorded in the rocks for animal species. Each cycle occurred at an interval from the catastrophe preceding of about five hundred years or fifteen generations, and from each was born a new era in France. And this, too, is similar to the cycles involved in the evolution of the species. In short, since the invasion of Cæsar, five successive Frances can be traced, and this evolution is quite like those separate stages in the evolution of such a species as the horse and the elephant.

But now we know that cataclysms among the species are followed by a rejuvenation and a new development. After each one of the cataclysms there is an upward and higher progress. And in the case of the four past cycles in French history this is also known to have happened to France. Therefore, on the basis of this close resemblance, Gautier infers that France can look forward to a new cycle of rejuvenated national energy. This is the explanatory theory which he reaches as a result of his analogy.

What, now, is to be thought of this analogy? It is highly suggestive, interesting and original. But does it prove the conclusion? Certainly not. Only time can prove his theory that France is entering a period of rejuvenated national energy. He has only advanced a pleasing tentative hypothesis. The analogy is suggestive, but he does not establish his conclusion. Now this is true of every analogical conclusion, unless it is supported by the complete method of explanation. The great value of this

method of analogy is that it serves as a guidepost and directs the way to valid principles of explanation. To prove that they actually are valid it is not within the power of this method to do.[1]

The Defects and Fallacies in Analogical Reasoning

Valuable though it is in suggesting fruitful explanatory principles, analogy is subject to serious limitations. In fact, it is a form of reasoning which scientists generally scathingly condemn, but which they all find it necessary to use. One of its chief defects is that it does not take account of differences. It throws the whole emphasis upon resemblances, but the differences between objects are just as significant as the likenesses. Then, too, as we have already noted, analogy is frequently based on extrinsic and superficial resemblances instead of being based on fundamental and intrinsic ones. Moreover, in the example just given, we also have a good illustration of basing the analogy on too few facts. Aspects of the evolution of species which are frequently important are ignored. Moreover, there are highly complex factors entering into social cataclysms which do not enter into biological evolution at all. Gautier oversimplifies and ignores numerous factors. On the basis of a comparatively few facts he makes what has been well called a *hasty generalization*. This is a vice which frequently besets those who resort to analogical reasoning.

Then there is the special vice known as *false analogy*. This expression is sometimes used to mean attaching more weight to an argument from analogy than is warranted in view of the defects in that form of reasoning. As Mill points out, it is the peculiar vice of unimaginative people to

[1] M. Gautier's articles were entitled: "Interpretation Biologique des Grandes Catastrophes," and they first appeared in the *Mercure de France* (1920). I owe my account to a résumé, published in the Boston *Transcript*.

overrate the importance of such similarities or analogies as they happen to observe. "We always find that those are the greatest slaves of metaphorical language who have but one set of metaphors" (Mill).

However, the real meaning of false analogy is drawing an inference from analogy which is no more probable, or perhaps not even as probable, as the reverse or contradictory of that conclusion would be. Now since some lines of evolution are known to have culminated in the extinction of an animal species, Gautier's argument, as sketched above, is also a good illustration of false analogy in this precise logical sense of the word. Think, for instance, of the conspicuous example of the Irish reindeer, which developed such stupendous horns that they became too great a strain on the organism and led to the extinction of the species. May there not be an analogy between the final stages of this line of evolution and present-day France? May not an overdevelopment of militarism, imperialism and chauvinism be to France what the Irish reindeer's horns were to it? How do we know that every national catastrophe will be rejuvenating? Other nations have been buried in oblivion. May not France also become extinct?

Here is another very interesting example of false analogy in this real logical meaning of the term. Dr. Brill, distinguished expositor and defender of Freudian psychology or *psychoanalysis,* suggests an explanation of why a street sweeper he chanced to know chose the realm of dirt for his life work. When challenged to give his reason the man said: "What does it matter what you do? This job is as rotten and dirty as any other." Dr. Brill comments: "He was an avowed anarchist, and took every occasion to decry the rottenness of our social system. May we not assume that his choice of a vocation was an unconscious effort to clean up the rottenness of society, which seemed to trouble him so much?" Now it must be admitted that this sug-

gested explanation is entirely consistent with psychoanalysis. Nevertheless it is an excellent example of false analogy. For a non-psychoanalyst might very well retort: "How do you know that the man turned to scavenging because he was dissatisfied with the established order of society? Perhaps he became dissatisfied because scavenging was the only job he could get."

Analogy and Circumstantial Evidence

As Bode has pointed out, there is a very close connection between circumstantial evidence and reasoning by analogy. Suppose such a crime as murder has been committed. The prosecutor studies the facts and circumstances of the crime and, on the basis of this study, forms a clear conception of the *hypothetical* criminal. He then finds striking similarities between this hypothetical criminal and some suspected individual. Because a certain person, X, resembles the hypothetical person who committed the murder in having in his possession property of the murdered person, in having been seen near where the crime took place at about the time it was committed, and in possessing a blood-stained weapon exactly like the weapon with which the person was killed, it is highly probable that X is the murderer. Resembling the hypothetical criminal in so many respects that are fundamental, X must resemble him in having done the deed—must be identical with him.

Such reasoning is essentially analogical and is subject to all of the defects of analogy expounded above. Yet many a suspect has been punished by our courts solely on the basis of circumstantial evidence. In many cases those punished were doubtless guilty, but the logician must question the right of the state to administer capital punishment on such evidence alone.[2]

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[2] See Felix Frankfurter's *The Case of Sacco and Vanzetti.* The exercise on analogy is at the end of the next chapter.

CHAPTER XXIV

OTHER METHODS OF EXPLANATION

The General Meaning of Hypothesis

The word hypothesis in science usually refers to a definite interpretation of a given set of facts, which is put forth as a tentative suggestion and remains partly or wholly unverified. After it is once established it ceases to be an hypothesis, and becomes a theory or explanatory principle. Such tentative and unproved suggestions are frequently referred to as *working hypotheses*. And in spite of Francis Bacon's condemnation of using general axioms which have not been established by an appeal to facts, it is now generally conceded that science would be reduced to mere classification of observed data, if the use of hypothesizing were forbidden. Practically every great explanatory theory, now accepted as valid in science, was first advanced by some great scientist as an hypothesis. Later it was verified and became an established theory. Verification, however, is not a part of the method of hypothesis. When we pass to verification we have entered the domain of the complete method of explanation. The method of hypothesis is restricted to the formulation of good working suggestions, good tentative principles of explanation. Hence we may distinguish between an hypothesis and the method of hypothesis by defining the former as any well-knit principle which is put forth as a tentative explanation of a group of facts, and by regarding the method of hypothesis as the procedure, including the rules used, in reaching such a well-knit principle.

Although this is the strictly logical meaning of hypothe-

sis, there are other current interpretations of the word which should be briefly differentiated from it, if confusion in the use of the term is to be avoided. In the first place, we must distinguish the scientific use of the term, which has just been stated, from the *popular meaning*. According to the latter, any fact used to explain another fact is called an hypothesis. Whenever something unusual strikes the attention of the average unscientific person, he immediately puts forth the hypothesis that it must have been due to something else. If, for example, he finds a bridge out, he will explain it by the hypothesis that there has been a heavy downpour of rain in that locality, or that a heavily loaded truck or a threshing machine has broken the bridge down. Thus, in such a popular hypothesis, there is really no attempt to formulate a highly general principle of explanation to interpret a complicated mass of data. The contrast between such hypotheses of popular thought and a genuine scientific hypothesis, such as the nebular hypothesis in astronomy, is so great that the distinction between the two meanings needs no further elucidation. For even though it be admitted to be only a difference in degree, the difference is so vast that for all practical purposes it is a difference in kind.

But in the second place a genuine scientific hypothesis must be distinguished from what Jevons well called "purely *descriptive hypotheses.*" Indeed the contrast here is so great that the latter might almost be called *pseudo-scientific hypotheses*. They are just convenient names for something very unusual, and about which we do not have sufficient knowledge to enable us to comprehend its nature. Thus, to use Jevons' illustration: "If we are to speak of what constitutes electricity . . . the motion of a fluid of a very subtle character presents itself as appropriate. There is the *single-fluid* and the *double-fluid* theory of electricity, and a great deal of discussion has uselessly been spent upon

them. The fact is, that if these theories be understood as more than convenient modes of describing the phenomena, they are altogether invalid.''[1] Scientists are now beginning to suspect that the theory of a *luminiferous ether,* as the bearer of wave motions, is another purely descriptive or pseudo-hypothesis, and that, to use the ungrammatical, but expressive, words of the rustic who looked for the first time upon a circus giraffe, ''there ain't no such animal.''[2]

The Method of Hypothesis

There is a great deal of truth in the contention that only great geniuses have the ability to formulate good scientific hypotheses, and that consequently no definite procedure can be laid down by following which they can be reached. Given a sufficient store of information in a certain field, and the scientist, gifted with intuitive insight and high powers of the imagination, simply sees connections among the data which the ordinary man never even so much as dreams exist. Now it must be admitted that it is thus that many great hypotheses are produced. ''With accurate experiment and observation to work upon,'' says John Tyndall, in his famous essay on *The Scientific Use of the Imagination,* ''imagination becomes the architect of physical theory. Newton's passage from a falling apple to a falling moon was an act of the prepared imagination; out of the facts of chemistry the constructive imagination of Dalton formed the atomic theory; Davy was richly endowed with the imaginative faculty, while with Faraday its exercise was incessant, preceding, accompanying and guiding all his experiments. . . . Without the exercise of this power, our knowledge of nature would be a mere tabulation of co-existences and sequences.''

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[1] W. S. Jevons, *Principles of Science,* 2d ed., revised, p. 522.

[2] See A. N. Whitehead, *An Inquiry Concerning the Principles of Natural Knowledge,* Ch. I.

Not every scientific hypothesis, however, springs full-grown from the Jovian mind of the scientific genius. Some are only gradually formulated after years of research and painstaking study. Moreover, it is easy to exaggerate the importance of intuitive insight on the part of the genius in the invention of hypotheses. They must be based upon facts. And in the formulating of the hypothesis, the hard work which even the genius must go through with as a disciplinary training is just as important as his crowning flash of insight. In any case, certain general requirements or rules to be followed in reaching an hypothesis are usually given by logicians, and I shall treat them as a part of the method of hypothesis.

1. *The hypothesis should be conceivable and not absurd.* Or, in other words, it must be capable of being brought into the received body of knowledge. This rule, however, must be carefully interpreted. It does not mean that all hypotheses are to be thrown out as worthless which seem absurd to ordinary minds. Few, indeed, were the physicians in England, contemporary with Harvey, who did not look upon his theory of the circulation of the blood as highly absurd! Instead of being left to individual taste, the test of the conceivability of an hypothesis must be determined by its power to explain the facts. Only experts capable of weighing the hypothesis have a right to say whether it is conceivable. Similarly, we cannot demand that hypotheses fit into the received system of knowledge in a purely mechanical or wooden way. We must never forget that knowledge is a living organism, and that new hypotheses can grow into the system, even though they appear, when first set forth, to be so novel as to be entirely out of accord with previous knowledge.

2. *The hypothesis must be of such a character that deductions can be made from it.* This is of the utmost significance. We cannot move a step toward verifying an hy-

pothesis until deductions are made which are capable of empirical verification. Consequently every hypothesis is entirely valueless which is not stated in such a way that various deductions can be made from it.

3. *An hypothesis should not contradict any of the known laws of nature.* But here, again, we must be on our guard against throwing out hypotheses which *appear* to contradict these laws. Perhaps a slight modification in the hypotheses or a reformulation of the laws may reveal the fact that the supposed contradiction does not exist. Every *new hypothesis* is likely to appear contradictory to old established natural laws simply because of its strangeness. Think, for instance, of the new theory of relativity. Yet such an hypothesis cannot be thrown out entirely until scientists are able to establish a flat contradiction between it and the known laws of nature. Here, as in many other matters, appearances may be deceiving.

But the most important element in the method of hypothesis still remains to be mentioned. It is what is known as the process of *molding the hypothesis*. This involves two things. In the first place, a good scientific hypothesis is usually the product of a number of different relatively vague guesses which have been fused together by a process of elimination of the irrelevant ideas, and of reconstruction of what are left into a clear and definite hypothesis. Hypotheses are thus gradually constructed out of various tentative ideas. But in the second place, the hypothesis must also be shaped to fit the facts. Even though more than one hypothesis be found to explain a set of facts, all which are not molded into the facts so that they actually explain them must be thrown out. By the molding of an hypothesis is meant this process of fusing vague and indefinite ideas into clear and precise principles of explanation, formulated so as to fit the facts under consideration.

As an illustration of this process of molding the hy-

pothesis, consider the following recent account of investigations into the nature of the disease known as cancer:

> Keeping in mind the facts that cancer begins in a circumscribed area and increases in size by a continuous multiplication of cells, the crux of the problem presented by it is seen to be: How are the cells in the localized area concerned, started on their altered tempo of growth, and why are they unaffected by the restraining influence which normally holds the cells of the rest of the body within proper bounds? It would be very natural to assume that a weakening of the restraint which the cells of the body exert on each other is merely a part of the general weakening of the natural forces accompanying old age. A special circumstance has enabled us to eliminate this suggestion, rendered unlikely by the fact of the circumscribed origin of these growths. (Note that this suggestion is rendered unlikely because it does not fit the facts.) The circumstance alluded to is the occurrence in certain animals of cancerous growths which can be grafted successfully into normal animals of the same species. Most of this work has been done with mice and rats. A careful day-to-day study of the site of the inoculation shows that there is no question of the cells of the inoculated animal's body becoming cancerous. What happens is that the cells introduced continue to grow until they form large masses of cells. The process can be repeated indefinitely in a succession of mice. Clearly, then, the rapid and continuous growth of cancer cells cannot be due solely to the weakening of the restraining influence on growth in the body of the cancerous subject. On reflection, there seems to be no escape from the hypothesis that the cancer cells behave as they do, because they are different from their neighbors. They have undergone a change in becoming cancerous, by which they no longer respond to the influences which restrain and regulate cell division and growth in the body.[3]

Note that the last sentence but one states the molded hypothesis, which has been reached by an elimination of the tentative suggestion stated in the second sentence of the quotation. While the hypothesis has been molded to fit

[3] Adapted from an article by J. A. Murray, entitled, "The Biological Problem of Cancer," *Discovery*, Vol. I, pp. 71 *f*.

the facts it must be especially emphasized that it is **not** verified. It is put forth simply as a working hypothesis to be established, if at all, by further investigations.

The Complete Method of Explanation

To effect a complete explanation of a phenomenon it is necessary to carry on the investigation until the hypothesis is fully verified. Then it ceases to be a mere hypothesis, however well molded, and becomes an established theory. The goal of explanation is to reach such a comprehensive proof of a well-molded hypothesis as will establish it on an impregnable foundation and bring it into the system of knowledge. Thus, Harvey's theory of the circulation of the blood may be regarded as such a complete explanation of the facts with which it deals. Four stages may be distinguished in the complete method, and in some of them other methods are employed.

1. *Collecting the Data.* Given a certain problem, the first step is to get a sufficient body of material together to carry on the investigation. This stage of the method will involve the general basic methods of enumeration or simple counting and observation, and in some cases it may even involve sampling and statistical methods. Every possible means at the scientist's disposal must be employed in getting the data assembled and properly arranged.

2. *The Formulation of Hypotheses.* These may be suggested by such methods as agreement, analogy and concomitant variations, and they may be gradually molded by the method of hypothesis which has just been explained. It is quite essential to get one or more good working hypotheses as tentative explanations of the data. Frequently many weeks or years are spent on a certain problem in the effort to formulate fruitful hypotheses. For instance, as we have just seen, the attempt to explain the nature of cancer is still in this second stage.

3. *Deductions from the Hypothesis, or Hypotheses.* In putting hypotheses to the test, various deductions have to be made from them. These may take the form of hypothetical propositions, such as, "If the hypothesis is true, then this and that and the other consequences should follow." From a single fruitful hypothesis, many deductions may be made. Hence, hypotheses from which deductions cannot be made are worthless. It is absolutely impossible to establish them as theories. This shows how far away from the truth Francis Bacon was in trying to exclude deduction from scientific method. The strictest form of inductive inquiry always involves a deductive element. The scientist cannot make any headway without formulating general working hypotheses as the major premises for deductive inferences. It will be remembered that we emphasized the importance of hypothetical reasoning in science in our account of the hypothetical syllogism.

4. *Verification of the Separate Deductions.* This involves observation and experimentation. Hence, Mill's method of difference would frequently be used here. Every means must be used to discover whether the facts support the deduction or deductions. If not, the hypothesis must be rejected. No hypothesis is ever directly verified. Its verification is *via* the separate verifications of the deductions. If a hypothesis yields fruitful deductions, which hold good when an experiment is performed to test them out, or when closer observation of the data is made, the hypothesis is *ipso facto* verified. It is by such a tedious process, in which many failures must be expected, many blind alleys explored, that the really valuable explanatory principles of modern science are raised from the level of mere guesswork to that of proven theory.

As an example of the complete method of explanation, consider the very recent experimental investigations which have revealed the nature of the Röntgen, or X-ray. All

other light rays were already known to be wave motions. Hence, the natural hypothesis was that the X-ray is also a wave motion. Hence, this hypothesis was already molded and ready for use as a result of investigations of other kinds of light rays. The deduction made from it was: If the X-ray is a wave motion, then it should be possible to diffract it, and, when it is diffracted, it will show the same effects as are shown by rays of the sun which diffract into lines of light, playing fantastically about and producing unusual color effects. Then came the problem of putting the deduction to the test by actually diffracting the X-ray to observe whether it would produce these effects. But the extreme shortness of this ray baffled every attempt to diffract it by any of the known diffracting media. "Then it occurred to Max von Laue to employ crystals as a grating through which Röntgen rays could pass. Crystals had for some time been regarded as geometrical in structure, and there seemed reason to infer that they had three dimensions in space. Such a grating, with minute intervals between each constituent part, seemed just suited for the diffraction of X-rays. An X-ray was duly isolated by means of a dark screen, and then sent through a crystal and caught on a photographic plate. After its development it revealed the expected picture of an atomic structure or arrangement. All about the imprint of the primary ray was developed a pattern of the diffracted rays." [4]

Inasmuch as Von Laue's experiment was a direct verification of the deduction from the hypothesis, it follows that the theory that the X-ray is a wave motion is now established on an impregnable foundation. Moreover, establishing it has thrown a flood of light on a whole nest of problems. For it explained the nature of crystalline structure,

[4] From an article in *Current Opinion*, Vol. LXXII, pp. 800 *f*. A fuller popular account of the Laue experiment is given in *Discovery*, Vol. I, pp. 35 *f*. See the references there given.

established the fact that the X-ray is a wave motion and proved conclusively the reality of the atom. Here, then, is a simple and clear illustration of a discovery made by the complete method of explanation. The data include X-rays and various diffracting media, especially crystals. The hypothesis was that the X-ray is a wave movement, and the deduction and its verification have been clearly stated. All the steps of the complete method are thus exemplified by this illustration.

The Nature of Verification and the Ways of Verifying

It is important to distinguish the *process of verification* from the results of that process. When thinking of the latter it is necessary to distinguish between results which confirm the hypothesis and those which refute it. The former I call *positive verification* and the latter, *negative verification.* Each hypothesis can be regarded as the antecedent of an hypothetical proposition, the consequents of which are the deductions. This means that it takes this form: if hypothesis W is true, then x, y, and z follow. Verifying positively consists in showing that x, y, and z do follow, and the conclusion is that the hypothesis is probably true. Note that the conclusion is only probable because positive verification commits the technical fallacy of the hypothetical syllogism known as *affirming the consequent.* Negative verification does not commit this fallacy, since it *denies the consequent.* (See pp. 166 *f.*)

This interesting difference between positive and negative verification shows how important it is in verifying hypotheses to establish the truth of as many deductions as possible. Several fundamental deductions that are all positively verified give great weight to any hypothesis and make it practically certain. Hence it is important to make and to verify every deduction an hypothesis will yield. On the other hand. deductions that are verified negatively may

indicate that the hypothesis needs remolding rather than that it is entirely false.

We may also distinguish five different ways of verifying deductions made from hypotheses.

1. *Observation and Experimentation.* This involves making a detailed study of the data on hand, gathering more data, or performing some kind of an experiment, all with the definite purpose of finding out whether the deduction which is being tested really holds. In making such tests, Mill's methods may be used.

2. *Mathematical Calculation.* When quantative matters are involved, mathematical calculations may be made and the facts shown to be either consistent or inconsistent with these computations. This way of verifying is frequently combined with experiment or observation, the calculations being regarded as a more exact and detailed interpretation of the deduction. Sometimes such mathematical elaborations of a deduction furnish the basis of a *prediction* of the precise way in which something will happen in nature. When such a theoretical prediction is later verified, the proof of the hypothesis from which it has been deduced is highly conclusive.

3. *Statistical Verification.* Closely allied to mathematical calculation is statistical enumeration. When experiment is out of the question, a statistical study may be made for the purpose of ascertaining whether the deduction always or generally holds. This may also bring into play the method of sampling.

4. *Verification by Elimination.* An hypothesis may be strengthened by eliminating all competing hypotheses. The negative verification of competing hypotheses becomes a positive verification of the hypothesis which cannot be eliminated. To be sure, other evidence can usually be found favoring such hypotheses, but elimination greatly strengthens the remaining hypothesis. Here again, as in

Mill's method of difference, negation is shown to be highly significant in the growth of knowledge.

5. *Verification by Disjunctive Reasoning.* Sometimes, the hypothesis is verified by showing that it is the only one consistent with what is already known in the field of knowledge to which the data under investigation belong. The reasoning may take the form of a disjunctive syllogism—either this hypothesis is true or else what we know in this field is false. But what we know in this field is not false and hence this hypothesis is true. Consistency with previous knowledge in its field is always an excellent criterion of the truth of any hypothesis. That is why scientists always try to show that the hypothesis which they defend fits in with what we know better than any of the competing hypotheses.

In actual research, scientists combine in one way or another all of these methods of verifying hypotheses. Hence it is a mistake to think of them as being mutually exclusive. The scientist verifies his deductions in any way that he can.

The Historical Method of Explanation

There is a method of explanation, widely used in the social sciences which goes by the name of the *historical method.* It is really a combination of two other methods—the *genetic* and the *comparative* method. Each of these may be used as a separate method. But they may also be employed together, and when they are synthesized and fused into a single method they become the historical method.

The genetic method of explanation consists in tracing a complex phenomenon to its origin. The phenomenon under investigation may be an organ or function of a living body, a social institution or custom, a religious ceremony or doctrine, a legal code, a language, a form of literature, a geological deposit, or what not. Hence this method can be

profitably employed in any science where complex phenomena can be shown to have had a history or to have undergone a temporal development from a lower to a higher form, from a relatively simple to a more complicated structure. To follow such a process of unfolding back to its beginning, to isolate the various *genetic forces*—those forces coöperating to produce the phenomenon under investigation, to understand a later stage by connecting it with an earlier—this is the genetic method of reasoning.

This genetic idea may be said to be the universal methodological assumption of all the social sciences. The basic principle of social organization in any particular social order is known as the *social pattern* of that group. The genetic method, as employed in the social sciences, consists in tracing human institutions, customs, and beliefs back to the social order in which they originated. The social pattern of some particular group is regarded as the essential causative factor in the formation of such customs and doctrines.

The genetic method passes imperceptibly and by logical necessity into the comparative method, which is the explanation of a phenomenon by comparing and contrasting it with other phenomena or by comparing and contrasting one of the stages in its development with another stage. As Ernst Mach has well said:

Comparison, as the fundamental condition of communication, is the most powerful inner vital element of science. The zoölogist sees in the bones of the wing-membranes of bats, fingers; he compares the bones of the cranium with vertebræ, the embryos of different organisms with one another, and the different stages of development of the same organism with one another. The geographer sees in Lake Garda a fjord, in the Sea of Aral a lake in process of drying up. The philologist compares different languages with one another, and the formations of the same language as well. If it is not customary to speak of comparative physics in the same sense that we speak of comparative anatomy, the reason is that in a science of such great experi-

mental activity the attention is turned too much from the *contemplative* element. But like all other sciences, physics lives and grows by comparison.[5]

The historian, the sociologist, the economist, the legal theorizer, the philologist all make use of the genetic and comparative methods in their researches. But they synthesize and fuse them together into a special method of research. This is the historical method. Unique events which may never recur in human experience are thus dealt with scientifically. Their genesis is traced and they are compared and contrasted with events in other cultures or civilizations. All of this elucidates their meaning, brings to light their organic relations, and makes clear their complex ramifications. Hence the historical method is a basic instrument of the social sciences. In importance it must be ranked alongside the statistical method, these two methods having made possible the exceptional growth and extension of the social sciences during the last half-century.

The historical method also includes the criticism of texts and documents to determine their authenticity, reliability, date, and the circumstances connected with their origin. The *internal evidence* is that which is derived from a detailed analysis of the text or document under investigation. The *external evidence* is that obtained from other sources bearing in any way on the events and ideas dealt with in the original document. This special form of the historical method is basic in such sciences as history and philology.

Subsidiary Processes

To complete our account of the methods of explanation it will be well to discuss together certain processes, to some of which we have already alluded. Abraham Wolf calls these "types of explanation," but from our viewpoint they

[5] *Popular Scientific Lectures* (Open Court Publishing Co.), p. 239.

are rather subsidiary processes or ways of explaining.[6] However, they are of special value and importance, and the student will find it interesting and profitable to compare them with the basic methods of explanation expounded above.

1. *Reference to a Class.* A phenomenon may be explained by referring it to some class or by creating a new class for it. For example, a lichen of a type not hitherto known may be put in some existing class of lichens or a new class may be made for it under some wider class.

2. *Reference to an Evolutionary Series.* A class of phenomena or a single phenomenon may sometimes be explained by locating it at some definite place in an evolutionary series. For example, newly discovered bones of an "ape-man creature" may be located in the evolutionary series of *homo sapiens.* This process is subsidiary to the genetic method discussed above.

3. *Reference to Mediating Conditions.* If we want to show how facts seemingly remote are really connected, we may try to bring to light the intermediate facts or events to bridge the gap. For example, ether-vibrations and nerve currents are used in physiological psychology to explain perception.

4. *Reference to Laws.* We may refer a new event to some relevant law, theory, or general principle of explanation already accepted. For example, "the bent appearance of a stick partly immersed in water is explained by reference to Snell's law of refraction." (Wolf.)

5. *Reference to Purpose.* One way to explain conduct is to refer it to the purpose of an agent. This is especially valuable in dealing with human volitions or with behavior

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[6] See Abraham Wolf's valuable article entitled *Scientific Method* in the new (14th) edition of the *Encyclopedia Britannica,* from which I have taken the names of these five processes, although I have expounded each in my own words.

such as that of the chimpanzees which was described at the end of Chapter I. Ethics, legal science, and other social sciences cannot dispense with this way of explaining, even though mechanistic and behavioristic psychology demands the abandonment of all explanations in terms of purpose. William MacDougall and others call this type of explanation *teleological explanation.*

Exercise XV

1. Analyze each of the following examples of the method of analogy by listing as many resemblances as you can, and by indicating (*a*) whether it is used to clarify exposition or to make a discovery, (*b*) whether it is between objects or between relations, and (*c*) whether a conclusion is drawn. In each case give one reason why you think the argument is valid or invalid.

(*a*) Imagine how difficult it would be to view a motion picture show through a pin hole in your theater program. That is analagous to scanning the heavens with the new 200 inch telescope at Mt. Wilson Observatory.

(*b*) An atom is like the solar system. The nucleus of the atom resembles the sun. This consists of a bunched mass of protons and neutrons. The electrons resemble the planets. There is an exact balance between the positively charged protons of the nucleus and the negatively charged electrons that revolve around it. Hence the movements of the electrons around the nucleus are like the movements of the planets around the sun.

(*c*) Radioactive isotopes serve as tagged atoms or tracers when mixed with common stable atoms of the same species. They "fly with the flock," and can later be identified as surely as banded birds. With these amazing tools of research, the course of any element or compound may be traced through the bodies of men, animals, and plants.

(*d*) According to the great Arabian philosopher, Averroes, the supreme mind of the universe, God, called the active intellect, in relation to a human mind, called the passive intellect, resembles the sunlight which, in passing through refracting media is broken up into variegated light rays such as make a rainbow, yet this sunlight recovers its perfect unity when the refracting media are removed. Dr. Stephen C. Tornay elaborates this analogy as follows: "Our bodies are like dewdrops, varying in size and shape. The quantitative differences of the glassy surfaces, observable on the dewdrops, may be compared to the passive intellect, that is, to our different individual dispositions.

When the sun, the active intellect, sends out his rays on the dewdrops, the smooth glassiness of these drops becomes luminous, capable of mirroring external objects, and this luminosity, a common character in every dewdrop, may be compared to the material intellect. The material intellect, in our comparison the sunny luminosity, is not an emergence from the dewdrops. Water can never turn into sunshine. Rather, the material intellect is to be conceived as identical with the sun which radiates actively and is luminous passively, although its luminosity can come into existence only in the presence of dewdrops. The radiance of the sun—the active intellect—in conjunction with the luminosity of the sun—the material intellect—results in a refraction, different altogether according to the individual drop of dew.'' *Philosophical Review,* Vol. LII (May, 1943), p. 277.

(*e*) For many years the Mendeleiff Table of the Elements consisted of the fixed number of 92 elements. The discovery of isotopes of various elements greatly complicated the table. Then through nuclear fission of atoms of radioactive elements four new elements were added to the table, including plutonium and neptunium. On the basis of these remarkable discoveries it has been suggested that a series of seven more elements from 97 through 103 will eventually be added to the table, and it is believed that each of the seven will be radioactive and will have a very brief half-life, so brief in fact that they will be so short-lived that they will be difficult to discover. These seven elements are called the actinide series.

(*f*) ''William Wundt, one of the greatest of German psychologists, under whom a large number of the early American psychologists received their training, proposed a 'tri-dimensional' theory of feeling (often extended to include emotions as well), which he hoped would enable one to give a complete account of the psychological dimensions of a feeling just as we state the dimensions of a rectangular solid in terms of its height, breadth, and thickness. According to Wundt, feelings vary in one dimension along a scale running from unpleasantness at one end up to pleasantness at the other, with a neutral zone in the middle. The second dimension has excitement—the greatest state of excitement imaginable—at one extreme, and numbness or depression at the other extreme, with varying degrees of excitement or its lack in between. The third dimension runs from tenseness to release or relaxation. A good many persons have objected to Wundt's scheme both on the grounds that it is incomplete, that feelings have other dimensions or qualities that are equally important (Woodworth in his *Psychology* mentions as examples, desire-aversion and familiarity-strangeness), and on the grounds that in the two dimensions

last named the extremes cited are not true opposites of each other.'' Florence L. Goodenough, *Developmental Psychology*, 2nd ed., pp. 516-517, D. Appleton-Century Co. Reprinted by permission.

2. Imagine that you have met a victim of amnesia who has completely lost his sense of identity, and explain how you would go about finding out who he is by the use of circumstantial evidence when the following facts are known: the victim's age, sex, profession, marital status, home, state and city, and the police information that a person from that locality and similar in age, sex, profession and marital status has been reported missing for a definite period of time. Discuss the validity of such reasoning.

3. Analyze each of the following examples into the four steps of the complete method of explanation. First, indicate the problem under investigation and the data. Second, formulate one hypothesis to explain the data. Third, make one or more deductions from this hypothesis. Fourth, show what evidence in the example supports or refutes this deduction, and tell whether this evidence is a case of positive or negative verification of the deduction. Carry your analysis as far as possible by stating other deductions, or other hypotheses, if you find more than one of each.

(a) *Penicillin-Resistant Bacteria*

When penicillin first came on the market it was regarded as a miracle drug capable of destroying any kind of bacteria. But recently hardy bacteria have been found that appear to be able to survive a treatment of penicillin. These hardy bacteria created a new problem for investigation. How could bacteria resist so powerful a drug?

One investigator surmised that this was due to the bacteria adapting themselves to an environment containing penicillin. Now, if this is true, patients under treatment with penicillin would have bacteria in their blood-stream that were immune to penicillin.

''That the appearance of penicillin-resistant bacteria is connected with the use of the drug itself has been established. A recent survey of blood-destroying staphylococcus bacteria found among patients in an Army hospital detected 40 different varieties of the micro-organism that would not be killed off by practical doses of penicillin. Significantly, most of these hardy bacteria were found in men who had at some time received penicillin.

''Laboratory evidence emphasizes this point. If bacteria are grown in cultures that contain small but increasing amounts of penicillin, they may eventually survive doses of penicillin, large enough to wipe out 'untrained' bacteria. In this way experimenters have trained pneumonia bacteria to tolerate 30 times the normally effective dose

of penicillin, and similar results have been obtained with other disease producers. Once the resistant forms have been developed, they retain their insensitivity indefinitely.''[1]

(b) *Bodies Smaller than Atoms*

In a paper entitled ''On Bodies Smaller than Atoms,'' published in the Smithsonian Scientific Report for the year 1901, Sir J. J. Thomson proved that there are objects more than a thousand times lighter than a hydrogen atom.

He started his investigations when he discovered that the particles of a cathode ray have an exceedingly small mass when electrically charged in comparison with an electrically charged mass of hydrogen atoms. His problem was to explain this difference. He surmised that it had to be due either to the mass of each cathode ray particle being very small compared with that of the hydrogen atom, or else to the electric charge carried by each particle being large compared with that carried by the hydrogen atom. Thus, it was necessary to determine the electric charge carried by one of these particles to find out whether that charge was excessively large.

Here is Sir Joseph's own account of how this was done. ''The problem is as follows: Suppose in an inclosed space we have a number of electrified particles each carrying the same charge, it is required to find the charge on each particle. It is easy by electrical methods to determine the total quantity of electricity on the collection of particles, and knowing this we can find the charge on each particle if we can count the number of particles. To count these particles the first step is to make them visible. We can do this by availing ourselves of a discovery made by C. T. R. Wilson, working in the Cavendish Laboratory. Wilson has shown that when positively and negatively electrified particles are present in moist dust-free air, a cloud is produced when the air is cooled by a sudden expansion, though this amount of expansion would be quite insufficient to produce condensation when no electrified particles are present; the water condenses around the electrified particles, and, if these are not too numerous, each particle becomes the nucleus of a little drop of water. Now Sir George Stokes has shown how we can calculate the rate at which a drop of water falls through air if we know the size of the drop, and conversely we can determine the size of the drop by measuring the rate at which it falls through the air. Hence by measuring the speed with which the cloud falls we can determine the volume of each little drop. The whole volume of water deposited by cooling the air can easily be calculated, and dividing the whole volume of water by the volume of one of the drops we get the number of drops, and hence the number of the electrified particles. We saw, however,

[1] ''Bacteria Become Hardy,'' *Science Illustrated*, Vol. 1, No. 4, p. 56, published by the McGraw-Hill Publishing Co. and reprinted by permission.

that if we knew the number of particles we could get the electric charge on each particle; proceeding in this way I found that the charge carried by each particle was about... 2.17×10^{-20} electro-magnetic units.... In the electrolysis of solutions... the atom of hydrogen will carry a charge equal to 2.27×10^{-20} electro-magnetic units.... These numbers are so nearly equal that, considering the difficulties of the experiments, we may feel sure that the charge on one of these gaseous particles is the same as that on an atom of hydrogen in electrolysis. This result has been verified in a different way by Professor Townsend, who used a method by which he found, not the absolute value of the electric charge on a particle, but the ratio of this charge to the charge on an atom of hydrogen, and he found that the two charges were equal.'' And this is Sir Joseph's conclusion: ''As the charges on the particle and the hydrogen atom are the same, the fact that the mass of these particles required to carry a given charge of electricity is only one-thousandth part of the mass of the hydrogen atoms shows that the mass of each of these particles is only about one one-thousandth of that of a hydrogen atom.'' *Smithsonian Scientific Series,* edited by C. G. Abbot, Vol. 12, p. 52 *f*. Reprinted by permission.

(*c*) *The Wilson Cloud Chamber*

In 1912 Mr. C. T. R. Wilson in Cambridge, England, formulated the hypotheses that supersaturated vapor will condense more readily on ions than on neutral molecules. It is known that alpha particles passing through an air chamber will produce ions. This deduction was made from the hypothesis: If the air in a chamber is saturated with water vapor, and is cooled by expansion immediately after an alpha particle has passed through it, tiny drops of water will condense on the ions formed by the alpha particle. This was verified by experimentation. Another deduction was: If these drops of water are made to reflect sufficient light, then the actual path of each alpha particle can be seen or photographed. And this deduction was also verified positively by experimentation. As a result the Wilson Cloud Chamber has been perfected, and ''has been enormously useful in studying the behavior of individual particles, alphas, protons, electrons, positrons, mesotrons, photons, and the fast atoms caused by collisions with alphas, protons, or neutrons.'' Adapted from Henry De Wolf Smyth, *Atomic Energy for Military Purposes,* Princeton University Press, p. 231. Reprinted by permission.

(*d*) *The Discovery of Neutrons*

''In 1930 W. Bothe and H. Becker in Germany found that if the very energetic natural alpha particles from polonium fell on certain of the light elements, specifically beryllium, boron or lithium, an unusually penetrating radiation was produced. At first this radiation was thought to be gamma radiation although it was more penetrating than any gamma rays known, and the details of experimental results

were very difficult to interpret on this basis. The next important contribution was reported in 1932 by Irene Curie and F. Joliot in Paris. They showed that if this unknown radiation fell on paraffin or any other hydrogen-containing compound it ejected protons of very high energy. This was not in itself inconsistent with the assumed gamma-ray nature of the new radiation, but detailed quantitative analysis of the data became increasingly difficult to reconcile with such an hypothesis. Finally (later in 1932) J. Chadwick in England performed a series of experiments showing that the gamma-ray hypothesis was untenable. He suggested that in fact the new radiation consisted of uncharged particles of approximately the mass of the proton, and he performed a series of experiments verifying his suggestion. Such uncharged particles are now called neutrons.'' *Idem.* pp. 9 *f.* Reprinted by permission.

(*e*) *The Use of Neutrons*

E. Fermi formulated the hypothesis ''that neutrons, because of their lack of charge, should be effective in penetrating nuclei, especially those of high atomic number which repel protons and alpha particles strongly. He was able to verify his prediction almost immediately, finding that the nucleus of the bombarded atom captured the neutron and that there was thus produced an unstable nucleus which then achieved stability by emitting an electron. Thus, the final, stable nucleus was one unit higher in mass number and one unit higher in atomic number than the initial target nucleus.'' *Idem.* pp. 13 *f.* Reprinted by permission.

(*f*) *The Discovery of Uranium Fission*

Early in 1939, when Niels Bohr left Copenhagen for Princeton, New Jersey to work with Einstein, two of his colleagues, O. R. Frisch and L. Meitner suggested to him the hypothesis ''that the absorption of a neutron by a uranium nucleus sometimes caused that nucleus to split into approximately equal parts with the release of enormous quantities of energy, a process that soon began to be called nuclear 'fission.' The occasion for this hypothesis was the important discovery of O. Hahn and F. Strassmann in Germany (published in *Naturwissenschaften* in early January, 1939) which proved that an isotope of barium was produced by neutron bombardment of uranium. ...As a result of conversations among Fermi, J. R. Dunning, and G. B. Pegram, a search was undertaken at Columbia for the heavy pulses of ionization that would be expected from the flying fragments of the uranium nucleus....Several other experiments to confirm fission had been initiated, and positive experimental confirmation was reported from four laboratories (Columbia University, Carnegie Institution of Washington, Johns Hopkins University, University of California) in the February 15, 1939, issue of the *Physical Review.*'' *Idem.* pp. 24 *f.* Reprinted by permission.

(g) *The Discovery of the Personal Equation*

"Experiments in psychology had their beginning in the important discovery that no human being can observe and record with absolute accuracy. In 1796, Maskelyne, an astronomer at Greenwich, found that Kinnebrook, his assistant, was observing and recording the time of stellar transits almost a second later, on the average, than Maskelyne himself did. This was a very serious error indeed, since upon these observations depended the calibration of the clock by which the world's time was regulated, as well as all astronomical calculations about time and space. Although Kinnebrook strove to correct the error after his attention was called to it, he was unable to do so. If anything it grew worse. Maskelyne therefore decided that Kinnebrook could not be following the accepted method of observation but must have 'fallen into some irregular and confused method of his own.' Kinnebrook was accordingly dismissed.

"Several years later, Bessel, the astronomer at Königsberg, became interested in the matter and decided to find out whether the Maskelyne-Kinnebrook affair was a unique case or whether other astronomers might not also disagree in their observations if put to the test. In 1820, he found an opportunity to compare his own observations with those of Walbeck. It was found that Bessel always observed a transit earlier than Walbeck and that the average difference between their observations was even greater than that found between Maskelyne and Kinnebrook. This discovery led to a number of further investigations from which it became evident that the time required to observe and report any external event will differ from person to person, even when the utmost efforts to secure accuracy are made. This difference came to be known as the 'personal equation,' and while it was at first considered a problem of interest chiefly to astronomers, its wider significance soon became recognized, and many important investigations aimed at determining its physiological and psychological attributes were undertaken.

"Since that time, psychologists have been devoting much time to the question of errors of observation, and many important discoveries about the peculiarities of human nature have had their origin in attempts to account for the prevalence of certain kinds of mistakes." Florence L. Goodenough, *Developmental Psychology,* 2nd ed., pp. 33-34. D. Appleton-Century Co. Reprinted by permission.

4. Use the following example to illustrate the genetic and comparative methods, by indicating how each item of evidence proves that there have been cultural infiltrations into the Yellow River Valley in China. Try to trace each item to its origin in some other culture. This evidence has been brought to light recently by archeological excavations and it has pushed the twilight zone of history back some 500 years.

I. Objects found that are unknown in the Valley:
   (*a*) Jade, found in Tarjm basin.
   (*b*) Glass beads, similar in design to those of earlier Mediterranean artisans.
   (*c*) Painted pottery, strikingly similar to that of Eastern Europe.

II. Other indications of Cultural Borrowing:
   (*a*) Ox-drawn plows like those in the Near East.
   (*b*) Coffins similar to those in ancient Egypt.
   (*c*) Water clocks.
   (*d*) Parthian boots and trousers.
   (*e*) Sythian caps and belt buckles.
   (*f*) Iron swords, metals, and bronze castings, probably imported from the West.
   (*g*) Ivory.[2]

5. The following is an example of the historical method applied to a document known as *The Protocols of the Wise Men of Zion,* which has been widely used for anti-Semitic propaganda. Distinguish the internal and external evidence against the theory that this book is what it claims to be.

The Protocols claim to be the records of a number of secret meetings of leading Jews, at which plans for the complete subjugation of the whole world to a Jewish monarch were carefully made. They have been published in several languages, first in Russian in 1903, and again in English in 1920.

However, *The Protocols* have been proven to be substantially identical with a satire on the tyranny of Napoleon III, which was published in 1865. It was written by Maurice Joly and was entitled: *A Dialogue in Hell between Machiavelli and Montesquieu* or, the *Politics of Machiavelli in the Nineteenth Century.*

John S. Curtiss in *An Appraisal of The Protocols of Zion* (Columbia University Press, 1942) finds 176 points of resemblance between *The Protocols* and Joly's *Dialogue;* and many of these he prints in parallel columns. In 1921 Philip P. Graves obtained a copy of Joly's *Dialogue* from a Russian landowner who, in turn, had bought it from a former police officer of the Czar. In the London *Times,* August 16, 17, and 18, 1921, Graves pointed out a number of similiarities between *The Protocols* and Joly's *Dialogue.* These conclusions were confirmed and amplified by other writers, notably by Benjamin Segel (Berlin, 1924).

Thus the evidence obtained from a study of *The Protocols* and the

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[2] Adapted from the Abstract of Adlaid Alber Esteb's Ph.D. thesis: *A Study of the Influence of the Yellow River in Chinese Civilization. Abstracts of Dissertations.* The University of Southern California, 1945.

Dialogue, joined with that of the original appearance of *The Protocols* in Russia under the Czarist régime as justification of Jewish pogroms, proves that *The Protocols* are not what they claim to be. They were fraudulently compiled from Joly's *Dialogue* to further anti-Semitism in Russia under the czars. Alfred Rosenberg, the well-known Nazi philosopher, used *The Protocols* to further anti-Semitism in Germany, although he must have known of Benjamin Segel's book. Adapted from Professor Albert R. Chandler: *Rosenberg's Nazi Myth.* Cornell University Press, 1945, pp. 36-41.

6. Here is an example of the genetic method. Make an outline of the evidence that Mexican cultures influenced the North American Indians beyond the Rio Grande River.

The Influence of Mexican Indian Cultures

How much did the cultures of Mexico, rising and falling for more than 2,000 years, influence United States Indians? Recently Dr. Richard MacNeish of Chicago reported on an ancient trade route he had just discovered. The civilized Huasteca people, said Dr. MacNeish, left identifiable camp sites all the way up the Gulf Coast from their center near Varacruz to Texas. It is likely, he thinks, that Huasteca traders carried their goods and customs into what is now the United States as much as 1,000 years ago. Perhaps few Mexicans went farther than Texas, but there is overwhelming evidence that their culture, passed from tribe to tribe, influenced much of North America.

At Spiro, Oklahoma, are large low mounds, built by a long-forgotten people. In them have been found articles of stone, shell, clay and copper, many of which are so characteristically Mexican that their patterns could have come from nowhere else. Objects from a mound at Etowah in Georgia show similar influence. In far away Wisconsin, a small dead Indian city looked so "Aztec," even to early natives, that a Judge N. F. Hyer named it Aztalan. In what is now the North Eastern United States, the light of Mexican civilization shone with distant dimness. There were no great mounds, no templed pyramids. But some archeologists believe that the superior economic and political system of the Iroquois was of ultimate Mexican origin. Adapted from *Time,* Vol. XLVIII. No. 15 (Oct. 7, 1946), p. 68.

SECTION IX

RECENT TENDENCIES IN LOGICAL THEORY

CHAPTER XXV

A SKETCH OF THE DEVELOPMENT OF LOGIC WITH SPECIAL REFERENCE TO CONTEMPORARY SCHOOLS

The opening sentence of John Stuart Mill's great *System of Logic* is especially applicable to contemporary logicians: "There is as great diversity among authors in the modes which they have adopted of defining logic as in their treatment of the details of it." For present-day logic is actually almost an inextricable maze of more or less antagonistic doctrines, and the phrase, *bellum omnium contra omnes* can, as Edmund Husserl aptly says, be applied to modern logicians. Consequently, any method of ordering this chaos of conflicting interpretations is bound to be inadequate in some respects. But since it is desirable that the student, before quitting an elementary course, form some idea of contemporary tendencies and schools, it seems best to approach a classification through a sketch of the historical development of the science.

Traditional Aristotelian Logic

Logic really began when Zeno, the Eleatic philosopher (lived about 450 B. C.), formulated his celebrated paradoxes to prove the inconceivability of motion. The "flying arrow" cannot move because either it must move in the place where it is or in the place where it is not, and neither is conceivable. In a similar manner he argued that Achilles or the hare could not overtake a tortoise. Here is involved the use of the logical principle of contradiction which Zeno learned from his master, Parmenides. Motion is impossible because it leads you into the flat contradiction of

asserting that a thing can both be and not be in the same place at the same time. Thus arose the first logical question, namely, the question as to what methods of reasoning are trustworthy. Later on this type of argument was developed into the dilemma.

The next important contribution to logic was made by itinerant Greek teachers called the Sophists. They held truth to be relative to the individual. Protagoras, one of their leaders, enunciated the famous saying: "Man is the measure of all things, of things that are that they are, and of things that are not that they are not." Consequently, these men took a special interest in the *art of persuasion.* Since anything is true which any one believes, high value is attached to the ability to convince others. As early as the age of the Sophists (400 B. C.), rhetoric and logic began to take form as separate studies. Logic, then, very early appears in the history of thought as the art of arguing, this being one of the accomplishments essential to the success of a young Greek in public life. Needless to say, it has always retained something of this character.

Socrates raised the question whether there are not universal meanings or ideas which must be accepted as true by every thinking being. In seeking for such ideas he abandoned the view that truth is relative to the individual, and turned away from logic as an art of persuasion to logic as a science of definition. What do we mean by goodness, justice, or beauty? What is the common or universal meaning behind these words? This was the all-important question for Socrates. And he wanted and demanded something more for a definition than a mere example of an act of justice. He sought for the constituent elements in the thing itself. He thought that it was possible, and essential to clear thinking, to get at the common qualities possessed by all individuals of the group in question. Otherwise a knowledge of that group is impossible. We have already

seen that definition is also an essential part of present-day logical doctrine.

In order to define more accurately, Plato carried the investigations of Socrates further, and developed the methods of *analysis* and *logical division or classification.* In getting at the common qualities of an object, Socrates had already hit upon the method of analysis. Plato, however, greatly extended and made systematic use of it in his own teaching in the Academy of Athens. But not only did he formulate the method of analysis with greater precision and definiteness than Socrates had done, he also added to it the method of division. In the logical sense of the word, division is an instrument for classifying our knowledge of a subject. This instrument was carefully developed and applied in the investigations carried on in the Academy.[1]

Aristotle organized and arranged all of this material which had been worked out by his predecessors, and added to it his own doctrine of the syllogism. In the syllogism, he thought he had discovered the method of reasoning which is common to all branches of knowledge. There are, then, two reasons why Aristotle is usually called the "Father of Logic," (1) because of his having been the first to systematize the logical doctrines already in existence and (2) because of his having originated that form of reasoning known as syllogistic reasoning. His most important logical writings are: the *Categories, Interpretation, Prior Analytics, Posterior Analytics, Topics,* and a treatise on fallacies entitled, *Sophistical Difficulties.* His followers collected these writings into a single book, and gave it the title *Organon.* This was because they accepted Aristotle's

[1] The Academy was a higher educational institution founded by Plato in Athens. It was the most famous university of antiquity, although the Lyceum, which was later founded by Aristotle, was a close second.

interpretation of logic as a code of rules or principles used in the investigations carried on by science. The word *organon* means an instrument of thought, and to Aristotle and his followers the syllogism is the instrument of thought *par excellence.*

We learned above that there are four different figures of the syllogism. Aristotle only worked out the first three figures. The fourth is said to have been introduced by Galen, a celebrated physician, some time during the later half of the second century, B. C. Another important addition was made by the Stoic logicians of the Roman period when the doctrine of the hypothetical and the disjunctive syllogisms was worked out. Otherwise the Aristotelian logic has remained substantially the same as he left it.

Many modern logicians regard the adoption of the Aristotelian logic by *the schoolmen,* the orthodox church philosophers of the Middle Ages, as being, at least in some important respects, a step backward rather than forward. For these men did not fully comprehend Aristotle. Hence, their conception of the function or purpose of logic was radically different from his. As a result, the deeper side of his thought was subordinated to the purely formal use of the syllogism. They omitted especially the scientific foundation upon which Aristotle built his theory of the syllogism. Hair-splitting distinctions, for example, whether the pig that is being driven to market is held by the man or the rope, whether a shield that is white on one side and black on the other may be called either white or black, were defended and opposed by disputants in long strings of formal syllogisms. This way of using logic came to be known as *logic chopping,* and it has brought considerable discredit on syllogistic reasoning.

That this belief in the infallibility of purely formal logic as an art of thought died hard is shown by the following interesting quotation from Graham Wallas: "During the

years of the American and French Revolution Oxford students were still required, in order to receive their official certificates as trained thinkers, to repeat long Latin 'strings' of syllogistic affirmations and denials on some questions in moral or natural philosophy. Here is a translation of a part of such a string:

> *Opponent.* What think you of this question, whether universal ideas are formed by abstraction?
>
> *Respondent.* I affirm it.
>
> *Opponent.* Universal ideas are not formed by abstraction, therefore you are deceived.
>
> *Respondent.* I deny the antecedent.
>
> *Opponent.* I prove the antecedent. Whatever is formed by sensation alone, is not formed by abstraction, but universal ideas are formed by sensation alone, therefore universal ideas are not formed by abstraction.
>
> *Respondent.* I deny the minor premise.
>
> *Opponent.* I prove the minor. The idea of solidity is a universal idea, but the idea of solidity is formed by sensation alone, therefore universal ideas are formed by sensation alone, etc., etc." [2]

To-day it is generally recognized that scholastic logic is not pure Aristotelian logic, but more or less of an adulteration. In recent years logicians have gone back to a careful study of the Aristotelian writings to purge the traditional logic from its scholastic glosses. Hence criticisms of scholastic logic are not necessarily valid of Aristotelian logic. But too many critics of scholastic logic fail to take into consideration the general level of culture of the medieval period. We could not reasonably expect logical science to be far in advance of general knowledge, and in an age before the rise of modern science, and before the renaissance of classical learning, scholastic logic was a natural product. The schoolmen kept alive an interest in

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[2] *The Great Society*, p. 177. Quoted by Wallas from Godley: *Oxford in the Eighteenth Century*, p. 177.

logical questions before the enrichment of knowledge by the discoveries of the great scientists, and thereby bridged the gap between ancient and modern logic. For this they deserve great credit.

The Development of Methodology

Under the influence of the distinguished scientists of the fifteenth and sixteenth centuries, especially William Gilbert and Galileo, there was initiated a new movement out of which a supposedly new type of logic developed. Francis Bacon (1561-1626), contemporary and friend of Gilbert, worked out into a general logical theory of method the procedure Gilbert had employed in his remarkable experiments into the nature of magnetism which laid the foundation for the great science of electro-magnetism. Bacon followed Gilbert in condemning the scholastic method of reasoning. He set forth what he called a new method—a *Novum Organum,* to replace the *Organon* of Aristotle:

> There are (he writes) but two ways of investigating and discovering truth. The one hurries on rapidly from the senses and particulars to the most general axioms, and from them, as principles and their supposed indisputable truth, derives and discovers the intermediate axioms. This is the way now in use. (Bacon here refers to the traditional logic in the corrupt form in which it existed among the schoolmen of his day.) The other constructs its axioms from the senses and particulars, by ascending continually and gradually, till it finally arrives at the most general axioms, which is the true but unattempted way.[3]

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[3] *Novum Organum,* Bk. I, Aphorism XIX. That Bacon was indebted to Gilbert will be obvious to any one who reads the Preface of the latter's *Treatise on Magnetism,* published nearly twenty years before the *Novum Organum.* He there refers to his own method as "a new style of philosophizing," as against the method of "constructing certain ratiocinations on a basis of mere opinions and of old womanishly dreaming the things that are not." He insists upon the importance of beginning with what "we may handle and may perceive with the senses," taking first "facts of less rare occurrence, from

He then attempts to map out this new way by distinguishing instances where the thing or phenomenon under investigation is present, from those instances in which it is absent and those in which it is present in varying degrees. He arranged these different types of instances of the thing in (1) *a table of presence*, (2) *a table of absence*, (3) *a table of degrees*. David Hume took over these three principles later as rules for determining cause and effect. It remained for John Stuart Mill to develop Bacon's tables and Hume's three rules into his famous experimental methods—*agreement, difference, joint method of agreement and difference, concomitant variations and residues*, which were expounded in detail above. Thus, one line of development of scientific method took its departure from William Gilbert and Francis Bacon, and culminated in the experimental methods of Mill.

At the same time methodology was enriched by a renewed emphasis on deduction as it takes place in such rigid disciplines as the mathematical sciences. Descartes, the great French mathematician and philosopher, emphasized the importance of deductive procedure in scientific investigation. In this he was strongly supported by his contemporary in England, Thomas Hobbes, and by his successors on the continent, Spinoza and Leibniz. Incidentally, this movement led to a revival of the Aristotelian logic in the so-called *Port Royal logic*. Newton, in England, and Laplace, in France, worked in actual science along the same line that those great logicians worked in developing the logical theory of scientific method. The great weakness in the Baconian method was the neglect of hypothesis, whereas this was the strong feature of the Newtonian

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these proceeding to facts of a more extraordinary kind," and by means of repeated "experiments," reaching "hidden causes." See above, pp. 285 *f*. In fact, it is my opinion that Bacon got his *Novum Organum* from Gilbert.

method. To-day these two separate lines of development are being brought together, and a real science of method is in process of development. This science is called *methodology* of *scientific method,* and it was sketched in Part Two. It is being greatly enriched by contemporary investigations into the actual thought processes and methods which are proving fruitful in scientific research. As we have already seen, modern methodologists recognize the value of hypothesis and deduction, but they emphasize with equal force the need of experimentation, and of taking into consideration both positive and negative instances of the thing or phenomenon under investigation, as did Bacon and Mill.

Contemporary Logic

In the light of this brief historical survey, let us make a rough classification of the more important contemporary schools of logic.

1. *Traditional Aristotelian logic* is to-day represented by two different groups of logicians whose conceptions of it are diametrically opposite. (*a*) The *neo-scholastic logicians* have revived the logic of Aristotle as expounded by the schoolmen, on the assumption that this contains the essentials of logic and only needs to be purged of its medieval glosses and corruptions. Such a presentation of the subject is supposed to facilitate the study of logic by sparing the student the inevitable perplexities involved in any attempt to adapt the system of Aristotle to modern modes of thought which are wholly alien to it. P. Coffey's *Science of Logic,* and G. H. Joyce's *Principles of Logic,* both ably represent this standpoint. (*b*) *Modernized formal Aristotelian logic* is the doctrine of Aristotle, modified to meet criticisms, and to include a discussion of the actual thought methods employed in exact scientific research. Since Francis Bacon inaugurated a reform of the school logic it has always been

customary to include in a textbook some discussion of the experimental methods of science. Consequently, there has developed an important type of logic which contents itself with combining scientific method and Aristotelianism. H. W. B. Joseph's *An Introduction to Logic* is a thorough presentation and vigorous defense of this modern point of view.

2. *Symbolic Logic.* Since about 1850, under the influence of George Boole and John Venn, distinguished English logicians, a logic has developed which operates wholly with symbols instead of written words. Logicians of this school generally hold that exact thinking demands a special language because ordinary language is too clumsy to express the nice distinctions necessary to precise thought. Following a suggestion of Leibniz, these logicians have elaborated such a language frequently called *logistic*. Its elements closely resemble mathematical symbols. In fact, it may be regarded as an elaboration and extension of mathematical symbolism. Consequently this type of logic is frequently referred to as *mathematical logic*, and representatives of it have high hopes of its supplanting all other types, a position which only lovers of symbols are ready to take. It goes without saying that all the logical writings of this school of logicians are highly technical. The contemporary leaders in this movement are Bertrand Russell and A. N. Whitehead, whose *Principia Mathematica* is so far the most important contribution to the literature of this logic; Couturat, the late French logician, whose *Algebra of Logic* is perhaps the best introduction to symbolic logic; Peano, a great Italian logician; and C. I. Lewis, of Harvard, who has written a history of the movement. Owing to the technicality of symbolic logic, I have not attempted to deal with it in this book, but it is undoubtedly one of the most significant additions that has ever been made to the body of logical science.

3. *Modern Non-Symbolic and Non-Aristotelian Logic.* There is also an extremely important, a large and a growing logical literature, which is non-symbolic and largely hostile to the syllogistic logic of Aristotle. From our point of view this is the most substantial part of contemporary logic. It consists of three radically different schools, the differences being primarily due to their underlying philosophical positions.

(*a*) *Realistic non-symbolic logic* is nearest to and sympathetic with symbolic logic, but it operates with ordinary written language instead of with symbols. It is especially interested in and has made important contributions to scientific method. The important writings of the German logician, Alexius Meinong, and L. T. Hobhouse's work entitled *The Theory of Knowledge*, belong to this type of logic, as do also the coöperative volumes, *The New Realism* and *Essays in Critical Realism.* In general it is hostile to traditional logic, and emphasizes especially the relativity of human knowledge.[4]

(*b*) *Pragmatic logic,* sometimes called the *logic of personal idealism,* treats logic as the science of thought, but regards thought as only a stage in the temporal development of human experience. John Dewey is the outstanding representative of this type of non-symbolic logic, but he is ably supported by F. C. S. Schiller, Boyce Gibson and A. W. Moore. John Dewey's *Essays in Experimental Logic* is perhaps the most substantial piece of work of this school. Schiller's *Formal Logic* is largely a tirade against Aristotelian Logic. Boyce Gibson's *Problem of Logic* is a valuable introduction to the science from the standpoint of pragmatism, as is also the coöperative volume, *Creative Intelligence.* Mark Baldwin's *Genetic Logic* is somewhat apart from this movement but is closer to this than to

[4] See Ch. XVII, on "Probability."

any of the types which I distinguish. As a competent critic has said: "Though there are some important differences between Professor Baldwin's views and those of the pragmatic evolutionists, they belong together in general standpoint and aim. Not only do they both approach the problem of logic from the psychological point of view, but both alike derive their working conceptions from the biological formulation of evolution rather than from post-Kantian idealism."[5]

(*c*) *Idealistic non-symbolic logic* goes back to Kant and Hegel, two great German logicians, who did their chief work from 1750 to 1830, and attempts to conserve the important logical doctrines of these writers in modern logic. Hence it is traditional but non-Aristotelian. This type of logic has eminent defenders in Germany, among whom the late Windelband, Husserl, Rickert and Cassirer are especially important. Josiah Royce's valuable logical articles in the *Encyclopedia of Religion and Ethics*, and in Ruge's *Encyclopedia of the Philosophical Sciences*, are written from this standpoint. In England its staunchest defenders are F. H. Bradley, H. H. Joachim and Bernard Bosanquet. The latter's two volumes, *Logic*, and his smaller *Essentials of Logic* and *Implication and Linear Inference* are admirable expositions of this position. According to Dr. Bosanquet, the task of logic is to "understand the process of understanding, the growth and transformation of thought," but he means by thought any thinking whatsoever and not just that of a finite mind.

In succeeding chapters the theories of truth, the interpretations of the basic assumptions of logic, and the attitude toward traditional Aristotelian logic of these three schools will be briefly outlined.

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[5] J. E. Creighton, *Psychological Review*, Vol. XVI, p. 179.

CHAPTER XXVI

CURRENT THEORIES OF THE NATURE OF TRUTH

Truth as the Central Problem of Logic

Each branch of philosophical knowledge may be distinguished by the central question or questions which it attempts to answer. Thus, the basic question of *metaphysics* is: What is the nature of ultimate reality? And the chief questions of *ethics* are: What is goodness or value? What is duty or obligation? And *æsthetics* asks: What is the nature of beauty? While the *philosophy of religion* asks: Who is God? What are the proofs for the existence of God? Is man immortal? Is evil real? In like manner, the most fundamental question of logic is: What is truth? Every treatise and monograph written on the subject of logic is dominated by some theory of the nature of truth. To be sure, it may either be implicit or explicit, but in some form or another it is sure to be there. To be a logician, one must have an answer to the absorbing question which the Fourth Gospel has Pilate put to Jesus: "What is truth?" It is doubtful whether there is any question known to man which is any more fundamental than this one, or any the answer to which has farther reaching consequences. Hence, each contemporary school of logic has its own theory of truth.

We have been presenting one of these current theories of truth in one form or another throughout this textbook. When we first presented the doctrine of the implicative system or the inferential whole as the unit of knowledge, in Chapter I, we were really presenting an answer to the question: What is truth?

And again, we were dealing with it in various other forms

in our account of the nature of judgment, of inference, of the general theory of induction as scientific analysis which reveals the actual relations in reality, and in our account of explanation as reaching out toward a complete synthesis of all the facts in some comprehensive theory. In brief, our entire exposition of the subject matter of logic has been based upon, and thoroughly permeated by, the theory that truth is what a proposition gets from its relation to its implicative system, and that whatever truth any particular assertion or inferential process possesses is wholly constituted by its place in this system. And this holds, regardless of whether it is an inductive generalization reached by one or more of the scientific methods, or a conclusion reached by the use of syllogistic reasoning. Sometimes we have referred to this objective system of order as being knowledge, and sometimes as being reality. The fact is that, from a logical point of view, no ultimate distinction can be made between the whole system of knowledge and ordered reality. For the system of knowledge is reality, as viewed and interpreted by the human mind. Hence any analysis of the nature of knowledge, viewed as a system independent of any particular human being, is at the same time an analysis of the nature of reality. Now that system, as a whole, is the truth, and the truth of any judgment whatsoever is determined by its relation to and place in that system. If any judgment claims truth, and every judgment does, the adjudication of its claim must rest upon a determination of its relation to the system of reality.

This, then, is the answer to the basic question of logical science which dominates this textbook. It is the theory of the idealists, and is known as the *coherence theory of truth*. In order to set it in the clearest possible light, it seems altogether fitting and proper that it should here be presented anew, along with the main opposing theories advocated by living logicians.

The Pragmatist Theory of Truth

Pragmatists are the bitter enemies of the coherence theory, because they think that it is highly impracticable in that it makes the truth of any proposition dependent upon a system of things, which, *as a whole,* no man can know. As a substitute, they offer a theory which is usually called the *instrumental theory of truth.* According to this view, truth is the intellectual process of verifying a judgment. The truth is made by the human mind whenever it follows a suggested idea through to the practical value to which it leads. Suppose, for instance, that some one is lost in the mountains, nearly starved and frozen, and, coming upon what looks to be a cow path, he makes the judgment: "There is a house where I can find food and shelter and my way back to civilization at the end of this cow path." Here is a judgment arising out of a practical situation, whose truth is by no means rooted in the system of reality. For it is neither true nor false until it is verified. And by verifying it is meant going through a certain definable set of activities. If following the path actually leads to the house, the judgment gets verified and thereby becomes true. This process of proving the judgment useful, of tracing out its practical consequences, is a process of verification in which truth is made. Again, if I say, "He had better consult a physician," the truth of my assertion will be the process set up by its formulation, since this leads the man in question to go to a physician and get cured from an ailment. Truth is not a static and fixed quality of a judgment. It is very human and notoriously variable. It is what happens to judgments when they lead to good practical consequences.

Thus, William James says that the pragmatic method is:

To try to interpret each notion by tracing its respective practical consequences. What difference would it practically make to

any one if this notion, rather than that notion, were true? If no practical difference whatever can be traced, then the alternatives mean practically the same thing, and all dispute is idle. . . . Truth means that ideas (which themselves are but parts of our experience) become true just in so far as they help us to get into satisfactory relations with other parts of our experience. . . . Any idea upon which we can ride, so to speak; any idea that will carry us prosperously from any one part of our experience to any other part, linking things satisfactorily, working securely, simplifying, saving labor; is true for just so much, true in so far forth, *true instrumentally.* [1]

This sounds very good until it is subjected to criticism. Then it falls like a house of cards. For instance, it involves such statements as these: (1) that we can verify all those of our ideas, which are true; (2) that all those among our ideas, which we can verify, are true; (3) that all our true ideas are useful; (4) that all those of our ideas, which are useful, are true; (5) that beliefs may be true at one time, and not true at another; (6) that human beings not only make their beliefs, but that they make them *true*.[2] It is very easy to show that all of these assertions are false, and yet the instrumental theory of truth commits its advocates to maintaining them.

Moreover, James himself appears to have taken for granted the truth of a certain form of the coherence theory. For even though he defined truth as that which satisfies our human needs, he insisted that the need for *consistency* is the greatest claimant among all our needs. "Truth in science is what gives us the maximum possible sum of satisfactions, taste included, but consistency both with previous truth and with novel fact is always the most imperious

1 William James, *Pragmatism,* pp. 45, 58.

2 These statements are adapted from G. E. Moore's *Philosophical Studies.* See the essay entitled, "William James' 'Pragmatism,'" pp. 100 *f.*

claimant'' (p. 217). This *consistency with previous truth and with novel fact,* which James here demands of every truth, is nothing more nor less than a form of the coherence theory of truth brought in by James at the end, without his even being aware that it is entirely different from his instrumental theory. For his appeal to consistency really admits that we only recognize an idea or judgment as true which fits into the consistent system of truth and fact. Now this is the coherence theory of truth stated psychologically, with the emphasis on our human need for consistency. But a real logical development of this view would lead to the theory that truth is an actual integrated system of facts, irrespective of whether it satisfies human needs.

Every pragmatist has to do what James has here done, or else go back to a purely relative theory of truth, such as that of the sophists mentioned in the last chapter. The instrumental theory cannot be twisted so that it can give an adequate account of mathematical concepts or of judgments about past events. Advocates of this view either have to justify such truths by going over to the coherence theory, or they have to deny their independence of the human mind making use of them. But to say that the truth of such a judgment as, ''Two plus two equals four,'' or ''Cæsar crossed the Rubicon,'' depends in any way on the mind of the individual now using the judgment is a vicious relativism, which, in the light of the difference between our stage of knowledge and theirs, beggars the relativism of the Greek sophists. The proposition, ''There are planets revolving around the north star,'' is certainly either true or false, and yet its truth or falsity is absolutely independent of any of our human processes of verification and of any utility its being true might conceivably have for us.

Yet it must not be thought that the pragmatist theory of truth is utterly valueless. For it has rendered a real service

in contemporary philosophy, which I cannot express better than in the words of Hocking:

> The pragmatic test has meant much in our time as a principle of criticism, in awakening the philosophic conscience to the simple need of fruitfulness and moral effect as a voucher of truth. It is this critical pragmatism which first and widely appeals to the intellectual conscience at large. *Negative pragmatism,* I shall call it; whose principle is, "That which does not work is not true." The corresponding positive principle, "Whatever works is true," I regard as neither valid nor useful. But invaluable as a guide do I find this negative test: if a theory has no consequences, or bad ones; if it makes no difference to men, or else undesirable differences; if it lowers the capacity of men to meet the stress of existence, or diminishes the worth to them of what existence they have; such a theory is somehow false, and we have no peace until it is remedied. I will even go farther, and say that a theory is false if it is not interesting: a proposition that falls on the mind so dully as to excite no enthusiasm has not attained the level of truth; though the words be accurate, the import has leaked away from them, and the meaning is not conveyed.[3]

Thus practicality is one good criterion of truth. To be sure pragmatists, who refuse to go to the extreme of saying that the truth of a belief *is* its practical consequences, substitute the idea that such consequences are the supreme and ultimate and hence, really, *the only criterion of truth.* This is sometimes called *essential pragmatism,* to distinguish it from the extreme view, which is designated *hyper-pragmatism.*[4] There are, however, other criteria of truth, as for example, the *interest* to which Hocking referred, so that this view is also too extreme. All the truth which can be extracted from the pragmatist theory is that practicality is one among other criteria of truth.

[3] W. E. Hocking, *The Meaning of God in Human Experience* (Yale University Press), Preface, p. 13.

[4] This useful distinction was first made by D. C. MacIntosh in his *Theory of Knowledge*, p. 409.

The Correspondence Theory of Truth

There are two forms of this theory of truth. One is an older theory no longer current among logicians, although held by many naïve persons who are not conscious of holding it, and sometimes attributed to those who hold the coherence theory by critics who do not fully understand the coherence view. This is the copy theory of correspondence, recently styled, *one-one correspondence.* True ideas are those which agree with the facts is another way of expressing this view. The other type of correspondence may be designated, the *representative theory* of truth, to distinguish it from the one-one correspondence view. In this case an idea is true which correctly represents the facts, even though it cannot be said to correspond entirely with them.

1. *The Copy or One-One Correspondence Theory of Truth.* According to this view we have ideas in our minds which copy or agree with objects in nature or with facts. An idea which exactly copies or agrees with its object is true, one which only partly copies it is partially true, and one which entirely fails to agree with its object is false. The exact, point-for-point agreement is what is known as one-one correspondence. For example, suppose I have an idea of a certain table. If my idea is complete it will contain in it color, size, shape, style, etc. If it contains each of these exactly as they exist in the table it is a true idea. Its truth is determined by the extent to which it corresponds with its object. Only one-one correspondence gives complete truth.

The difficulties with this copy theory of truth are too numerous to mention. But we may point out that there are numerous ideas for which there are no physical or sense objects. Mathematical concepts are conspicuous examples. This theory of truth cannot account for the truth of an idea of a non-sensuous object. Moreover, such a theory in-

volves what is known as *epistemological dualism*, that is to say, it separates entirely the object of knowledge from the idea. Many present day logicians think that such an absolute separation makes knowledge impossible, since the only means we have of knowing the existing table is our idea of it. *We cannot get outside of the idea to observe whether it corresponds with its object or not.* Hence the theory adopted by such logicians is *epistemological monism*, which denies that the object of knowledge and the idea we have of it can be wholly disparate. Now epistemological monism is inconsistent with a one-one correspondence view.

2. *The Representative Theory of Correspondence.* Various attempts have been made by logicians to escape the difficulties in the copy, or one-one correspondence theory of truth. All of these attempts recognize that exact copying is an interpretation of the correspondence theory which is logically indefensible. They accordingly operate with the notion that there is something in knowledge which *represents* the objective fact. Hence there are various forms of the *representative theory of correspondence.* I shall deal with the two most significant forms: *(A)* the representative theory of critical realism, and *(B)* the representative theory of Bertrand Russell, which is widely held by logicians of the school of *neo-realism.*

(*A*) *The Representative Theory of Critical Realism.* Some of the most distinguished logicians and philosophers in general of our day call themselves critical realists. A number of these men, including George Santayana, A. K. Rogers, R. W. Sellars, C. A. Strong, and J. B. Pratt have recently published a coöperative volume entitled, *Essays in Critical Realism.* But the best exposition of their theory of truth is to be found in A. K. Rogers' more recently published *What is Truth?* (Yale University Press).

According to Rogers, the universe is a dualistic affair, with psychical existents on the one side and physical ex-

istents on the other. True knowledge or *trueness* consists in a definite relation of correspondence between a psychical existent and a physical existent. Or, more exactly, any belief is true which expresses an actual correspondence between some definite psychical existent and some definite physical existent. But this correspondence is not so simple an affair as the one-one type of correspondence, according to which the psychical existent simply *copies* exactly the physical existent. The matter is much more complicated. What actually happens is that the mind abstracts from the psychical existent a certain "character" or set of "characters." Taken together, these "characters" constitute an "essence," or "meaning," or "idea" of the psychical existent as distinct from its "existence." And then the mind *refers* this essence or idea to some physical existent. The act of reference is a judgment, and the judgment is true when the "essence" is referred to the right physical existent, otherwise it is false. Rogers asks the question: "If on certain occasions we are led to react at the same moment that we find ourselves experiencing a sensation of redness, why should we *not* automatically characterize the existent to which the reaction points by redness, and so have a mental tool for future discriminations in conduct?" (p. 66).

This suggests that we know the right physical existent from the fact that it is that to which the mental state or psychical existent causes us to react, and presumably the reference of the essence abstracted from this mental state would be more likely to be referred to the right physical existent the nearer in time the act of reference stood to the act of experiencing the mental state. But this is a detail. Let me quote passages setting forth the theory as a whole:

The true object of knowledge cannot accordingly be understood except in terms of an intimate union of two aspects. In its construction we have to distinguish two separate processes or phases

—the apprehension, or direct presence in psychological experience, of the character or essence which describes it, *and* the outgoing reference which locates this as an attribute of an independently real world (p. 60).

This character of the psychical state which the mind "intends" in its ideas must really be identifiable with the character of the object to which it is referred, or else in so far our knowledge is in error; and if the essence in the two cases is identical, the things which have such an identical essence "correspond." In this way we may answer the familiar objection (to the correspondence theory) that if by definition an object lies outside experience, there is no method of getting hold of it to compare it with the mental state, and so to discover the correspondence. Correspondence is *discovered* not in the original act of knowing, which is a unitary act of reference or identification, but through a subsequent reflective thought, to which both the terms alike are on the side of their *existence* external, but also both object and mental state alike are now present in *idea,* that is, in their essence, and so can be compared (p. 68).

The valuable part of this theory is to be found in its insistence that the essence of the psychical existent is *identical* with the essence of the physical existent. By setting up a common idea or meaning, this theory has a substitute for our notion of the implicative system, and approaches very closely to the coherence theory of truth. All that is necessary to turn this theory into the coherence theory is to deny that the essence is abstract, and to identify it with the concretely known object.

In holding that the essence is an abstraction, dissevered from the existence aspect both of the psychical and physical existents, this theory is really involved in an impossible triparte dualism. In the first place, there is the dualism between psychical existents and physical existents, which is held to be bridged in knowledge by the identity of essence. But in the second place, *essence* is strictly other than existence, and really belongs to another category than existence. Rogers wrestles with this distinction to very

little purpose. For if essence is strictly separate from existence it must fall in the realm of non-existence. Hence, any true reference of an essence to a physical existent would be false, since such a reference would *ipso facto* confer existence on that essence, and, belonging to the realm of non-existence, it is incapable of existence. Existence is always particular and concrete, whereas essence is universal and abstract. How, then, can an essence exist? But it must exist both in the psychical existent and in the physical existent, or the correspondence between them would be entirely fictitious. Dr. Rogers tries to avoid this difficulty by a further distinction between the essence "as embodied in the object," and the essence as "attended to in abstraction from it." Suppose we designate the former *concrete essence*, and the latter *abstract essence*. The very making of this distinction introduces a third dualism which throws grave doubt on the correspondence. How am I to know that the abstract essence which I refer to a physical existent will not be altogether different in its concretion in the existent from what it is in the abstract form in which it is identical with a meaning or an idea? It now appears that at least three different correspondences are implied in this theory: (*a*) a correspondence between the abstract essence and the concrete essence as embodied in the mental state; (*b*) a correspondence between the abstract essence and the concrete essence as embodied in the physical existent; (*c*) a correspondence between the mental state or psychical existent with its concrete essence and the physical existent with its concrete essence. To know that this last type of correspondence holds, one must first know that both the others hold, for the correspondence between the two existents is based on the assumption that no change in the essence is involved in passing from the mental state, with its concrete essence, to the pure abstract essence, and from the pure abstract essence to the physical existent, with its

concrete essence. Now, to say the least, this is a very generous assumption. It seems, then, that such a complicated correspondence, involving as it does the crossing of three chasms, is less free from difficulties than simple copying, which offers but one chasm to cross. But in neither case is it possible to discover whether the correspondence actually holds.

Ultimately Rogers holds that we know that the correspondence holds because of certain prerational instinctive beliefs. But when he faces the significant question: "How can beliefs be *justified,* so as to separate out the sheep from the goats?" he is compelled to say:

> The answer I should give to this last question is the familiar one of "coherence." Coherence I think must be rejected as a sufficient definition of truth, or a sufficient reason for belief. . . . But with belief presupposed, it does seem to be the case that coherence is the only test by which we can justify belief to the intellect, outside the very insignificant field where intuitive certainty holds. This does not necessarily mean that we ought to abandon all beliefs that we cannot so justify. Nature will probably be too strong for us in any case. But nevertheless we do find on the whole that rational belief is the better and more satisfying sort. And so long as we play the game of reason, and profess to have passed beyond the first naïve and non-reflective stage of experience, "justification" may be taken as meaning "inclusion within a coherent system" (pp. 11 *f.*).

(*B*) *Russell's Theory of Correspondence.* In his little book entitled *The Problems of Philosophy,* to which we have already referred, Bertrand Russell presents a popular exposition of the correspondence theory of truth which has been widely accepted by logicians of the school of neo-realism. According to him there subsist, in a non-human, non-physical, but logical world, certain complex entities called fact complexes or propositions—as many, in fact, as are needed as points of reference for our human judgments.

A human judgment or belief is a purely subjective assertion of some mind. It is true when it correctly represents or corresponds to the proposition or fact complex to which it refers in that independent world of subsisting propositions. Now, both the human judgment and the non-human proposition are highly complex, involving both terms and relations. The relation which ties together the *constituents* of the proposition is known as the *object relation,* and the relation which ties together the terms in the judgment is known as the relation of *believing.* Now the judgment is true when the relation of believing ties together the constituents in the same way in which they are tied together in the fact complex by the object relation. Otherwise, the judgment is false.

To take Russell's illustration: The judgment of Othello that Desdemona loves Cassio is true if, in the subsisting world of propositions, there is a fact complex having the constituents: Desdemona, Cassio and loving, the latter being the object relation, and if these two terms are actually related by loving in the way Othello relates them by his act of believing. "Thus, a belief is *true* when it *corresponds* to a certain associated complex, and *false* when it does not. . . . Judging or believing is a certain complex unity of which a mind is a constituent; if the remaining constituents, taken in the order which they have in the belief, form a complex unity (fact complex or proposition), then the belief is true; if not, it is false. . . . If we take such a belief as 'Othello believes that Desdemona loves Cassio,' we will call Desdemona and Cassio the *object-terms* and loving the *object-relation.* If there is a complex unity 'Desdemona's love for Cassio,' consisting of the object-terms related by the object-relation in the same order as they have in the belief, then this complex unity (fact-complex or proposition) is called the *fact corresponding to the belief.* Thus a belief is true when there is a cor-

responding fact, and is false when there is no corresponding fact."[5]

The strong point in Russell's theory is to be found in his recognition of the fact that falsehood is not objective, not a part of the fact complex itself, but that a false belief is any belief that objects are related otherwise than they actually are. But this is entirely consistent with the coherence theory. Indeed, T. H. Green, one of the strongest advocates of the coherence theory, was wont to refer to falsehood as "a confusion of relations."

The defect in Russell's theory is best brought to light by asking the question: How are we to know whether a judgment complex corresponds with a fact complex, assuming the two to be entirely different? In his effort to explain the nature of knowledge, Russell wrestles with this question to very little purpose. For he reaches the conclusion that *we can never really be sure that the two complexes correspond.* He defines knowledge as being what is validly deduced from *known* premises. And then he explains that the *known premises* are known intuitively. "But in regard to intuitive beliefs," he writes, "it is by no means easy to discover any criterion by which to distinguish some as true and others as erroneous. In this question it is scarcely possible to reach any very precise result; all our knowledge of truths is infected with *some* degree of doubt." And again, he writes, "Knowledge is not a precise conception; it merges into probable opinion." These passages show clearly that Russell denies that we can ever be sure about any particular belief because we can never know whether there is a fact complex which corresponds to it. And to admit that knowledge is not a precise conception is to admit that truth is not a precise conception. For how can real knowledge be distinguished from truth? Knowledge,

[5] Bertrand Russell, *Problems of Philosophy* (Henry Holt), pp. 201 *f.* See also his *Philosophy*, pp. 254-263.

for Russell, practically reduces to *probable opinion,* with the recognition that there are degrees of probability. And he falls back on the notion of coherence as a test of probability or measure of the degree of probability. "In regard to probable opinion, we can derive great assistance from *coherence,* which we rejected as the *definition* of truth, but may often use as a criterion. A body of individual probable opinions, if they are mutually coherent, become more probable than any one of them would be individually."[6]

Each form of the correspondence theory really makes truth inaccessible to the human mind. If truth is an agreement between a mental belief and an absolutely independent object, be it physical existent or propositional subsistent, there is no conceivable way in which one could ever know whether this agreement is ever actually present in any particular case. Now, defining truth in such a way as to make it inaccessible begs the whole question. If we never know when we have truth, how can we know that our theory of truth is, itself, true?

The Real Meaning of the Coherence Theory of Truth

We have seen that advocates of pragmatism, critical realism and neo-realism, accept coherence as the chief criterion of truth. But they interpret coherence to mean consistency among our human beliefs, and when we define truth in terms of coherence, something more than mere consistency must be understood. To all advocates of the coherence theory, an identity of nature between knowledge and reality is presupposed. There is a coherence of our beliefs with reality, as well as a coherence among our beliefs about reality. What we know is always some actual implicative system in reality itself. The truth or trueness

[6] *Loco citato,* p. 218. Russell's theory of probability is expounded above. Ch. XVII.

of any belief is what it gets by virtue of its embodying that actual system. Thus the truth of the judgment that the blood circulates through the human body is to be found in the fact that it embodies within itself an actual implicative system, namely the circulatory system of the human body as known to physiologists. There is no dualism between a belief and the content in reality which it embodies, for, viewed from the logical side, that content is the belief. Much of the confusion in discussions of truth is due to the fact that the problem is approached from the point of view of biological psychology instead of from that of logic. Unless we hold that human science is progressively feeling its way into the actual nature of the universe, we are really shut up in utter relativism and scepticism. Either the coherence theory is true or human knowledge is a fairy tale.

Let him who can think that what Newton and Laplace, Faraday and Davy, Darwin and Pasteur, and all the other honest seekers after, and contributors to, human knowledge, have told us about the universe is only a fairy tale. There will always be those who prefer to hold that the universe is actually what it is known to be, and that the investigations of such scientists take us deeper and deeper into its essential nature. Newton was not far wrong when he remarked, concerning his own discovering of the law of gravitation, that he was like a child gathering pebbles and shells along the seashore, while the great ocean of truth lay all unexplored before him. But was it not something to recognize that the shells and pebbles came out of that ocean, and that it was there to be explored? Human knowledge is probably much less important and much more fragmentary than we think. To repeat the dying words of Laplace: "What we know is little, but what we do not know is immense." Yet shall we abandon the little we know in the interest of a false conception of certainty?

And shall we kill the interest of truth seekers by telling them that no one has ever reached the truth, and that there is a great gulf fixed between human ideas and concrete facts? Yes, certainly, if we know that it is true that there is such a gulf. But why *assume* that there is? And why put forth this assumption as our basal certainty? The value of the coherence theory of truth is that it makes the other assumption, and by so doing justifies the splendid faith of men of science that there is an ocean of truth which can be, and is being, charted.

CHAPTER XXVII

MODERN INTERPRETATIONS OF THE LAWS OF THOUGHT

Postulates in Deductive Systems

It is chiefly to the mathematical logicians that modern logic owes a debt of gratitude for the elaboration of an extremely valuable *theory of postulates*. But, on the other hand, it is primarily to the idealistic logicians that credit belongs for the best modern interpretation of certain basic logical principles, traditionally known as *laws of thought*. Now perhaps the best way to reach an understanding of the problem involved in interpreting these laws is to approach it through the modern conception of a postulate.

Every orderly or systematic body of knowledge, which is sufficiently well organized to be capable of relative independence from the whole system of knowledge, is necessarily based upon certain principles which are taken for granted for the purposes of that particular system. This need not be interpreted to mean that they cannot be proven, but only that the attempt to establish them takes one outside the system of knowledge in question. We must start somewhere, and in deductive procedure we start with certain assumed general principles and see what we can get out of them. Such assumed principles are now ordinarily called *postulates* or *axioms*. Thus, for every deductive system there is a definite set of postulates or axioms on which the system as a whole is based. Plane geometry is a good example of a deductive system. The axioms one has to learn at the beginning of plane geometry are the postulates of that deductive system. The whole body of knowledge included in plane geometry rests upon these postulates. Disprove them, and the whole system will collapse like a house

of cards. The only proof for them is their fruitfulness, unless they are deduced from more fundamental postulates such as the laws of mathematics, and that would mean going outside the system of plane geometry. It is in this sense that we are to understand the precise definition of a postulate given by Royce:

> An axiom, or postulate, is a principle which lies at the basis of a certain selected system of propositions, and which is not demonstrated in the course of that system. This is the sense in which the term *axiom* is still most serviceably employed in modern theory. . . . Axioms, in the language of modern theory, are best defined, neither as certainties nor as absolutely first principles, but as those principles which are used as the first in a special theory.[1]

In concluding our exposition of syllogistic inference, we pointed out that logicians have attempted to reduce traditional syllogistic reasoning to a single principle—the *dictum de omne et nullo*. And it was there pointed out that this principle, the rules of the syllogism and the special rules of the separate figures, could be regarded as a set of postulates for traditional logic. On the other hand, the law of the universality of nature or the principle of causation can be treated as the basic postulate of inductive reasoning. But there is also a still more general set of postulates, which go by the name, *laws of thought*. The three of these laws which were formulated by Aristotle are (1) the *law of identity*, (2) the *law of contradiction*, and (3) the *law of excluded middle*. Leibniz, the greatest logician of the eighteenth century, added a fourth, the *law of sufficient reason*. Now there is a sense in which these laws are the presuppositions behind every branch of knowledge whatsoever, or the set of postulates of knowledge

[1] From the article entitled, "Axioms," in Hastings' *Encyclopedia of Religion and Ethics*. Quoted by permission of Charles Scribner's Sons.

itself taken as a whole. For knowledge is impossible unless they are adhered to by the thinker. The schools of logic with which we have been dealing differ widely in their interpretations of these laws. I will first formulate them to accord with the idealistic interpretation, and then indicate three significant points of divergence.

The Idealistic Interpretation of the Laws of Thought

The idealistic interpretation is dominated by the coherence theory of truth. Although recognizing that the laws are capable of being treated as relatively independent, the chief trend of the idealistic interpretation is in the direction of reducing all of the laws to the one principle of sufficient reason, which, in turn, is held to be identical with the principle of the implicative system. And, just as essential to this point of view, is the theory that these laws are not only the axioms on which all human thinking rests, but express the actual nature of reality itself. Both of these principles of the idealistic interpretation will come to light as we proceed to discuss the separate laws.

1. *The Law of Identity.* This law states that every object of thought is identical with itself: "A is A." "Business is business." "War is war." "Evil is evil." It seems absurd to insist that thinking is based upon the assumption that every thought content or meaning is identical with itself. But it seems absurd because it is a truism which no one doubts. Yet, once you become conscious of the existence of this postulate, you will be surprised to find that it is frequently violated. The fallacy which results when it is violated has already been discussed under the name of ambiguity and equivocation. For example, a political speaker may begin an address by using the word "republican," or "democrat," in that broad sense in which we are all republicans or democrats, and gradually lead over to a sense in which the word is a designation of

a definite set of partisan policies. When he does this *A* has become *B*, or something else. It is no longer the *A* with which he started. The law of identity requires that *A* be *A*, that the thinker shall be consistent throughout and call a spade a spade. Rigid thinking gives way to haziness whenever this law is violated. Logic cannot tolerate the shifting of the meaning of a term. Whatever a thing is, it is, and thought depends upon every one taking things for what they are. This is the ultimate demand of the law of identity.

2. *The Law of Contradiction.* Aristotle stated this law as follows: "The proposition *A* is *B*, and the proposition A is not-B, cannot both be true together." And again he said: "It is impossible that the same predicate should *both belong and not belong to the same thing at the same time and in the same way.*" In other words, according to this law we are forbidden to hold that a thing both is, and is not, of a certain definite character. It either is that, or it is not that. It cannot at the same moment both be that, and not be that. Suppose I make the assertion: "John is a dog," and then, speaking of the same John, I say, "John is a man." According to the law of contradiction, one only of these statements can be true. John is one or the other, but he cannot possibly be both at once. You cannot say that any object is at one and the same time poison and non-poison. It is one or the other but not both. And this applies to all opposites. To assert two opposites of the same subject at the same time is to be guilty of self-contradiction. Now this must not be interpreted to mean that an object may not include within itself various differences, for neither the law of identity nor the law of contradiction excludes the possibility of change. It only means that a thing cannot be conceived as being the exact opposite of itself and still be itself. Exact opposites cannot be affirmed of the same subject at the same time. And

yet violations of this law are too common even among educated people. It is not easy to avoid contradiction. Nevertheless, scientifically exact thinking is thinking which is not infected with the disease of self-contradiction.

3. *The Law of Excluded Middle.* According to this law, which is really a corollary of the law of contradiction, of the two propositions *A* is *B*, and *A* is not-*B* one must be true and the other false. *There is no middle ground or third possibility.* If *B* and not-*B* are exact opposites, they divide the world between them in such a way that anything whatsoever is either *B* or not-*B;* it cannot possibly be something different from both; it cannot possibly be partly *B* and partly not-*B, if thought is to be possible.* Behind this law is the assumption that there are two sides of every reality, its positive and its negative side. *B* is the symbol for its positive side, and not-*B* is the symbol for its negative side. Now any mentionable object will fall on one side or the other, it will either have the qualities in question or it will not have them. There is no middle ground. "John is a man." "John is a non-man." If John has the qualities of a man the first proposition is true, and if he does not the second is true. And of any particular John one or the other of these assertions is true. John may be an idiot, he may be lacking in some of the attributes of reasonable beings, yet he is a man. All thought depends upon the principle that everything which is asserted to be true of a given subject involves or carries with it the denial that something else is true of that same subject. If a dog is a four-footed animal, it is not a two-footed animal. The real meaning of the law of excluded middle is this principle that opposites cannot both be true of the same subject at the same time. It is simply a more explicit formulation of the law of contradiction.

4. *The Law of Sufficient Reason.* Leibniz stated this law as the ultimate postulate of all thinking. He formulated it

as follows: "No fact can be found real or existing, no statement true, unless there be a sufficient reason why it should be so and not otherwise." And he adds that this postulate holds, even though "these reasons very often cannot be known to us." But the law may also be stated, *For everything that exists or is real we must assume that there is a full explanation or reason for its being what it is rather than otherwise from what it actually is.* So stated, this principle or law is the ultimate postulate of all thinking. To take our former illustration: "If John is a man it is because he has the attributes of a man, and if John is a dog it is because he has the attributes of a dog." The sufficient reason for John's being a man is the system of things to which John belongs and of which the judgment, "John is a man" is an expression. To explain what any object whatever is *we must go behind the particular object to the implicative system or inferential whole to which it belongs.* What it is, it is because it is grounded in some larger system than it is itself. This larger system is the sufficient reason for its being what it is, and for its not being something other than what it actually is. Ultimately, this sufficient reason will be the system of reality as a whole.

Hence, this law of sufficient reason has been expressed by the idealistic school of logicians as the assumption that the ultimate subject in any judgment is reality itself—the whole system of things. Thus, there is the immediate subject "John," and the ultimate subject "reality," so that the judgment should be written: "Reality is such that John is a man." The sufficient reason for his being a man is the complete implicative system, which is reality itself or the nature of things.

This law has also been expressed as *the principle of consistency.* This really assumes that reality is a consistent system, and that in reality everything is what it is because

of its place in the system. Hence valid thought must be consistent. Unless we think things as they are in the consistent system of reality, our thoughts are mere vulgar opinion and lack truth. Logic has to assume that there is a consistent system of truth which thinking reaches. So Creighton writes: "The laws of identity and contradiction are simply the expression, in positive and negative forms respectively, of the principle of consistency. The one fundamental postulate of all thought is that it must be consistent with itself."[2] But no matter whether we call it the principle of consistency or the law of sufficient reason, or whether we say that the ultimate subject of every judgment is reality or the nature of things conceived as an order system, all thinking which aims at objective truth certainly depends upon the postulate that the universe taken as a whole is an implicative system, and can be known as such. It is for this reason that Professor Royce would define logic as the *science of order,* order being for him the logico-mathematical structure of the whole existing universe.

Divergent Interpretations of the Laws of Thought

There are three significant divergent interpretations of these laws:

1. The pragmatist school, as well as some other contemporary logicians, treat the laws of thought in a thoroughgoing relativistic fashion, regarding them as purely man-made prescriptions for the use of names. They hold that mutual understanding and communication of thought is impossible unless we use words with the same meaning. In the slow process of social evolution man has discovered this fact, and the laws of thought are his way of dealing with it. Consequently they are a relatively late product in the

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[2] J. E. Creighton, *An Introductory Logic,* new ed., p. 38. See, also, pp. 343 *f.*

evolution of intelligence, and are nothing more than human conventions governing the use of names. By no means do they indicate the structure of ultimate reality. If, a few thousand years hence, intelligence succeeds in evolving a better set of conventions, these laws will be regarded as obsolete, petrified truths, esteemed only for the pragmatic value they once had, but no longer binding upon thinking people. This interpretation is thus a corollary of the pragmatist relativity theory of truth.

W. Curtis Swabey has recently subjected this pragmatist interpretation of the laws of thought to a penetrating analysis. He points out that it is really based upon a dogmatic and uncritical acceptance of the laws of natural science, whereas scientific analysis reveals the fact that they are strictly subordinate to the laws of thought. He has well expressed the idealistic answer to the contention of the relativists:

> Psychologism or sceptical relativism cannot be refuted in case its advocate refuses to accept the authority of the laws of thought, and of the other principles of logic. In the end there can be no argument with those who repudiate the authority of clear logical insight. But . . . the logician can strengthen his own confidence by observing how his opponents contradict themselves at every step. Relativism involves the assertion of the existence of causal connections between the mind's empirical constitution and human beliefs. It thus involves not only the formal laws of thought, but also the principle of causality or of the lawfulness of nature. If there were no causality, no uniformity of nature, our beliefs could not depend on our mental constitutions or on our particular temperaments. Again, if the law of non-contradiction were false, relativism would itself be both true and false, and consequently neither true nor false. The ultimate laws of thought, in other words, have the peculiar character of being involved in their very denial and thus of automatically maintaining themselves against every attack.[3]

[3] *Philosophical Review,* Vol. XXXII, p. 220. Swabey has been answered, from the point of view of critical realism, by Charles E. Hooper, in the same volume, pp. 531 *f.*

2. The realistic school is in general agreement with the idealists in regarding the laws of thought as basic aspects of the universe itself, but they object to their reduction to one fundamental postulate. While recognizing their importance, these logicians tend to regard the laws as only a subordinate part of the whole set of logical principles which must be taken for granted by the science of logic. Russell, for example, considers the principle, "every proposition which follows from true premises is true," as an equally fundamental logical axiom. Thus, referring only to the first three laws, he writes: "For no very good reason, three of these principles have been singled out by tradition under the name of 'Laws of Thought.' . . . These three laws are samples of self-evident logical principles, but are not really more fundamental or more self-evident than various other similar principles: for instance, the one we considered just now, which states that what follows from a true premise is true. The name 'laws of thought' is also misleading, for what is important is not the fact that we think in accordance with these laws, but the fact that things behave in accordance with them; in other words, the fact that when we think in accordance with them we think *truly*."[4]

Russell is right in holding that the name "laws of thought" is misleading. But he is wrong in implying that the three laws are independent axioms, for our discussion has already shown that they are inherently related. Hence, the reduction of them to the principle of sufficient reason is justifiable. Indeed, another able realist asks, with an implicit affirmative answer, "Is it not possible to include the interpretation of perceived meanings, together with the foundations of inductive inference concerning

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[4] Bertrand Russell, *Problems of Philosophy* (Henry Holt), p. 113

nature, in some general law of analogy or principle of sufficient reason?"[5]

3. The realists also hold that the laws of thought, as well as all other basic postulates or principles of logic, are self-evident or intuitive truths, and that they are, therefore, incapable of proof. Thus, in the passage just quoted, Russell refers to them as "samples of self-evident logical principles." Moreover, the critical realists are at one with other realists in adopting this doctrine of self-evidence. On this vital issue, the idealists are committed to the view that self-evidence is a dangerous principle to which a logician should never appeal. They hold that there are two kinds of proof—internal and external. *Internal proof* is equivalent to the fruitfulness of a set of postulates in ordering the various facts within the system in question. In this sense a set of postulates can be established by pointing out the ways in which they actually function in ordering the concrete factual details in the system to which they belong. *External proof,* on the other hand, consists in deducing a set of postulates from a still more general postulate or set of postulates than the set of the system in question. To be sure, this involves going outside of that system. Now, the laws of thought cannot be established by external proof, for the very simple and good reason that there is no system more general than the whole of knowledge, and consequently, no more funda-

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[5] John Laird, *A Study of Realism,* p. 123. Laird, however, is not sure of himself, for he later adds: "This final step seems more seductive than solid." This wavering attitude also characterizes the authors of the *Introduction to Reflective Thinking.* But they lean strongly toward the idealistic interpretation, as is shown by their statement: "The so-called laws of thought seem to us to be more deepseated (than geometrical axioms). . . . They seem to be part, not only of the very structure of our minds, but also of the structure of the universe to which thousands of years of biological experience has adapted the mind of man." (Pp. 114 *f.*) But the last clause indicates their pragmatist leanings.

mental set of postulates than these laws. The most that can be done is to show that the first three laws are corollaries of the law of sufficient reason, thus in effect deducing them from it. But when we take into consideration the fruitfulness of these laws in ordering the whole of knowledge, we at once recognize that they are capable of internal proof. Hence, they may be said to be "self-mediated by the system of truth" (Royce). Thus we see that the idealistic interpretation of the laws of thought is part and parcel of the coherence theory of truth.

CHAPTER XXVIII

RECENT CRITICISMS OF LOGIC

A Statement of the Criticisms

Although they are at swords points on some issues, as we have learned in the last two chapters, contemporary logicians are, curiously enough, near unanimity in condemning traditional logic as little more than a poor exercise in mental gymnastics. Thus the authors of the *Introduction to Reflective Thinking* deliberately adopt an entirely new approach which excludes all of the Aristotelian logic, because, "The older so-called formal logic seems strangely technical and remote to the student of the present day. Its nice distinctions and mathematical precision may be appreciated and enjoyed, but they do not seem to be carried out of the textbook into everyday life" (p. 16). In fact, most pragmatists and realists regard traditional logic as a more or less barbarous relic from antiquity, incurably corrupted in its transmission to us by the schoolmen of the Middle Ages. As Whitehead puts it: "A science which hesitates to forget its founders is lost. To this hesitation I ascribe the barrenness of logic."[1] Moreover, many of the idealists appear to hold much the same view. Thus, in a very recent article R. C. Lodge, an avowed follower of the greatest of the idealistic logicians,—the late Bernard Bosanquet, writes concerning traditional logic that, "while it is still largely taught" and, "with its tricks, puzzles, fallacies and repartees, can be made entertaining to elementary students," nevertheless, "its chief educational value con-

[1] A. N. Whitehead, *The Organization of Thought*, p. 115.

sists in the exercise it gives in paraphrasing poetical and rhetorical passages, with minimal loss of meaning.'' And he adds the comment: ''It is trivial and superficial.'' His own textbook, prepared especially for elementary students, only mentions the syllogism once and that reference is in a footnote.[2] But that some of the idealistic logicians are less radical in their criticisms of traditional logic is shown by the following comment, from the article quoted in the last chapter, by W. Curtis Swabey: ''The historic tradition, which goes by the name of Aristotelian logic, has weathered many a storm and will probably continue to play its rôle after many of its more ambitious rivals have come to figure only in the history of philosophy.'' This discriminating estimate is shared by the majority of the idealistic as well as by some other logicians. For it goes without saying that the neo-scholastic school of logic is committed to a high valuation of traditional logic. It is also interesting to note that an attempt has quite recently been made to rehabilitate the historic tradition by giving to it a symbolic interpretation.[3] On the whole, however, traditional logic is rather out of favor just at present. ''Away with this mummy from antiquity, make room for something living and modern!''—this is the far-flung battle-cry of many doughty champions of novelties in logic.

As we learned above, Bergson, the distinguished French philosopher, is the leader of a significant school of contemporary philosophy known as anti-intellectualism, which exalts intuitive insight far above logical analysis. To be sure, Bergson himself acknowledges, in the closing pages of his *Introduction to Metaphysics*, that the highest intuitive insights cannot be reached without an extensive prepa-

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[2] *Philosophical Review*, Vol. XXXII, p. 589. Compare F. C. S. Schiller: *Formal Logic*, and the article entitled, "The Reformation of Logic," by A. W. Moore, in the pragmatist volume entitled *Creative Intelligence*.

[3] See Bradford Smith. *First Book in Logic.*

ration in logical analysis, but in so far as he subordinates concepts to intuitions and insists that logical explanations falsify the facts, he is an enemy of all logic. There can be no doubt that the anti-intellectualist movement has contributed to the reaction against logic. Moreover, it has wide ramifications outside of the ranks of technical philosophers. Literary enthusiasts have always heaped scathing denunciations upon logic. This anti-intellectualist attitude of the poetic and imaginative temperament is well expressed in the interesting lines of Dana Burnet's little poem entitled "To a Logician":

> Cold man, in whom no animating ray
> Warms the chill substance of the sculptor's clay;
> Grim reasoner, with problems in your eyes,
> Professor Sage—however do they call you?
> Far-seeing Blindman, fame shall yet befall you;
> Carve you in stone—that winter of the wise!—
> And set you up in some pale portico
> To frown on heaven above, on earth below.
>
> I shall make songs and give them to the breeze,
> And die amid a thousand ecstasies!
> I shall be dust, and feel the joyous sting
> Of that sweet arrow from the bow of time
> Which men call Spring.
> And out of my dead mouth a rose shall come like rhyme!
> But you, in your eternal state of snows,
> Shall thrill no more to life's resurgent flood,
> Nor cast death's laughter into April's rose!
> You shall be marble who were never blood.

A Defense of Logic against These Criticisms

We may dismiss the criticism of all logic of the anti-intellectualist, which comes to clear expression in this poem, with the words of the great French interpreter of Aristotle, Barthélemy Saint-Hilaire: "Without logic the mind of man can admirably energize, admirably reason; but without it does not know itself through and through; and

ignores one of the fairest and most fruitful of its faculties. Logic brings to the mind self-acquaintance. Such is its use and it cannot have any other."[4] For even though logic is not the whole nor even the better half of life, even though there are bloodless and spineless logicians who come near fitting the poet's sarcastic description, yet logic has its rightful place. Cultivated modern life is really not complete without it, bloodless though it be. Nor is it necessarily bloodless. Even a logician may have his ecstasies and "cast death's laughter into April's rose." And surely all who know something of the contributions of modern science to human knowledge must recognize the educational value to elementary students of a study of the logical procedure involved in making these discoveries. Whatever one may think of traditional logic, surely every one who knows anything about the matter at all must concede that logic, in the sense of methodology, is an indispensable adjunct of modern science.

Recognizing the demand for a new interpretation, which shall take account of modern logical developments, one need not accept the wholesale and practically unqualified condemnation of traditional logic. For, in the first place, the critics overlook the fact that elementary students are simply barred from advanced work in this field until they know the rudiments of traditional logic. The whole technique of logical science is beyond the reach of him who has not fully mastered Aristotelianism. There is hardly an advanced treatise on logic, if we except the new symbolic logic, which does not bristle with technical terms whose roots are in the Aristotelian logic. Moreover, the authors of these treatises presuppose a knowledge of these terms on the part of their readers. Even the critics' own

[4] Barthélemy Saint-Hilaire, *De la Logique d'Aristote*, p. 42. Quoted by J. Brough, in the article entitled "Logic," in Hastings' *Encyclopedia of Religion and Ethics.*

condemnations cannot really be understood or fully appreciated apart from such knowledge. Hence, teachers owe it to their most gifted students, who may later develop into logicians, to present the content of traditional logic.

The critics may answer that a teacher is not justified in loading onto the whole class content which will only benefit the occasional and exceptional student, who may prepare for graduate work in this field. Hence, in the second place, it must be mentioned that these technical terms of traditional logic have entered into our language as a whole to such an extent that it is hard to appreciate literary masterpieces and scientific treatises without a knowledge of them. Nor can adequate knowledge be reached by the slavish process of repeatedly consulting dictionary definitions. These terms must needs be learned in their context. The cultural value of a mastery of traditional logic is a sufficient justification for its inclusion in an elementary course. And note that this is entirely different from its disciplinary value, on which the critics assume a defender of Aristotelian logic must rely. Without any question, the open-eyed student who has been through Part One of this textbook, has already repeatedly met words and passages in his other reading on which a flood of light has been thrown by what he has learned. Before studying logic such passages were read superficially, without comprehension.

In the third place, the study of traditional logic does have a disciplinary value which cannot be destroyed by the jibe of "mental gymnastics." Why aren't "mental gymnastics" just as legitimate as "physical gymnastics"? The fact that one does not use certain muscles in ordinary life is no reason for neglecting them in the gymnasium. Nor is the fact that we do not use daily, every logical process dealt with in traditional logic, in the precise formal way in which they are studied in an elementary course,

any argument against spending a few hours of one's life mastering such processes. The observant student will find in his own experience full justification for the testimony of John Stuart Mill as to the value of a study of these traditional logical processes. He writes: "There is no more important intellectual habit, nor any the cultivation of which falls more strictly within the province of the art of logic, than that of discerning rapidly and surely the identity of an assertion when disguised under diversity of language. . . . The student of logic acquires habits of circumspect interpretation of words and of exactly measuring the length and breadth of his assertions, which are among the most indispensable conditions of any considerable attainment in science, and which it is one of the primary objects of logical discipline to cultivate" (Bk. II, Ch. I). And again, in *Autobiography:* "I am persuaded that nothing in modern education tends so much, when properly used, to form exact thinkers, who attach a precise meaning to words and propositions, and are not imposed on by vague, loose, or ambiguous terms."

Nor need the student go outside of his own college or university environment to find ample verification of the truth of the words of another great English logician, De Morgan, who said that it is the neglect of formal or Aristotelian logic which gives us "swarms of legislators, preachers, and teachers of all kinds, who can only deal with their own meaning as bad spellers deal with a hard word." And the wise college student will not forget that he is frequently going to be called upon throughout his life to express to others, clearly and concisely, his own meaning.

Moreover, if traditional logic is worthless, trivial and superficial, the empirical fact, known to every teacher of logic who knows why students enroll in the elementary course, that so many successful professional men and scientists warn their own sons and daughters against the neglect

of elementary logic, is simply inexplicable. For two or three decades ago, when these men were undergraduates, traditional logic made up the bulk of the content of courses in elementary logic. Yet this is one of the few courses which numerous successful men single out from among all that they had, as having been the most helpful to them in their professional career. It is highly questionable whether, twenty or thirty years hence, successful men will be able to pay a similar tribute to the substitutes for traditional logic which some of them are now being offered in our colleges and universities. In this age of iconoclasm beware lest the critics persuade you to sell your intellectual birthright for a mess of pottage!

The mere fact of the antiquity of Aristotelian logic does not justify substituting some other content for it, any more than the antiquity of Euclidian geometry would justify a teacher of mathematics in substituting non-Euclidian geometry for it in high schools and academies. Indeed, the fact that logic comes to the modern college student clothed in the wondrous garment of antiquity should make the subject peculiarly fascinating to a beginner. What a superb privilege to study that which formed a vital part of the educational equipment of the immortal Cicero, and of countless other immortals down to our own day! The persistence of the Aristotelian logic as a part of the program of studies since the days of its founder proves its imperishable value. Nothing worthless could survive the acid test of criticism through so many centuries. What is needed to-day is not the abandonment of Aristotelian logic, but a reinterpretation of it which will embody the significant modern contributions to the science. To accomplish this purpose has been one of my dominant aims in writing this textbook. I may have failed, but I relinquish the task with full confidence in the power of traditional logic to weather the assaults of its contemporary critics.

CHAPTER XXIX

THE LOGICAL SIGNIFICANCE OF RADAR PIPS

Statement of the Problem

The spectacular sinking of a Japanese warship, after two salvos were fired by one of our cruisers eight miles away, was made possible because the radar gave the Captain of our ship exact knowledge of where to fire, even though none of his men could even locate the enemy craft by using the highest powered range finders and binoculars. This novel type of "action at a distance" certainly proves that a radar gives those who are qualified to use it important and accurate factual information. Because it possesses certain unique features, such knowledge is worthy of the attention of logicians and students of logic. In preparation for a brief examination of this kind of knowledge certain facts need to be stated.

A radar sends out a pulse whose waves are only a fraction of a meter in length. Within a few thousandths of a second this wave pulse may return. This will happen if it is reflected back by an obstacle which it encountered. It is then amplified and fed into a cathode ray tube. A stream of electrons within this tube is sprayed on a fluorescent screen or radarscope, producing on it a representation of the obstacle which the radar pulse contacted. Such a representation is not like an image on a television screen which copies its object, but it is a flickering patch of light called a "pip." This pip tells the operator who knows how to interpret it the exact location and distance the obstacle which reflected the radar wave pulse is from the transmitter from which the pulse was sent, and if the object is a ship, the pip will reveal its speed and course. Thus, within a few seconds, a skilled radar operator who rightly interprets radar pips

knows the distance and location, and the speed and range of every ship on the surface of the ocean within the area of operation of his radar. To be sure, the possibility of an erroneous interpretation has to be taken into account, but let us rule out this possibility by assuming that the operator correctly interprets the pips that appear on the radarscope when it is properly functioning. At least for the case under consideration this is a valid assumption, since the Japanese ship was actually sunk. Keeping in mind these facts about radar detection of distant obstacles, including ships, let us summarize some basic logical distinctions, and certain general conceptions about nature as these have been used in recent philosophy of science.

In his stimulating and thought-provoking *Essay on Man*, Professor Ernst Cassirer writes: "Symbols—in the proper sense of this term—cannot be reduced to mere signals. Signals and symbols belong to two different universes of discourse: a signal is a part of the physical world of being; a symbol is a part of the human world of meaning.... Signals, even when understood and used as such, have nevertheless a sort of physical or substantial being; symbols have only a functional value."[1]

Professor Cassirer used the word *sign* as a synonym for signal, and says that "we must carefully distinguish between *signs* and *symbols*." He uses Pavlov's famous experiments on conditioned reflexes as examples, in one of which a bell ringing becomes a "sign for dinner" to a dog.

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[1] *An Essay on Man*, Yale University Press, p. 32. The death of Professor Cassirer in April, 1945, ended a distinguished career in which he rose to a high position of eminence among the philosophers of the twentieth century. The criticisms which follow show that the author cannot accept certain of his basic assumptions and logical tenets which are widely accepted among logical positivists, but the originality and high standing of this truly great philosopher are acknowledged. He has made permanently valuable contributions to the literature of philosophy. (See the bibliography below, p. 400, for his works available in English.)

And he rightly insists that "all the phenomena which are commonly described as conditioned reflexes are not merely very far from, but even opposed to the essential character of human symbolic thought." Doubtless the words *sign* and *signal* are correctly used in this context of conditioned responses. Nevertheless, men do use signaling and signs a great deal, and transmitting and receiving radar wave pulses appear to be a form of human signaling.

There is some ambiguity in Professor Cassirer's conception of nature and the physical world, both of which ideas are comprised under the term *existence*. It is clear that mathematical symbols belong to the realm of meaning which is denied the status of existence in either a metaphysical or a physical sense. He refers to a process of objectification which "begins in language, but in science it assumes an entirely new shape. For the symbolism of numbers is of quite a different logical type from the symbolism of speech" (p. 211). But further on, he writes, "If we speak of the objectivity of number, we do not think of it as a separate metaphysical or physical entity. What we wish to express is that number is an instrument for the discovery of nature and reality" (p. 218). And what are nature and reality in this context? Evidently they include what every scientist knows to exist, namely, "very large fields of phenomena which it has not yet been found possible to reduce to strict laws and to exact numerical rules."

Now is this realm of unreduced phenomena all that is meant by nature? Consider this passage: "Nature is inexhaustible—it will always pose for us new and unexpected problems. We cannot anticipate the facts, but we can make provisions for the intellectual interpretation of the facts through the power of symbolic thought" (p. 218). This means that an inexhaustible realm of particular facts constitutes objective nature, beyond the objectivity possessed by the world man has reduced to law and order. No doubt

the reduced portion of nature is what Professor Cassirer calls our physical nature when he writes, "The material of our physical world is composed of sense data" (p. 214), and "language is the first attempt of man to articulate the world of his sense experience" (p. 209). But objective nature that lies beyond our sense data must be meant in this statement: "Nature as such only contains individual and diversified phenomena" (p. 209). Perhaps this is that realm of inexhaustible nature which will always give man a new supply of sense data.

Thus we reach a stratified view of reality. The inner core is the ordered world of number that has neither metaphysical nor physical existence, but which is pure symbolic meaning. The next layer is the mathematical-physical world of completely ordered and systematized sense data that results from the human categorizing process of objectification. But beyond this is a fully existential realm of particular facts called inexhaustible nature that has not yet been reduced and ordered. The categorizing process of objectification creates our physical world by fusing factual sense data into a mathematically systematized and ordered unity. We never exhaust the inexhaustible nature beyond our objectified physical world, because our realm of symbolic meaning always remains incomplete. But as we improve our basic instrument, mathematical symbolism, we enlarge our physical world by making it more inclusive of those phenomena not hitherto reduced to order by our symbolic forms.

In such a conception of nature and reality the world of existence has a double aspect. First, it is the existential content of the mathematical physical nature which man has ordered and systematized with his symbolic forms. Secondly, it is inexhaustible Nature filled with individual and diversified facts, including those "very large fields of phenomena which it has not yet been found possible to

reduce to strict laws and to exact numerical rules.'' But it seems to be implied here that both of these aspects constitute the Kantian phenomenal world, since a complete phenomenology has to comprise both. This leaves ''nature as such,'' as the shade of Kant's *ding-an-sich,* somewhere behind all phenomena, both ordered and unordered, as the inexhaustible poser of problems, and continuous creator of fresh sense data. But the scientist must assume the possibility of these being ordered by his symbols whenever they occur.

Are Radar Pips Signs or Symbols?

Interesting questions arise when Professor Cassirer's statements are applied to radar wave pulses and pips. Undoubtedly these are physical entities. They are produced and controlled by man through the instrumentality of a highly complicated scientific invention. Hence, they are not to be classified as falling within that part of the world of being or existence that is entirely unreduced through the instrumentality of human symbolic meanings. On the contrary, since radar measures the time of its pulses with an accuracy impossible on any other human measuring device, giving precise measurements down to less than a millionth of a second, it beautifully illustrates Professor Cassirer's contention that number is our best scientific tool for obtaining exact knowledge of nature. Thus radar wave pulses and pips would certainly fall within the physical world that has been reduced to order by human symbols. This much seems ineluctably obvious. And it raises this question: Are radar wave pulses and pips signals or symbols? And if they are signals, rather than symbols, do they convey meaning in detecting a ship and referring to it those who are seeking this information? In answering these questions, a fuller analysis is necessary.

The radar pip is a physical blob of light, but we know

that the radar operator perceives this blob and interprets it to mean, say, a ship eight miles at right angles to the starboard beam. In making this interpretation, he refers the blob to a far distant ship. Or does he? Is it really the radar pip that he refers to the distant ship? Since that is a physical entity in a world of being and the interpretation of the operator falls in the human world of meaning, it would seem to be impossible for him to refer the radar pip to the ship. Does he then use his sensum of the blob of light as the bearer of the meaning? The sensum is the last member of a series of neural and cortical events initiated on the two retinas of the eyes of the operator by the blob of light acting as a stimulus, and that makes it a physiological event in our physical world in the sense of Professor Cassirer's theory.

As a matter of fact, to call the radar pip a blob of light and the sensum a single cortical event is actually an oversimplification. For each radar pip is a continuous succession of blobs of light no two of the members of which are identical, and the operator's visual experience is the result of a continuous series of stimulations each of which has a last member cortical event or sensum. And this makes understandable a logician's hesitancy to treat either a radar pip or an operator's sensum of that pip as a symbol. What, then, does serve the operator as a symbol? Could it be verbal symbols or scientific symbols he learned when he acquired the knowledge and skill to operate a radar? It seems impossible that verbal symbols constituting such general knowledge could mean a particular ship at a particular location on the surface of a well-defined area of the Pacific Ocean. Undoubtedly such verbal symbols are essential in the operator's interpretation, but so are the particular radar pips. Thus we seem to be compelled to accept the view that radar pips, after all, serve better as symbols than anything else in the entire context that could be named. It would

be much simpler to hold that the radar operator is interpreting radar pips than to say that he is interpreting something else. Undoubtedly, the operator's knowledge of the distant ship is mediated to him by the radar pips he perceives on the radarscope.

Two characteristics of symbols are especially emphasized by Professor Cassirer—universality and variability or versatility. By the former, he means form or architectural structure regardless of the sense material in which this is expressed, such sense material always being particular whereas form is general or universal. By the latter, he means that many different symbols can be used to express the same meaning. And he differentiates this aspect of a symbol from a sign or signal in these words: "A sign or a signal is related to the thing to which it refers in a fixed and a unique way. Any one concrete and individual sign refers to a certain individual thing" (p. 36). Now neither of these two characteristics of symbols seems to apply to radar pips, or to the sensa of an operator who interprets those pips. Consequently neither sensa nor blobs of light can be symbols on this theory. Natural scientists call the wave pulses sent out by a radar transmitter wave pulse signals, and it seems necessary to regard as the signal either this wave pulse or the radar pip it creates on the radarscope when it functions in the cathode ray tube. However, this implies that radar pips actually function as symbols, rather than as signals, whereas on Professor Cassirer's view they could only be signals, since they are particular and not universal, and unique and not versatile. When they are interpreted to mean the distant ship radar pips do have the character of universality in the sense that any qualified interpreter would give them the same meaning, and doubtless the particular shape they make on the radarscope is of no consequence, and could be changed without their referential meaning being modified. But if these two char-

acteristics of radar pips give them a symbolic character, why should they not be called symbols even though they are physical entities? And if radar pips are not symbols, what is it that unites these two characteristics and becomes the designator of the distant ship? What is the symbolic form of this important meaning?

If the radar pip is a signal it must be regarded as quite unique. For it is the answer to a signal, namely, the radar pulse signal reflected back from the distant ship. In other words, the signal sent out by the radar transmitter is a self-answering signal, and the radar pip is this answer to an expert operator who knows how to interpret it. How does this differ from other human signals? From the signal bridge of the ship three kinds of signals are sent, each of which requires a code used by both sender and recipient who returns the answer in the same code. The semaphore signals are gestures and fixed positions of the arms of the sailor, who is answered by another sailor using the same semaphore code of arm movements and positions. The pennant signals use a variety of colored pennants, hoisted to a yard arm, and answered by similar pennant signals on the ship for which the communication is intended. Searchlight signals use a code system of light blinkings or flashes. Thus in all three cases a code message is sent and a code answer is returned, and the code meanings are true symbols in Professor Cassirer's sense of the word. But such codes are not used in radar wave pulse signaling, and no answer is returned by any recipient of the signal. Hence radar pips are not signals like those used to communicate symbolic codes. Nevertheless, a radar pip conveys meaning to an expert operator. Must it not, then, be a symbol rather than a physical counterpart of a symbol, as are the physical entities used in communicating coded messages from the signal bridge? What logical character is to be attributed to a self-answering radar wave pulse signal? Whether we

regard it as a symbol, or as a sign, its logical status needs to be determined. For in either case it has the characteristics of a crucial instance for logical theory.

Wise critics have suggested four examples of experiences which they think are cases of the same kind as are radar pips being interpreted by a radar operator. These are: (1) receiving back the echo of one's voice in a mountain chasm; (2) the injury to a man who shoots a pistol into the outer darkness, only to have it ricochet and strike him on the rebound; (3) or a ball that is bounced against a wall; for example, a phosphorescent ball thrown against a gymnasium wall in pitch darkness; and (4) pimples that mean measles to a doctor making a diagnosis of a patient's illness. Are these four cases identical with the reference of radar pips to a distant ship? Certainly not. In case one there is no control whatsoever over the echoing sound waves, whereas the radar gives practically complete control to the operator sending out and receiving the waves that carry the meaning he is interpreting when he comprehends what the radar pips are telling him. In case two there is no control either, and whatever meaning there is seems purely accidental. No predictive knowledge could possibly be based upon such a sporadic occurrence. Hence radar pips can not be classified with accidental events such as this second example represents. In the case of the ball rebounding from a wall where is there a factor comparable to radar pips? There is nothing in it to interpret and to refer to any complicated object which it can be said to mean. In such a case neither signalling nor symbolic reference appears to be in any way involved. But in case four there is an interpretation of pimples as measles. Yet is this comparable to the interpretation of radar pips as meaning what they mean? Measle pimples always mean measles, but radar pips sometimes mean ships, sometimes coast line, buoys, light-houses, whales or what not. Measle pimples

mean measles because they are constitutive elements of the case of measles which they mean. When radar pips reveal the presence of an object, whatever it may be, are they constituent elements of the ship or object they mean, or do they perform a purely logical and referential function?

At this point in the discussion we might seek help from a physicist who is also a trained logician. Professor Victor F. Lenzen writes:

> In building microphysical theories, however, objects are posited that according to physical theory can be known only through their effects. The visual datum of a natural thing is ascribed to radiation that it emits or reflects. Scintillations on an appropriate screen, condensation tracks in a cloud chamber, and indications of Geiger counters are attributed to the ionizing action of electrified particles. The only means of cognizing such hypothetical objects is through perceptible phenomena that they serve to explain. Accordingly, such objects have been viewed as fictions. Now the positing of microphysical elements is founded on the postulate that in an interaction between microphysical and macrophysical realms both factors have the same status in reality. Hence the reality ascribed to the microphysical realm depends on that accorded objects of perception. If the latter are only conceptual instruments, so are the former; if the latter are independently real in the sense of traditional metaphysics, so are the hypothetical objects of physical theory. Regardless of interpretation, the repeated confirmation of predictions made from hypotheses for microphysical objects eventually results in the acceptance of perceptions of specific phenomena as observations of such objects. The positing of hypothetical objects demonstrates the relativity of the concept of reality to theoretical schemes of explanation. The most general definition of reality for science is that it is the universe of discourse of a conceptual system that serves to correlate and predict, deterministically or statistically, the data of experience.[2]

How much does this help us in the search for an answer to the question of whether radar pips and radar wave pulses are signals or symbols? It tells us that the pips are

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[2] *Twentieth Century Philosophy,* pp. 114-115. Edited by Dagobert D. Runes, New York, Philosophical Library, 1943.

"scintillations on an appropriate screen," that they are the effects of unperceivable microphysical elements, namely, the radar wave pulses, and that these are purely hypothetical. Some writers consider them to be "fictions" and they undoubtedly are "posited." Professor Lenzen also tells us that the intellectual act of positing them rests upon the postulate: "In an interaction between microphysical and macrophysical realms both factors have the same status in reality." But he does not tell us what that status is, whether radar pips and microphysical elements are both independently real or are merely conceptual instruments. However if they are conceptual instruments they certainly would have to be symbols. If they are members of a "conceptual system that serves to correlate and predict, deterministically or statistically, the data of experience," then they must be symbols. Even though they are physical entities, why not posit for them a symbolic function?

Thus we reach this conclusion. In radar communication there is a highly complicated and continuous correlation and interaction between two macrophysical realms, (1) the radar instruments and radar pips on the ship where the operator uses them, and (2) the distant ships and other objects which deflect the wave pulses the operator sends against them. And the *via media* of this interaction is the microphysical realm constituted by the imperceptible wave pulses. Thus when a series of radar pips mean to an operator a distant ship the meaning is dependent upon a continuous stream of imperceptible wave pulses, which stream the operator completely controls with the radar. He knows that certain pips mean a particular ship because his machine controls the microphysical wave pulses which produce pips. But according to Professor Lenzen these imperceptible wave pulses may be pure fictions! Or they may be physical existents! Certainly the radar pips, which are perceptible, are not fictions but are definite physical

existents possessing a meaning. Are they, then, not functioning as symbols?

At this point consider Professor Feigl's pronouncement: "The question as to whether any thing not given within the range of immediate experience has that same quality or 'raw feel' of existence as that which is given—a question which pervades the reality problem—must forever remain undecided. And this must be, not because it surpasses human powers to answer the question but because the very way the terms are used logically excludes any decision whatsoever. To seek an answer is to chase a will-o'-the-wisp."[3] Does this settle the matter? Is it a misuse of terms to ask whether the imperceptible microphysical elements or radar wave pulses that produced the radar pips are "given within the range of immediate experience"? And is the unseen ship, which the radar pips mean and which is far beyond the horizon, "given within the range of immediate experience"? Regardless of how Professor Feigl might answer such questions, surely it would be difficult to convince the gunners who sank the Japanese ship that it did not have the same quality or "raw feel" of existence as did the radar pips perceived by the operator who gave them the ship's location and range. But perhaps common sense plus science can tell plain men more than logic or science or both can tell logicians!

Radar Pips as Members of Implicative Systems

Suppose we abandon the unwarranted dogma that a physical entity cannot function referentially as a meaning in mediating knowledge, since here we seem to be confronted with a case where such a dogma impedes understanding. There seems to be no reason other than prior dogma against the view that radar pips are points of reference in an implicative system which includes in the case

[3] *Loco citato,* p. 391. *Twentieth Century Philosophy.*

under consideration the whole context of radar wave pulse signal, reflecting distant ship, cathode ray tube, radarscope, operator, gunnery officer, and whoever else had a part in the discovery and the sinking of the Japanese ship by the use of radar. On such a view knowledge obtained by a radar can be comprehended. But on a view, which sharply separates the physical events and objects from the symbolic form or meaning within a given context, it does not seem possible even to know that there was a ship or that it was sunk. On a theory which divorces meaning from existence, knowledge that an expert operator gets from a radar becomes highly questionable, and possesses only a certain degree of probability at best. A realistic logic would refuse to make this separation, and whatever metaphysical consequences this refusal might entail, it would at least make comprehensible the logical significance of radar pips as mediators of accurate knowledge.

This raises the whole question of the logical status of all technological scientific knowledge. Pure science may be regarded as existentless meaning, providing a logician wishes to carry the abstracting of the form from the content of knowledge that far. Technological knowledge is different, because the artifacts it creates, such as radar wave pulses, and radar pips, are the products of instruments in which human knowledge is so indissolubly interwoven with physical existents that it cannot conceivably be abstracted from them.

Another dogma of scientific empiricism must also be abandoned, if we are to comprehend the kind of knowledge that is obtained from scientific inventions such as the radar. Professor Cassirer states categorically: "The material of our physical world is composed of sense data" (p. 214). If this factual assertion is true then what are the sense data that constitute such a physical object as the Japanese battleship which our gunners sank? Certainly the sense

data of those gunners and of the radar operators are not the constituents of that part of the physical world designated by the Japanese ship. Nevertheless these men knew the ship was there because they sank it with some well directed salvos. Here, then, was a physical object that was not composed of sense data. Or, if it was, then let it be revealed whose sense data or what sense data composed the enemy vessel and all its trained personnel.

Two dogmas of scientific empiricism, neither of which can be anything more than an unwarranted assumption, or an assumption warranted only by the complete system of ideas constituting scientific empiricism, have to be completely rejected. They are: (1) Meaning and existence are two separate universes of discourses, one referring only to human symbolic forms, and the other to a realm of being; and (2) The material of our physical world is composed of sense data. Both of these assumptions lead to complete scepticism. Mr. Bertrand Russell writes: "Sometimes it would seem as if astronomers considered that the only real occurrences with which they are concerned are the observations of astronomers."[4] Well, sometimes it would also seem as if scientific empiricists considered the physical world to be a conceptualistic system of symbols, subsuming the observations of scientists. In the light of recent technological developments such a conception of physical nature is wholly untenable.

Lest these conclusions be thought to be merely the author's opinion they can be supported by high logical authority. Surely if any one knows what Charles S. Peirce meant by a sign, Dr. John Dewey knows. In his recent suggestive criticism of Professor Morris' misinterpretation of Peirce, which misinterpretation is evidently due to the influence of logical empiricism, Dr. Dewey wrote:

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[4] Bertrand Russell, *The Scientific Outlook*, p. 87.

Peirce uniformly holds (1) that there is no such thing as a sign in isolation, every sign being a constituent of a sequential set of signs, so that apart from membership in this set, a thing has no meaning—or is *not* a sign; and (2) that in the sequential movement of signs thus ordered, the meaning of the earlier ones in the series is provided by or constituted by the latter ones as their interpretants, until a conclusion (*logical* as a matter of course) is reached. Indeed, Peirce holds so consistently to this view that he says, more than once, that signs, *as such,* form an infinite series, so that no conclusion of reasoning is forever final, being inherently open to having its meaning modified by further signs.... That to Peirce the movement of signs, while it *has* form, is itself material or factual, *not* formal, appears clearly in the following passage: "To say, therefore, that thought cannot happen in an instant, but requires a time, is but another way of saying every thought must be interpreted in another, or that all thought is in signs." [5]

When this important statement is applied to radar pips we can say that these constitute a series of signs in Peirce's sense, and that this moving series "while it has form, is itself material or factual, *not* formal." Such a conception of signs makes radar pips understandable, but how can a purely formal logic explain their function of mediating accurate factual knowledge to the operator who interprets them? Hence, radar pips refute the basic underlying assumption of logical positivism, and support those types of logic which deny the possibility of completely separating the form from the content of factual knowledge, or in other words, which affirm the inseparability of meaning and existence in human experience and in scientific knowledge of the natural world. It would be much nearer the truth to say that the physical world is composed of signs in Peirce's sense of the word, than to say that it is made up of sense

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[5] John Dewey, *The Journal of Philosophy,* Vol. XLIII, No. 4, p. 88, Feb. 14, 1946. The reference from Peirce is *Collected Papers,* Vol. V, p. 161. Dr. Dewey's note: "The presence of the word 'time' in this passage while speaking of a sign-sequence, is sufficient evidence of the fact that to Peirce 'the relation of signs to one another' is not just formal."

data. Signs that convey knowledge of nature are a mixture of meaning and existentiality, and that is exactly what radar pips are.

As Professor Cassirer uses the terms radar pips are not signs because they are saturated with meaning, and they are not symbols because they are physical existents. But in Peirce's sense of the word *they are signs because they have meaning.* Radar pips are constituent members of objective order systems. They are so supercharged with the meaning of the unique and concrete order system to which they belong that they function almost instantaneously to convey this meaning to the mind of a qualified interpreter. Any competent radar operator can immediately interpret a given set of radar pips as meaning a ship moving at a precise speed, in a specific direction, and at a definite distance from the operator just because the pips which he thus interprets are constitutent members of such an objective order system. Knowledge mediated by radar pips obviously combines arbitrary with objective factual elements of meaning. The arbitrary elements make false the assertion that the operator sees part of the distant ship when he perceives a pip, but the statement that the pip is a constituent member of its objective order system is true. The fact that blobs of light were arbitrarily selected to serve as pips, when various other physical existents might have been used, in no way affects the fixed referential meaning which the pip has as a constituent of its objective order system. This referential meaning is the logical significance of radar pips, and it is an integral part of the marvel of the invention called radar.

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No attempt is here made to give an exhaustive list of books and articles on logic. For the best extant bibliography, see Benjamin Rand's in Baldwin's *Dictionary of Philosophy and Psychology,* Vol. III, Part II, pp. 647-703. My aim is to indicate some books and encyclopedia articles of more recent date, which should be helpful to students and teachers using this textbook, who may wish either to supplement my discussions, or to go more deeply into some of the problems which I have barely been able to suggest. Beginning students will find it to their great advantage to read regularly some other account parallel to what I have given, and teachers will do well to encourage this by keeping on an open reserve shelf in the reading room of the library, copies of the elementary textbooks listed below, and by making from time to time definite reference in the classroom to various subjects especially well treated in one or another of these books. For it goes without saying, that every textbook is clearer on some points than on others.

In order to orient the student in the general field of logic, and to get him into the habit of using other books, it is suggested that he be required to read the opening chapter in two or three elementary texts, and to write a discussion about one thousand words in length on some such topic as one of the following; this discussion should be prepared during the first week of the course:

1. The Definition of Logic.
2. The Relation of Logic to Psychology.
3. The Relation of Logic to Language.
4. The Relation of Logic to other Sciences.
5. Is Logic a Science or an Art?
6. The Unit of Logical Science.
7. The Sense in which Logic is Formal.
8. The Practical Value of Logic.

It is also recommended that students be asked to find illustrations of the various inductive methods, as well as of the logical processes dealt with in Part I, from their own reading. It is a good plan to require a brief written discussion, containing illustrations of the chief methods, worked out in steps, and based upon a reading of one or more well-written scientific essays in which the methods stand out fairly well. For this purpose the following will be found especially valuable: Huxley, *A Piece of Chalk;* Tyndall, *The Scientific Use of the Imagination;* F. W. Westaway, *Scientific Method,* Book III, *Famous Men of Science and Their Methods;* R. A. Gregory, *Discovery;* T. B. Strong (editor), *Lectures on the Methods of Science;* and various parts of the four volumes of J. A. Thomson (editor), *The Outline of Science.* Another valuable book, containing many fresh and interesting examples of the methods of science and written especially for beginners, is the *Introduction to Reflective Thinking,* so frequently referred to in the text, written by nine associates in philosophy in Columbia University. It is of the utmost importance in teaching logic, and particularly in teaching methodology, to get the student to see for himself the connection between the actual thinking done in scientific research and the statement of the methods given in this book. The more he is able to correlate logical processes with the methods actually employed by scientists, the better he will grasp the logical significance of methodology.

Elementary Textbooks

Bode, B. H., *An Outline of Logic.*
Columbia Associates, *Introduction to Reflective Thinking.*
Creighton, J. E., *An Introductory Logic* (new ed.).
Dotterer, R. H., *Beginners' Logic.*
Hibben, J. G., *Logic: Deductive and Inductive.*
Jevons, W. S., *Elementary Lessons in Logic.*
Jones, A. L., *Logic: Inductive and Deductive.*
Jones, E. E. Constance, *A Primer of Logic.*
Joseph, H. W. B., *An Introduction to Logic* (2d ed.).
Keynes, J. N., *Studies and Exercises in Formal Logic.*

Mellone, S. K., *An Introductory Textbook in Logic.*
Minto, Wm., *Logic: Inductive and Deductive.*
Sellars, R. W., *The Essentials of Logic.*
Welton and Monahan, *An Intermediate Logic.*
Welton, J., *Manual of Logic,* 2 vols.
Westaway, F. W., *Scientific Method* (2d ed.).

Other Elementary Books

The books in this list contain a great deal of valuable logical material, and they are all within the reach of the beginner in logic:

Dewey, John, *How We Think.*
Gibson, Boyce, *The Problem of Logic* (2d ed.).
James, William, *Pragmatism.*
——*The Meaning of Truth.*
——*Principles of Psychology,* 2 vols.
Jevons, W. S., *The Principles of Science.*
Pearson, Karl, *The Grammar of Science.*
Royce, Josiah, *The Problem of Christianity,* 2 vols. (The first volume contains the best extant account of the epistemology of Charles S. Peirce.)
Russell, Bertrand, *The Problems of Philosophy.*
Sidgwick, A., *The Use of Words in Reasoning.*
——*Fallacies.*
——*The Application of Logic.*
Thomson, J. Arthur, *An Introduction to Science.*
Whetham, W. C., *The Foundations of Science.*

Advanced Textbooks and General Treatises

The books in this list are more or less technical and abstract. They are beyond the reach of the average beginning student, but for advanced students and teachers they contain invaluable logical material:

Baldwin, Mark, *Genetic Logic,* 3 vols.
Bosanquet, Bernard, *Logic,* 2 vols. (2d ed.).
——*Essentials of Logic.*
——*Implication and Linear Inference.*
Bradley, F. H., *Principles of Logic* (2d ed.).
Dewey, John, *Essays in Experimental Logic.*

ENRIQUES, F. (K. Royce, trans.), *The Problems of Science.*
HOBHOUSE, L. T., *The Theory of Knowledge.*
HUSSERL, EDMUND, *Logische Untersuchungen,* 2 vols.
JOACHIM, H. S., *The Nature of Truth.*
JOHNSON, W. E., *Logic,* Part I and Part II.
KEYNES, J. M., *A Treatise on Probability.*
LAIRD, JOHN, *A Study of Realism.*
LOSSKY, N., *The Intuitive Basis of Knowledge.*
MACINTOSH, D. C., *The Problem of Knowledge.*
MACKENZIE, J. S., *Elements of Constructive Philosophy.*
MEINONG, ALEXIUS, *Gegenstandstheorie.*
——*Uber Annahme.*
MILL, JOHN STUART, *System of Logic.*
MOORE, G. E., *Philosophical Essays.*
PEIRCE, C. S. (M. R. Cohen, editor), *Chance, Love and Logic.*
PEIRCE, C. S., and others, *Studies in Logic.*
POINCARE, H., *Science and Method.*
——*Science and Hypothesis.*
RIEBER, C. S., *Footnotes to Formal Logic.*
ROGERS, A. K., *What Is Truth?*
RUSSELL, BERTRAND, *Scientific Method in Philosophy.*
——*Philosophical Essays.*
——*Mysticism and Logic.*
WHITEHEAD, A. N., *The Organization of Thought.*
——*Principles of Natural Knowledge.*
——*The Concept of Nature.*

The New Realism, Creative Intelligence, and *Essays in Critical Realism* are collective volumes containing valuable articles on logic by various authors.

ENCYCLOPEDIA ARTICLES

The following encyclopedia articles are of the highest value, and although they are difficult, the beginner should be able to get something from them. They are really indispensable to the teacher who wants a digest of the best thought in this field:

Encyclopedia Britannica (14th ed.). The article entitled "Logic," contains the best available survey of the history of logic.

Century Dictionary and Cyclopedia. The definitions of logical terms in this dictionary were written, for the most part, by C. S. Peirce, and they are very complete and sound.

Baldwin's *Dictionary of Philosophy and Psychology:* "The Laws of Thought," "Truth and Falsity," by C. S. Peirce.

Hastings' *Encyclopedia of Religion and Ethics:* "Logic," "Belief," "Concept," "Judgment," "Inference," "Method," by J. Brough; "Axioms," "Error and Truth," "Mind," "Negation," "Order," by Josiah Royce; "Epistemology," by James Iverach; "Probability," by J. G. Hibben; "Analogy," by G. C. Joyce; "Causality," by F. R. Tennant; "Science," by J. Arthur Thomson.

Ruge's *Encyclopedia of the Philosophical Sciences,* vol. I, "Logic." All of the articles in this volume are valuable, but those by W. Windelband and Josiah Royce on the "Principles of Logic" are unusually significant.

Consult also the *Proceedings of the Aristotelian Society* (published annually).

Symbolic Logic

The following are among the most important works on symbolic logic:

Boole, George, *The Laws of Thought.*

Couturat, L. J., *The Algebra of Logic.*

De Morgan, A., *Formal Logic.*

Dotterer, R. H., *Beginners' Logic* (contains a good chapter on Symbolic Logic).

Lewis, C. I., *Survey of Symbolic Logic* (the best historical survey).

Russell, Bertrand, *Principles of Mathematics.*

Russell, B., and Whitehead, A. N., *Principia Mathematica.*

Schroeder, E., *Vorlesungen über der Algebra der Logik,* 3 vols.

Shearman, A. T., *The Development of Symbolic Logic.*

Smith, Bradford, *First Book in Logic.* (Aristotelian Logic in symbolic form.)

Venn, John, *Symbolic Logic.*

Wittgenstein, L., *Tractatus Logico-philosophicus.*

Consult also F. P. Ramsey, *Proceedings* of the London Mathematical Society, Vol. XXV; H. M. Sheffer, *Transactions* of the American Mathematical Society, Vol. XIV; and Miss L. S. Stebbing on "Logistic" in the 14th Edition of the *Encyclopaedia Britannica,* Vol. XIV.

Supplementary Bibliography for the Third Edition

Since the first edition of this book was published, I have prepared a sourcebook in logic and scientific method, entitled *Illustrations of the Methods of Reasoning,* which contains numerous examples of each of the methods of reasoning expounded in Part II, as well as many examples of the various parts of traditional logic. In an Appendix will be found John Stuart Mill's own exposition of his Experimental Methods and John Tyndall's now classic essay entitled "The Scientific Use of the Imagination."

Since the publication of the first edition of this book, Logical Positivism, which is anti-metaphysical, and is based upon the radical empiricism of David Hume, *The Critique of Pure Reason* of Immanuel Kant, and the Positivism of Auguste Comte, has come rapidly to the front. Consult the following books for an understanding of this important new development in logic:

AYER, ALFRED, J., *Language, Truth, and Logic.*

CARNAP, RUDOLPH, *The Logical Syntax of Language.*

——— *Philosophy and Logical Syntax.*

——— *Introduction to Semantics.*

——— *Formalization of Logic.*

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See also the valuable articles in *The International Encyclopedia of Unified Science,* edited by Otto Neurath.

Some searching criticisms of Logical Positivism to supplement the discussion in Chapter XXIX are:

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KELLEY, T. N., *Statistical Method and Scientific Method.*

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SMITH, N. K., *Prolegommena to an Idealist Theory of Knowledge.*

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